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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

VOLUME IX.

Cambridge:
PRINTED BY J. AND C. F. CLAY,
AT THE UNIVERSITY PRESS.

PROCEEDINGS

OF THE

CAMBRIDGE PHILOSOPHICAL SOCIETY.

VOLUME IX.

October 28, 1895—May 16, 1898.

Cambridge:
PRINTED AT THE UNIVERSITY PRESS,
AND SOLD BY
GHTON, BELL AND CO. AND MACMILLAN AND CO. CAMBRIDGE.
BELL AND SONS, LONDON.

1898

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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

ANNUAL GENERAL MEETING.

Monday, 28 *October* 1895.

PROFESSOR J. J. THOMSON, PRESIDENT, IN THE CHAIR.

The following officers and new members of Council were elected:—

President:
Prof. J. J. Thomson.

Vice-Presidents:
Prof. T. McKenny Hughes, Mr F. Darwin, Prof. G. D. Liveing.

Treasurer:
Mr Glazebrook.

Secretaries:
Mr Larmor, Mr Newall, Mr Bateson.

New Members of Council:
Prof. Sir G. G. Stokes, Mr A. Sedgwick, Mr A. Scott.

The names of the Benefactors were read.

The President referred to the death of Prof. Babington and Prof. Huxley.

The following resolutions were proposed by Prof. Liveing seconded by Prof. Hughes, and carried unanimously.

"That this meeting desires to place on record its sense of the loss the Society has sustained by the death of Prof. Babington and to convey to Mrs Babington the expression of their sympathy and condolence in her bereavement."

"That the President be requested to convey to Mrs Babington the foregoing resolution."

The following resolutions were proposed by Prof. Newton seconded by Prof. G. H. Darwin, and carried unanimously.

"That the Fellows of the Cambridge Philosophical Society at this their first meeting since the death of the Rt. Hon. Thomas Henry Huxley, LL.D, desire to express their sympathy with Mrs Huxley and to record their sense of the depth of his influence on modern thought and on the progress of Biological Science."

"That the President be requested to convey to Mrs Huxley the foregoing resolution."

Monday, 11 *November* 1895.

PROFESSOR J. J. THOMSON, PRESIDENT, IN THE CHAIR.

The following Communications were made to the Society:

(1) *A method of measuring the loss of energy in Hysteresis*. By G. F. C. SEARLE, M.A., Peterhouse.

The extensive employment of transformers in the distribution of energy by alternating electric currents has caused the measurement of hysteresis in iron to become a matter of such commercial importance as to stimulate efforts to find a means for readily testing iron as to its quality with respect to hysteresis. The subject is important not only commercially but also from the purely scientific point of view, for it can hardly be doubted that our knowledge of the molecular structure of iron would receive some extension if a means were provided by which the effects of various physical conditions—stress, temperature, the passage of electric currents &c.—upon the loss of energy in hysteresis could be investigated with a comparatively small expenditure of time.

As is well known, the loss of energy occasioned by any cyclic change in the magnetization of iron is equal to $\int H dI$ in ergs per cubic cm. of iron per complete cycle, H denoting the magnetic force and I the intensity of magnetization, both in C.G.S. units. The waste of energy, $\int H dI$, I shall denote by W. The value of $\int H dI$ may be obtained by plotting in a curve the simultaneous values of H and I from a numerous series of observations, and

en measuring the area of the curve. Or again, since, if B is the agnetic induction, we have $B = H + 4\pi I$ so that

$$W = \int HdI = \frac{1}{4\pi}\int HdB \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

or $\int HdH$ vanishes when applied to a cyclic change), it follows at the hysteresis loss may be determined by plotting and easuring the curve given by simultaneous values of B and H. o obtain any satisfactory curve at least a dozen simultaneous servations of H and B are needed.

I have striven to design a method which should enable the steresis loss to be determined by a single observation of the hrow" of a spot of light along a scale. The method was far m being perfected when this communication was made to the ciety on 11 Nov. 1895, but it has been represented to me that ough at present imperfect it is of sufficient interest and promise justify its finding a place in the Society's pages.

A bar of iron of cross section A is placed in a solenoid wound th N turns of wire per cm. of its length. The current C which ws round the solenoid and magnetizes the iron, also passes nd the fixed coils of a sensitive electrodynamometer. A se- ndary coil of n turns is wound over the iron and is connected in ries with the suspended coil of the electrodynamometer, the sistance of the secondary circuit being R. When the primary rrent C changes, the magnetic induction B changes and an M.F. $AndB/dt$ is set up in the secondary circuit. If the time- nstant of the secondary circuit is very small compared with e time of a complete cycle, the effects of self-induction may be glected, and then there is a current γ in the secondary circuit amount $\gamma = \dfrac{An}{R}\dfrac{dB}{dt}$. Now the relation between C and H is pressed by $H = 4\pi NC$, so that $C = H/4\pi N$. Hence, if the uple experienced by the suspended coil when a current i_1 flows it and a current i_2 flows in the fixed coil is qi_1i_2 ergs, the couple perienced by the suspended coil at any instant due to c and γ is γ ergs, supposing that the deflection is not so large that the ue of q is appreciably different to its value in the equilibrium sition.

Now if the time of vibration of the moving coil is so great mpared with the time occupied by the double-reversal of the rrent that this process may be regarded as completed before the l has moved sensibly from its equilibrium position, the angular mentum acquired by the coil for a single double-reversal of the mary current is

$$K\omega = \int qC\gamma dt = \frac{qAn}{4\pi NR}\int H\frac{dB}{dt}\,dt = \frac{qAn}{4\pi NR}\int HdB \quad \ldots(2),$$

where K is the moment of inertia of the coil and ω its initial angular velocity. Thus since $\int HdB = 4\pi W$ we have

$$W = \frac{NR}{qAn} K\omega \quad \ldots\ldots\ldots\ldots\ldots\ldots(3).$$

Let T be the time of a complete vibration of the coil, θ its greatest angular displacement or "throw" and η the restoring couple exerted by the suspension per radian of displacement. We have, by the Conservation of Energy,

Initial kinetic energy = Potential energy at extremity of swing.

so that $$\tfrac{1}{2} K\omega^2 = \tfrac{1}{2}\eta\theta^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Also $$T = 2\pi\sqrt{\frac{K}{\eta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5).$$

From (4) and (5) we find

$$K\omega = \frac{\eta\theta T}{2\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6).$$

Thus from (3) we have

$$W = \frac{NR\eta T}{2\pi qAn}\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots(7).$$

The quantity η/q is easily determined as follows:—

Let currents i_1, i_2 flow in the suspended and the fixed coils of the electrodynamometer respectively and let ϕ be the observed steady deflection, then

$$qi_1i_2 = \text{couple} = \eta\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots(8),$$

so that $$\eta/q = i_1i_2/\phi.$$

If there is any appreciable damping the expression must be multiplied by $1 + \frac{1}{2}\lambda$, where λ is the logarithmic decrement. We have finally

$$W = \frac{NRTi_1i_2}{2\pi An\phi} \cdot \theta(1 + \tfrac{1}{2}\lambda) \quad \ldots\ldots\ldots\ldots\ldots\ldots(9).$$

Thus the "throw" of the coil is proportional to the energy lost in hysteresis.

It is evident that this affords a very ready means of testing the hysteresis of iron under different physical conditions for a given maximum value of H, the magnetic force, for all that has to be done after the constant of the instrument has once been determined is to observe (by means of a mirror and lamp and scale) the throw produced by a single double-reversal of the magnetizing current. For instance, a steady electric current may be maintained through the iron and the value of W for each value of this current may be

determined by simple observation of the "throw" in each case. If it is desired to investigate these effects for a given maximum value of B, the magnetic induction, the process is a little more complicated, because the value of B has to be found by the ballistic galvanometer method, and the strength of the magnetizing current so adjusted on each occasion that the value of B may be kept constant under the varying conditions imposed upon the iron.

In the electrodynamometer which I have constructed for this experiment, the suspended coil is a long narrow coil of 120 turns, about 8 cm. long and 1·5 cm. wide, suspended by a silver wire $\frac{1}{1000}$ inch in diameter. The fixed coils have 500 turns and are designed so as to fit closely round the suspended coil. With 1285 turns in the secondary coil and with the resistance in the secondary circuit about 100 ohms, the instrument is sensitive enough to measure the hysteresis loss in the case of an iron wire ·02 cm.^{2} in cross section. I have made the instrument sufficiently sensitive to measure W for so thin a wire with the view of investigating the effects of tensile and torsional stress upon the hysteresis.

Up to the present (3 Dec. 1895) I have not been able to make any accurate determinations owing to the fact that I have so far used an ordinary rocking mercury key for reversing the primary current. With this key, on breaking contact, the primary current is stopped very quickly, and when contact is re-established the current very quickly rises to its full strength, and each of these changes takes place in a time not very large compared with the time-constant of the secondary circuit. On account of this the instrument has given results about 100 % too large, as judged by the value of W obtained by a B-H cycle. When, however, a coil with a large coefficient of Self-induction was inserted in the primary circuit so that in spite of the key the primary current would not vary rapidly, the discrepancy was reduced to about 5 %. But I have designed a key (somewhat after the fashion of Prof. Ewing's "Sine Function" key) which will allow the current to be quite gradually reversed, and some preliminary observations have led me to expect that with this key the instrument may work quite satisfactorily.

Note added 18 Dec. 1895. The following method of determining the constant of the instrument is much more convenient than that described in the paper. Let a constant current C' flow in the primary circuit, and while this current is flowing suddenly produce a known change P in the number of lines of magnetic force passing through the secondary circuit. This is conveniently done by suddenly inverting an "earth inductor" which has been placed in the secondary circuit. The time-integral of the current

thereby produced in the secondary circuit is P/R, and thus the initial angular momentum of the suspended coil is

$$K\omega' = \frac{qC'P}{R} \quad \text{.....(10),}$$

where ω' is the initial angular velocity.

But, if θ' is the "throw" produced, we have by (6) $\omega/\omega' = \theta/\theta'$, so that by (3) and (10)

$$W = \frac{C'PN\theta}{An\theta'} \quad \text{.....(11).}$$

Two determinations of $C'PN/An\theta'$ on two different days agreed to within ·5 %.

The reversing key mentioned in the paper has been constructed and measurements of the hysteresis for a specimen of steel wire have given results agreeing, within the limits of accuracy of the observations, with the values obtained by B-H cycles. For very soft iron in which the maximum value of dB/dH is over 8000, and six times its average value, there is still a difficulty due to self-induction when a single wire about ·01 cm.² is tested. But I hope to overcome this difficulty shortly.

G. F. C. S.

(2) *On the forms of cubic surfaces containing* 27 *real straight lines.* By W. H. Blythe, M.A., Jesus College, Cambridge.

The following account of the transition between the various forms of these surfaces is a supplement to the paper in *Proceedings*, vol. VIII., p. 241.

A simple case of a cubic surface is that of a plane and a right cone shewn by one figure. Take a solid rectangular block, and let its upper surface represent the plane in question. Cut a right cone out of this block, axis vertical, having its vertex in the plane at a point A. Invert the solid cone, vertex at A. The surfaces of the solid and hollow cone, and that of the plane, represent a cubic surface. A straight line can be drawn to cut the surface in three points. Comparing this diagram (1) with (2) and (3), which are vertical sections of a cubic surface represented by the general equation, we see a similarity in form. In the case of (3) the upper hollow portion is connected with the lower. In passing from the form (2) to (3) and then from (3) to (2) it is evident that a "hole" is made, joining the upper to the lower hollow portion.

In the case of a cubic surface having twenty-seven real straight lines there are three such "holes" or openings.

Three drawings from photographs of a model shew these openings, the fourth being a view of the under part. [Plate I. Figs. I—IV.]

The lines on the solid upper portion which are the same as those on the lower hollow portion form a double six, these lines each pass through one opening.

The lines through each opening form a double six.

The lines between each opening and the next form a double six.

Twelve lines pass through two openings namely 17, 23, 13, 19, 1, 9, 7, 26, 11, 16, 4, 22.

Three lines 8, 18, 27 do not pass through any opening.

One conic node.

Take the lines 1, 13; 14, 6; 4, 11; 22, 16; 23, 17; 10, 2 and let them move up to one another namely 1 to 13, 14 to 6, &c. so as to form six lines only $r_1, r_2, \ldots, r_6$.

These lines will now all pass through a point, and the opening through which they go will close up to this point and become a conic node. We now have 15 mere lines and 6 rays of a conic node.

Two conic nodes.

If the two conic node rays r_3, r_4 now coincide, another opening closes up, namely that through which the double six 4, 22, 11, 16, 19, 9, 25, 5, 7, 26, 21, 12 passes. r_3, r_4 now coincide to form an axis A_1 joining the two conic nodes. The line 27 becomes a transversal meeting the axis. The lines 19, 9; 25, 5; 7, 26; 21, 12 coincide by pairs to form four rays of the second conic node r_7, r_8, r_9, r_{10}. Six more lines remain, each of which meets the transversal, namely 3, 8, 15, 18, 20, 24.

Three conic nodes.

Two rays from each conic node r_1, r_5; r_7, r_9 may be made to coincide. The third opening now becomes a point joined to the other nodes by axes A_2, A_3. 3, 20; 15, 24 unite to form r_{11}, r_{12} rays of the third node. 8, 18, 27 are the transversals.

Four conic nodes.

The six conic node rays in this case may be made to coincide by pairs. We then get a fourth conic node caused by the closing up of the opening at the base of the model. 27, 8, 18 still remain transversals.

Conic nodes may also have been found by considering the solid portions between two openings as diminishing in size, we then get a conical point formed by the meeting of two solid conical portions, instead of hollow ones.

A case of great interest is that in which one of the tangent planes to the surface is at an infinite distance.

The general equation in Cartesians is

$$xyz = Ax^2 + By^2 + Cz^2 + 2Fx + 2Gy + 2Hz + K.$$

The sections parallel to the coordinate planes are conics.

Four of these sections in each case become pairs of straight lines and two are parabolas. It is clear that we get twenty-seven straight lines, namely three sets of four pairs together with three at infinity.

Examining the meaning of this as regards the model we find that the three openings become infinitely elongated, we therefore get a central solid joined to four infinite solid conical shaped branches. These are so placed that a plane cannot cut both the central solid and the four branches at the same time.

A correct idea of the shape of this model may be obtained by placing four cones with their vertices at the angular points of a tetrahedron, their axes being in the line joining these points to the centre of the tetrahedron. If this construction be made in wood or wire it can be covered over with wax or plaster of Paris to shew the surface exactly.

If the equation is

$$xyz = Ax^2 + By^2 + Cz^2 + K,$$

we still get four sections parallel to the coordinate planes containing straight lines, but in each case two pairs consist of parallel straight lines and the parabolic sections do not appear, for they have coincided with them.

The symmetrical surface mentioned by Mr H. M. Taylor having four conical points, the equation to which is

$$x^2 + y^2 + z^2 + 2xyz - 1 = 0,$$

is here given. [Plate II.]

A, B, C, D are the conical points and are angular points of a regular tetrahedron. a, b, c are the middle points of BC, CA, AB. If we put x, y, or $z = 0$ we get circular sections, each bisecting four edges of $ABCD$.

Portions of these are shewn passing through a, b; b, c; c, a.

Next the curve passing through A and a is a parabola which also goes through D, having its vertex at a.

All sections at right angles to this parabola (that are also parallel to AD) are conics, and the extremities of their minor or major axes are upon it.

There is another parabola passing through B, C and the middle point of AD at right angles to this one, and is such that if the major axis of a section has its extremities on the first parabola, the extremities of its minor axis are on the second.

When looking at the diagram we see that a parabola (*e.g.* that through D, a, A) being visible on the face ABC passes through the point A, and being now on the further side of the solid branch at A passes out of sight. On each solid branch there are evidently three edges of the tetrahedron and three parabolas symmetrically placed, and if we cut any solid branch transversally by a plane at right angles to the line passing through its vertex and the centre of the tetrahedron we obtain a symmetrical cubic oval, a diagram of which is given, having its points of maximum and minimum curvature at M and m, where the edges and parabolas respectively meet it. There are also three infinite branches caused by the section of the plane and the three other solid portions.

Each edge of the tetrahedron coinciding with four straight lines on the surface and three being at infinity there are still of course twenty-seven straight lines on the surface.

Some of the simpler cases of binodes yet remain to be considered that have equations of the form

$$\alpha\beta\gamma = K\delta\mu\nu.$$

Construction of the surface.

Take $\alpha\beta\gamma = K\delta\mu\nu$ as the equation of a cubic surface where $\alpha, \beta, \gamma, \delta, \mu, \nu$ are of the first degree and K constant. Nine of its twenty-seven straight lines may be at once obtained as the intersections of the planes α, β, γ with δ, μ, ν respectively, and if we take them to be represented by the numbers 4, 2, 12, 13, 14, 15, 9, 8, 7 the planes and their intersections are shewn by the scheme

	α	β	γ
δ	4,	13,	9,
μ	2,	14,	8,
ν	12,	15,	7.

If we take the lines forming a diagonal of this table, *e.g.* 4, 14, 7, we can find three straight lines on the surface that meet all of these, and no more.

By interchanging rows and columns we get in all six such diagonals. Therefore there are 18 beside the 9 at first found, making a total of 27.

We can take it that

5,	16,	17	meet	4,	14,	7.	6, 26, 27 meet	4, 8, 15.
10,	18,	19	„	13,	8,	12.	3, 22, 23 „	13, 7, 2.
1,	20,	21	„	9,	2,	15.	11, 24, 25 „	9, 12, 14.

Take areal coordinates, the tetrahedron of reference being that formed by the planes α, β, γ, δ; and let

$$\mu \equiv (l_1\alpha + m_1\beta - \gamma)\, n_1 + \delta,$$
$$\nu \equiv (l_2\alpha + m_2\beta - \gamma)\, n_2 + \delta.$$

Let the lines 4, 9, 13 form an isosceles right-angled triangle, hypotenuse in 9. The planes 4, 12, 2 and 13, 14, 15 are perpendicular to 4, 9, 13.

15	cuts the edge	13	in the ratio	$l_2 : 1$,
14	„	13	„	$l_1 : 1$,
2	„	4	„	$m_1 : 1$,
12	„	4	„	$m_2 : 1$.

If $\gamma = 0$ cut $\alpha = 0$, $\beta = 0$ at a distance d_1 above $\delta = 0$, and 14, 2 and 12, 15 meet $\alpha = 0$, $\beta = 0$ at distances d_2 and d_3 above $\delta = 0$, then

$$n_1(d_2 - d_1) = d_2,$$
$$n_2(d_3 - d_1) = d_3.$$

We see therefore that by taking arbitrary positions for nine of the straight lines we determine all the constants in the equation $\alpha\beta\gamma = K\delta\mu\nu$, except K.

Take a point 0, β_1, γ_1, 0 on the straight line 4. Denote the ratio $\gamma_1 : \beta_1$ by ρ, we shall find that

$$\lambda_1\alpha = \delta,\ \lambda_2\beta = \mu,\ \lambda_3\gamma = \nu$$

will reduce to two independent equations both satisfied by 0, β_1, γ_1, 0 if

$$\left.\begin{aligned} \lambda_2 &= n_1(m_1 - \rho), \\ \lambda_3\rho &= n_2(m_2 - \rho), \\ \text{and} \quad \lambda_1(m_2n_2 - n_1\rho) &= -n_1n_2(l_1m_2 - l_2\rho) \end{aligned}\right\} \ldots\ldots\ldots\ldots\text{(A)}.$$

These two equations will represent a straight line on the surface if $\lambda_1\lambda_2\lambda_3\alpha\beta\gamma = \delta\mu\nu$ coincide with $\alpha\beta\gamma = K\delta\mu\nu$, that is if $\lambda_1\lambda_2\lambda_3K = 1$.

If K be known it is evident that using this equation with (A) we get a cubic equation for ρ.

When ρ is determined we get but one value of λ_1, λ_2, λ_3 for each value of ρ, and thus obtain three definite straight lines on the surface.

If, however, we consider K as unknown, and take an arbitrary value for ρ, say ρ_1, we find K from the equation

$$\rho_1(m_2n_2 - n_1\rho_1) = -Kn_1^2n_2^2(m_1 - \rho_1)(m_2 - \rho_1)(l_1m_1 - l_2\rho_1),$$

and the other values of ρ from the quadratic

$$\frac{(l_1m_1 - l_2\rho)(m_2 - \rho)(m_1 - \rho)}{(l_1m_2 - l_2\rho_1)(m_2 - \rho_1)(m_1 - \rho_1)} = \frac{(m_2n_2 - n_1\rho)\,\rho}{(m_2n_2 - n_1\rho_1)\,\rho_1},$$

obtained after simplification and division by $\rho - \rho_1$.

In this case take as an arbitrary point that at which 16 meets 4, we get a quadratic which determines the points at which 5 and 17 meet 4.

Similarly all the other lines may be found.

When the position of all the lines is fixed, the form of the surface can be found by taking sections through any straight line on the surface.

These sections are conics, the straight line forming part of the complete cubic section. Many points on these conics are known as being the intersection of the planes with lines already found.

The rules for the geometrical construction given in Vol. VIII., Pt. iv., page 241 may be deduced from the analytical method given above. For it has been shewn that the position of the lines 4, 12, 2, 13, 14, 15, 9, 7, 8, and the point at which 16 meets 4 may be placed in arbitrary positions, and that this construction determines all necessary constants.

When this has been done the facts used to determine the remaining lines are

(1) If three straight lines form a triangle this triangle is a section of the surface.

(2) Any two triangular sections meet each other in three points in one straight line, *e.g.* 3, 5, 7 and 13, 14, 15 are sections, 3 meets 13, 5 meets 14, 7 meets 15, and the three points of intersection are in one straight line.

(3) If we find four points on the surface in one straight line, this line lies wholly on the surface.

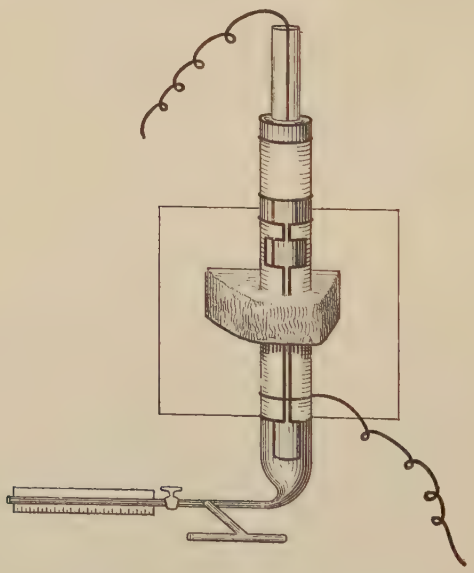
(3). *Expansion produced by Electric Discharge.* By MISS MARTIN.

Acting under Professor J. J. Thomson's directions, I have lately endeavoured to find some explanation of certain phenomena first mentioned, I believe, by Herr G. Meissner (*Abh. d. Gesell. der Wissensch. zu Gött.* 1871, Bd. xvi., p. 98). In the course of some experiments "über die elektrische Ozonerzeugung und über die Influenz-Elektricität auf Nicht-Leitern," Meissner observed that if a gas contained in a space between the plates of a condenser were subjected to a sudden rise or fall of potential the volume of the gas underwent a momentary increase. His apparatus consisted of two concentric glass tubes about 50 cm. long, both rounded off at the lower end and having no connection with each other in any way except that the inner tube was held in place at the upper end by a ring of wax. Between the two tubes was a space of 3—5 mm. The outer surface of the outer tube, and the inner surface of the inner were coated with tinfoil

having an area of 264 sq. cm. and 136 sq. cm. respectively. Between the two coatings there was—making allowance for the thickness of the tube walls—a space of 6—7 mm. Attached to the outer tube was a gauge to mark the variations of pressure in the gas under observation.

Having tried various modifications of Meissner's apparatus with no very satisfactory results, it seemed advisable to have his apparatus exactly copied—which was easily done as he gives very clear and careful measurements and descriptions. This was accordingly done, the only difference being that instead of the gauge used by Meissner a horizontal tube provided with a small quantity of sulphuric acid was used.

Observations with this led to the belief that, very frequently at all events, the movement of the acid in the gauge was accompanied by a luminous discharge through the tube due to induction. Before it could be ascertained if this was invariably the case the outer tube was accidentally broken and it seemed advisable to have the new one made of rather different dimensions. Accordingly a piece of tubing about 40 cm. long and having an outside



diameter of 3·3 cm. was used for the outer tube whilst the same tube as before (57 cm. long, outside diameter 1·2 cm.) was again used for the inside. Instead of sealing up the lower end of the outer tube as before, this was now bent and drawn out to serve as a gauge and provided with a tap to cut off the acid from the

tube when the latter was being exhausted and refilled with whatever gas might be under observation; for this purpose a short tube projected between the tap and the bend which communicated with a pump and with the source of gas employed. In this case the inner and outer tube, instead of being cemented together with wax, were provided with a rather loose ring of india-rubber between which and the glass melted paraffin was poured and allowed to harden so that the joint was air-tight. For about 1·5 cm. below the india-rubber the space between the two tubes was completely filled with paraffin. Instead of lining the inside tube with tinfoil it was filled with mercury; the outer tube as before being coated with tinfoil in which spaces were cut to enable the observer to note what went on inside the tube. This outer tinfoil which extended from just below the lower end of the paraffin stopper to within 4 cm. of the bend was 28 cm. long and had an area of 290·3 sq. cm.—not allowing for the small spaces cut out—the effective mercury surface of the inner tube was 87·96 sq. cm. The two surfaces were distant from one another 1·15 cm. and the air space between the two tubes had a width of ·9 cm. The two latter measurements may not be perfectly correct as of course it is the smallest distance between the two tubes that is required and, though every care was taken, the two tubes may not be perfectly concentric; there may be an error of 1 or even of 1·5 mm. The condenser thus formed was mounted on a square of glass by means of a thick band of paraffin. The whole was then enclosed in a light-tight case so arranged that the lower part of the apparatus forming the gauge projected from an opening beneath, and provided with an opening in front fitted with a small blackened tube through which the interior could be observed. A mm. scale was affixed to the gauge tube. The mercury in the inner tube was connected to the inner coating of one Leyden jar, the tinfoil on the outer tube to another jar, the outsides of the two jars being connected to earth. The maximum charge upon the two tubes could be regulated by means of the distance between the terminals of the Wimshurst by which the apparatus was charged.

With these arrangements it was easily seen that any movement of the acid in the gauge of not less than ·25 mm. was accompanied by a luminous discharge through the space between the two tubes, but it appeared at first as though smaller movements than this were unaccompanied by any luminosity. Greater care however proved that this was not the case, but that every movement of the acid is accompanied by a corresponding luminosity between the tubes. It requires great care and some practice however to see this when at its faintest; it is much easier to perceive the very slightest shake of the acid than the almost imperceptible luminous discharge which however un-

doubtedly accompanies it. On the other hand no luminous discharge ever occurs without a corresponding movement of the gauge, and it is only when the sudden change of potential of the two tubes is sufficient to cause a visible discharge in the space between them that any alteration in volume of the gas therein contained is to be observed.

Of course it might be urged that though a luminous discharge is always accompanied by an increase of volume in the gas the reverse may not be the case, but that sudden changes of potential too weak to give any luminous effect may produce a change in volume which the form of gauge used is not sufficiently sensitive to note.

I do not think this is the case. In the first place the gauge is so sensitive that if two fingers be laid against the top of the outer tube the acid in the gauge below begins to move in from 1″ to 1″·5 and in 17″ moves through a distance of 20 mm. There is however a much more conclusive proof that the gauge takes account of every change of volume and that every such change is accompanied by a luminous discharge through the gas. If the distance between the terminals of the Wimshurst be so arranged that the sudden fall of potential caused by the passing of the spark is sufficiently great to cause luminosity in the space between the two tubes upon every discharge there is always a movement of the acid in the gauge; but when the fall of potential does not give rise to a luminous discharge there is never the slightest movement of the acid.

When however the distance between the Wimshurst terminals is such that the sudden discharge causes sometimes a faint luminosity through the gas and sometimes none, the gauge sometimes moves and sometimes does not. It would perhaps be more satisfactory to have a second observer in order to be perfectly certain that the two effects synchronize, but I do not think there can be any real doubt upon the matter. For it is possible so to arrange things that the majority of discharges of the condenser shall be accompanied by a luminous induction discharge, in which case the acid generally but not always moves; if now the Wimshurst terminals be caused to approach very slightly nearer together so that the maximum charge possible upon the condenser plates is very slightly decreased, a smaller percentage of the discharge is accompanied by the luminous appearance in the tube, and the acid in the gauge moves a *correspondingly* fewer number of times.

This is the result of several hundreds of observations. It is of course only within extremely narrow limits that this uncertainty as to whether the sudden discharge will or will not cause the effects under consideration is observable. It seems

fair to presume that the luminosity and the increase in volume are strictly simultaneous. If the charge upon the condenser plates be so increased that two or three luminous induction-discharges occur before the discharge of the condenser by the passing of the spark at the Wimshurst, two or three movements of the gauge are correspondingly observable.

The minimum strength of charge capable of producing these effects varies slightly on different days according to temperature and pressure, but roughly speaking, at ordinary temperatures and pressures the distance between the Wimshurst terminals must be equal to the distance between the two tubes. That is to say, in my experience it has never required to be greater, nor have I been able to obtain any effect if the space between the terminals were more than 2·3 mm. less than the space between the tubes. Thus with the first tubes used, distant from each other about 4 mm., a spark 3·4 mm. was the smallest which produced any effect, whilst with the larger apparatus subsequently used having an air space = 9 mm. the minimum effective spark varied from 6·5 to 9 mm. but was generally about 8 mm.

These observations were repeated with carbonic acid and coal gas but for a long time with no very satisfactory results, both these gases appearing to give very much the same effects as air—except of course that the colour of the induction-discharge was different.

Meissner, however, is so very positive in his assertion that H gives distinctly smaller, and CO_2 distinctly greater, effects than air that I could not but think some essential point must have been overlooked. The cause for my failure in obtaining these differences at length became apparent. Meissner seems always to have first filled his vessels with the gas to be observed and then to have charged and discharged them. In my arrangements however it was found more convenient to do the charging and discharging by mechanical means, and the Wimshurst which was worked by a small motor was turned on at the beginning of the observations and left to run undisturbed to the end; thus the gas, which was always admitted gradually into the vessel, was subjected to any chemical effects of the discharge before being put into communication with the gauge, the top of which had of course to be kept closed till the vessel was full of gas. Under these circumstances the three gases used, dried air, carbon dioxide and coal gas seem to behave practically identically. But if now the Wimshurst were disconnected whilst the tube was being exhausted and refilled, it was found that, on connecting it up again and discharging frequently as before, air underwent a considerable diminution of volume, and whilst this diminution was in progress sudden changes of potential would produce marked

luminosity and movement of the gauge which after the air had assumed a steady volume were too weak to have any effect. Working with the same strength of charge it was found that coal-gas underwent practically no diminution—in comparison with air and CO_2, that is to say, there was actually a small diminution most likely due to the presence of a small quantity of air in the tube—and the discharging of the condenser thus filled was attended with no greater effects than observed at the close of the observations with air. With CO_2 the first movements of the gauge were enormously greater than those obtained with coal gas and appreciably greater than those noticed with air; and the time taken by this gas to assume a steady volume was at least twice as long as that required by air. This steady volume however having once been reached, the effects of any subsequent discharge were the same as with coal gas or air.

It would therefore seem that any sudden and temporary splitting up of the molecules of the gas which may take place under the influence of the induction discharge is more easily and thoroughly accomplished while any definite chemical change is in progress. When nothing of this nature is taking place, all gases seem to behave more or less alike and to require equal forces to cause any disintegration. It is quite easy to understand how this may have escaped observation; for if a vessel of the dimensions used has been filled with air whilst free from any electric disturbance, it requires about 600 discharges to reduce the air to its permanent volume when it gives the least movement for a given fall of potential; and to similarly reduce an equal volume of carbon dioxide at least 11000 discharges are necessary.

It would seem that the chemical change which produces this diminution of volume takes place more easily and quickly at low pressures; for though, as was before remarked, the gas to be observed was let in slowly, the rate was not nearly so slow as the above figures would require; nevertheless the volume was constant whenever the condenser had been exhausted and refilled whilst still connected to the Wimshurst.

I should remark that the above phenomena were also observed whenever the apparatus was suddenly charged, but the great majority of observations were made on the sudden discharge through the spark at the Wimshurst as this method was found to be rather more convenient.

Monday, 25 November, 1895.

PROFESSOR J. J. THOMSON, PRESIDENT, IN THE CHAIR.

L. A. BORRADAILE, B.A., Selwyn College, was elected a Fellow of the Society.

The following communications were made to the Society:

(1) *Exhibition of a curious specimen of Travertine lining a wooden pipe.* By Professor T. McK. HUGHES, M.A., F.R.S.

The interesting specimen exhibited was presented to the Woodwardian Museum by Mr Benjamin Holgate, of Headingley, near Leeds, who has furnished me with the following information respecting it.

It is a deposit of travertine formed inside a rectangular wooden pipe, used for conveying water from near the surface to the bottom of a mine under Magnesian Limestone. The wood decayed away and was entirely removed, leaving on the surface of the travertine the impression of every detail of the woody structure, so that it looks more like wood replaced than a cast of the interior surface of the pipe. Where a rusty nail projected into the travertine some of the iron scaled off, and the rust stained the deposit for a considerable distance round it so as to increase the deceptive resemblance to sawn timber.

Another curious point in this specimen is the manner in which there is always a tendency to break along what may be called the miter joints. The deposit is built up in layers parallel to the inside surfaces of the pipe, and when crystallization is set up, the crystals are formed at right angles to those surfaces. The faces of the parallel prisms readily coalesce to form one crystalline mass, but the faces of the pyramids do not so easily unite, especially under conditions in which, owing to the continuously reduced extent of surface, the marginal terminations are imperfectly developed.

The same phenomenon occurs in iron casting. If the metal cools against an irregular surface, which would tend to cause crystallization to start from surfaces arranged at various angles to one another, where the iron crystallized from one surface meets the iron whose fibres of crystallization have originated from another differently inclined surface, these two masses will not coalesce, but a plane of separation will extend some distance into the body, and be a source of weakness though the exterior may have been planed down so that the cause is not obvious.

In seeking the explanation two facts have to be borne in mind. First, that where the two masses meet at the miter their direction of increment does not coincide. And secondly, that the growth of the crystals is arrested along the margin by the ever-diminishing surface upon which they are built up.

(2) *On symmetry in the foliage of a branch of Mulberry with asymmetry in the individual leaves.* By Professor T. McK. Hughes, M.A., F.R.S.

The question of the forms of leaves has often been treated of, but I do not know whether the particular point to which I now call attention has been noticed. Looking at the mulberry trees in the Fellows' Garden at Christ's College, I observed that young branches thrown out from near the base of the trunk are apt to bear scalloped leaves something like those of a vine. When in the progress of the growth and development of the branch these scalloped leaves are replaced by the normal forms, the change does not take place by the gradual withdrawal of the lobes equally all along the margin of the leaves, but the inside of the leaf, that is the side next the branch, first returns to the undivided condition, while on the outside of the leaf the lobes are still conspicuous. On one flat twig all the leaves were shaped like gloves, which had a separate place for the thumb only, the other fingers being inclosed together in a sort of bag. In this case the thumbs were all turned outward, and the undivided margin was next the twig.

Some apparent exceptions were found in which a leaf lobed on the right was found projecting from the left side of a twig, or *vice-versâ*, but in all these cases, as far as I have been able to observe, the leaf which appears not to conform to the rule is growing from a small offshoot of the twig, so that a leaf which is left-handed in form with reference to the principal twig to which it at first seems to be attached, is right-handed with reference to the subordinate twig from which it really grows.

It would thus appear that the symmetry which has been lost in the individual leaves still exists in the branches, so that the twig with all its leaves may be regarded from this point of view as one large leaf lobed on its margin, and therefore only the outside margin of each individual leaf partakes of this scalloping, that is, the left side of those on the left of the twig, and the right side of those on the right of the twig.

(3) *The Origin of Vertebrates.* By W. H. Gaskell, M.D., LL.D., F.R.S., Fellow of Trinity Hall.

The problem of the ancestry of the Vertebrates can be attacked from the side of the Invertebrates or from that of the Vertebrates. To my mind the best chance of a successful solution is to be found in the study of the Vertebrates themselves, and especially of the variation which has occurred in each organ or group of organs within the Vertebrate phylum.

The principles upon which I have investigated the problem are based upon the continuity of the evolutionary process and may be stated as follows—

1. Whenever it is possible to state that the variation of an organ or set of organs within the vertebrate phylum has taken place in a well-defined direction, then, by tracing that same curve of variation a step further, we must arrive at the condition of the organ or set of organs in the immediate ancestor of the Vertebrate.

2. The same principle holds good whether the curve of variation is an ascending or descending one, i.e. whether the organ in question is becoming more important and more conspicuous as we ascend the vertebrate phylum, e.g. the cerebellum, or becoming less important and degenerate, e.g. the pineal eye and *ganglion habenulæ*. In this preliminary paper I propose to give a short sketch of the history of all the most important organs and tissues which I have investigated up to the present time and to shew that in the case of all of them the phylogenetic signpost points directly to the same group of animals, viz. the Limulus and its congeners, as the ancestor of the Vertebrates. When once the clue has been found then embryology confirms it and illustrates it; of itself it is a bad guide but affords an excellent illustration of what phylogeny teaches.

To illustrate my meaning I will rapidly sketch the downward path of the central nervous system and point out the conclusions to which such a study directly leads.

The history of the Vertebrate central nervous system.

The highest vertebrate nervous system possesses large cerebral hemispheres, a large cerebellum, a small pineal gland, choroid plexuses which are pushed into the ventricles and a small membranous roof to the IVth ventricle. As we descend the dorsal part of the brain becomes more and more membranous, the cerebral hemispheres are more and more confined to the ventral

side and the pallium becomes membranous; the cerebellar hemispheres are seen in Elasmobranchs to be formed by the invasion of nervous matter over the membranous folds on each side of the median lobe or worm, and in the Amphibians the whole cerebellum is reduced to a small band of nervous matter accompanying the IVth nerve across the dorsal part of the membranous tube; finally in Ammocœtes the whole of the dorsal part of the mid and hind-brain is formed by large membranous folds with the nervous system lying ventral to this membranous tube; at one place only is there any crossing of nervous matter to the dorsal side, viz. at the IVth nerve and commencing cerebellum. We see then that the phylogeny of the vertebrate central nervous system shows a distinct tendency towards the separation of the nervous system into two parts, viz. a non-nervous epithelial tube and a purely nervous part; follow that tendency one step further than the stage reached in Ammocœtes, separate the two entirely and we find immediately a nervous system placed in the same position with respect to this epithelial tube as the nervous system of an Arthropod with respect to the epithelial tube of its alimentary canal; we see moreover that in the lowest Vertebrate, the Ammocœtes, the old mouth and œsophagus, the old cephalic stomach and straight intestine are most easily seen and recognised. Further, in absolute accordance with this gradual and perfect evolution, is the history of the nervous system apart from its tube, for just as parts like cerebellum and cerebral hemispheres diminish in size as we descend, so other parts which are insignificant in the highest Vertebrates come out more and more conspicuously as we descend. This is especially true of the *ganglia habenulæ* and pineal eye, and we see that in Ammocœtes the pineal eye is most conspicuous, is arthropod in type and that the large right *ganglion habenulæ* is its optic ganglion. In fact, as already shown in previous papers, the central nervous system of Ammocœtes fits in both topographically, histologically and physiologically with the central nervous system of an invertebrate such as Limulus or Scorpio.

Embryologically the phylogenetic history of the old alimentary tube is represented by the formation of the medullary canal with its external anal opening by means of the neurenteric canal; while the history of the formation and growth of the nervous system round this canal is illustrated by the formation of the cerebral vesicles, the formation of the posterior fissure, and of the *substantia gelatinosa Rolandi* and of the raphé.

Again, trace downward the formation of the cavity of the cranium and spinal cord. In the highest Vertebrate a cranium so filled with nervous matter as to leave its impress on the confining walls and a spinal cord surrounded by membranes containing the cerebro-spinal fluid. As we descend, more and more room in the

cranium, until at last in Ammocœtes a large cranial and spinal cavity containing a small brain and spinal cord surrounded by a large mass of peculiar glandular tissue. One step further, enlarge the extent of this glandular tissue and make it functional and we find immediately that in position and relative distribution we are face to face with the old cephalic liver and generative organs of the Limulus and Scorpio, and that the same gradual evolution which has explained the nervous system itself gives the same explanation of the peculiar tissue which originally fills up the greater part of the cranium and is gradually squeezed out of existence as the brain grows.

This characteristic glandular tissue, the remains of the old generative glands and liver of the Limulus, is found nowhere else in the Ammocœtes except in the auditory capsule, where its presence will be explained later.

The history of the Vertebrate skeletal system.

Passing from the nervous system let us now consider the skeletal system.

In the highest Vertebrates a bony skeleton enclosing the nervous system to which is attached a facial skeleton, a thoracic skeleton and bones for the extremities. This bony skeleton can be traced step by step into the bony skeleton of the fishes and thence into the cartilaginous skeleton, where it is then found to form a cartilaginous cranium and vertebræ from which spring ventrally a series of cartilaginous bars associated with the branchiæ and viscera. Further downwards we can trace the whole system to its simplest beginning in the Ammocœtes, where we find the cranial cartilaginous skeleton consists of the basal trabeculæ, in front of which are the nasal cartilages, the parachordals with the auditory capsules and the branchial bars with a cartilaginous continuation of the parachordals along each side of the notochord. We may then conclude that the immediate ancestor of the Vertebrates possessed a simple cartilaginous skeleton consisting of a cartilaginous bar on each side of the axis which was in connection with an auditory capsule, extra-branchial bars and a nasal cartilage. We can go further than this, for the cartilages in Ammocœtes are of two kinds, hard and soft, the matrix of the first stains yellow with picro-carmine, and of the second red, and we find that the cartilages of the branchial basket-work and of the two disjointed rods along the notochord are of the soft variety, while the auditory capsules and trabeculæ are of the hard variety. Further, when transformation takes place the new cartilages are all of the hard variety so that the evidence points to the soft cartilage being older phylogenetically, a conclusion which is in accordance with Shipley's

observation that the branchial cartilages are the first to be formed in the Ammocœtes. We may then argue that in the direct ancestor of the Vertebrate two axial lines of cartilage ought to be found and from them a series of branchial bars should issue, which bars should lie external to the branchiæ. We ought in fact to find a cartilaginous skeleton exactly of the kind which exists in Limulus; where the two axial cartilaginous bars are found in the two entapophysial cartilaginous ligaments of Ray Lankester and the cartilaginous branchial bars are found in the cartilaginous rods which spring from each entapophysis into each branchial appendage as described by Gegenbauer. This is the only true cartilaginous skeleton of Limulus, for the ento-chondrite is not cartilaginous but is made up of tendon and muscle like the central tendon of the diaphragm (as shown by Schimkewitsch); it stains also like tendon and not like cartilage.

Again, tracing back the histology of the skeletal tissues we see how bone is formed from cartilage, and that cartilage is by no means always quite the same in structure. Among the different kinds we find that the branchial cartilages of Ammocœtes are quite distinct and different to any found in the higher Vertebrates. Further in Ammocœtes there is in addition to these cartilages a peculiar fibro-massive tissue, from which according to Schneider the cartilages arise, called by him muco-cartilage (*Schleim-knorpel* or *Vor-Knorpel*). This tissue-like cartilage gives a characteristic deep purple stain with thionin. It shows its ancestral importance by disappearing entirely at transformation, being either converted into cartilage or invaded by blood and converted into an areolar fat tissue.

I conclude then that the immediate ancestor of the Vertebrate possessed branchial cartilages of the structure of those of Ammocœtes, which themselves arose from muco-cartilage. In other words we are again led directly to the branchial cartilages and entapophysial ligaments of Limulus, for their structure is word for word the same as that of the branchial cartilages of Ammocœtes, they stain in the same way with thionin, and the cartilage arises from a fibro-massive tissue which resembles absolutely both histologically and in its staining deep purple with thionin the muco-cartilage of Ammocœtes. This muco-cartilage of Limulus is formed from cells of the chitinogenous layer belonging to the entapophyses, so that we see not only how the direct agreement of the cartilaginous skeleton of Limulus with that of Ammocœtes both topographically and histologically points to the identity of these two skeletal systems, but also how we thereby obtain striking evidence of the manner in which the chitinous skeleton of the Invertebrate gave origin to the cartilaginous skeleton of the Vertebrate.

The history of the Vertebrate muscular system.

It would take too long to trace out the changes which have taken place in the arrangement of the voluntary muscles as we ascend from the fish to man; it is sufficient to say that such a comparative myology has been largely worked out, with the result that the voluntary muscular system of Vertebrates falls into two well-marked groups, viz. the body or somatic muscles and the muscles connected with the branchiæ and viscera. This division is so marked and so characteristic in the cranial region that the segmentation of the body is looked upon as double, viz. Branchiomerie and Mesomerie, and the researches of v. Wijhe have shown that in the head region of Elasmobranchs segmental sets of well-defined ventral and dorsal plates of mesoblast give rise to the muscles of the splanchnic or branchial segments and to those of the somatic segments respectively. Passing to the Ammocœtes we see these two sets of muscles still more separated from each other, so that we may conclude that such double segmentation, being more and more conspicuous as we descend, is a marked characteristic of the immediate ancestor of the Vertebrate.

The importance of this division of the voluntary muscles of the body into a splanchnic and somatic group is still further increased by the fact that the nerve supply for these two sets of muscles is also well defined and separate in the cranial region. Thus we see that the splanchnic voluntary muscles are innervated by a segmental set of nerves having well-defined cell origins in the central nervous system, apart from the groups of cells giving origin to the somatic muscles. These nerves are V_m, VII, IX, X.

Thus we come to the conclusion that the double segmentation of the body was brought about because the immediate ancestor of the Vertebrates possessed a double muscular segmentation of which the ventral segments were connected with the movement of a branchial and allied apparatus, and that the cells of origin of this more perfectly segmented part of the body were grouped in well-defined groups, separate from the cells of origin of the less perfectly segmented somatic muscles.

Again, we see that the conditions which exist among the Crustaceans and Arthropods, as described by Hardy, exactly fulfil the requirements just mentioned, for he has shown that the well-marked segmented muscles of the appendages, whether branchiæ bearing or not, are innervated by nerves arising from well-defined cell groups in the ganglia, quite distinct from the more diffused cell groups which give origin to the less completely segmented somatic muscles.

We see, in fact, that just as the consideration of the skeletal tissues led directly to the conclusion that the branchial bars were

derived from the branchial bars of Limulus, so also does the consideration of the muscular system lead to the same conclusion, viz. that the branchial muscles are derived from the muscles of the branchial appendages of Limulus, and therefore to the further conclusion that the Branchiomerie is due to the segmentation of appendages: in other words, that V_m, VII, IX and X were all originally appendage nerves; the proof of this will be given later.

Again, we find that histological considerations point in a most striking way to the same conclusion: for we find in Vertebrates that these splanchnic muscles are not of the same structure as the somatic, and this difference of structure is most striking, as shown by Schneider, when we reach the lowest Vertebrate—the Ammocœtes. Here indeed we see the evidence of its ancestry in that its branchial muscles are to a large extent composed of tubular fibres such as are found in Invertebrates, very similar to many found in scorpions, and not found in Vertebrates. These muscles are as significant a finger-post as the pineal eye, and show their phylogenetic meaning by disappearing entirely at transformation.

The history of the respiratory system and of the IXth and Xth nerves.

Perhaps the best known phylogenetic development of any system in the Vertebrates is that of the respiratory system, for we can trace the lungs back to the swim bladder and pass from the air breathing to the gill breathing Vertebrate, and from thence to the Elasmobranchs with indications of extra-branchial cartilaginous bars in addition to the internal cartilaginous bars, until we reach the oldest known vertebrate respiratory system, that of Ammocœtes with its extra-branchial cartilaginous basket-work, and its segmental branchiæ dependent from the basket-work into the large open respiratory chamber. Throughout the water breathing Vertebrates the innervation is the same, the VIIth nerve supplying the 1st or hyo-branchial segment, the IXth the second or 1st branchial segment, the Xth the remaining segments and throughout, even in the highest lung-breathing animals, the respiratory centre still keeps its primitive position in connection with the origin of the IXth and Xth nerves.

In Ammocœtes we find in strict accordance with the arrangement and nature of the branchial cartilages and muscles that the separate branchiæ may be considered as paired branchial appendages hanging suspended into a chamber so as to leave a narrow slit between opposite pairs for the passage of water and food. Further, we find that the blood circulates in these branchiæ

not by means of capillaries as in the higher Vertebrate, but by narrow channels caused by the subdivision of a large blood space, just as in the branchiæ of Limulus or Scorpio.

The conclusion then is that the ancestor of Vertebrates breathed by means of lamellar branchial appendages which had united together so as to form a scorpion-like body surface, and that the nerves to these appendages were the VIIth, IXth and Xth.

In fact, the method of formation of the scorpion from Limulus as given by Macleod precisely illustrates the manner in which a respiratory chamber like that of Ammocœtes may be compared in the first place with that of Eurypterus, and so with that of Limulus. To this point I will revert later on.

The history of the thyroid and of the VIIth nerve.

If we compare the ventral surface of Eurypterus with that of Ammocœtes, we see that the comparison of the branchial segments with each other does not complete the ventral surface in either animal, for in each case a triangular projection passes down in the middle line and separates the 1st three branchial segments from each other. This projection in the case of Eurypterus is the middle projection of the operculum, and in the case of Ammocœtes is a middle plate of muco-cartilage underneath which lies the thyroid gland.

In all scorpion-like animals the respiratory segments of the body are terminated anteriorly by the operculum, just as the operculum is the first of the lamellar appendages of Limulus, and in all cases the operculum carries the terminations of the generative organs, while in a great number of cases, as in Eurypterus, Thelyphonus, Phrynus, the operculum is double, the posterior part being gill bearing, while the anterior part carries the organs in connection with the external orifice of the generative organs.

In Ammocœtes we find that the first of the branchial segments (the hyo-branchial) is not like the rest of the branchial segments, the gill surface being confined to the posterior part of the segment while the anterior part of the segment carries on its inner surface the ciliated groove called by Dohrn the pseudo-branchial groove; this groove as is well known passes from the dorsal to the ventral side and there joins with its fellow of the other side to form the so-called duct of the thyroid gland; this bilateral gland lying under the ventral median tongue of muco-cartilage which separates in so remarkable a manner the cartilaginous basket-work of either side. In position, then, the segment composed posteriorly of the hyo-branchial, anteriorly of the ciliated groove and the median thyroid corresponds exactly to the operculum of Thelyphonus or of

Phrynus or of Eurypterus with its anterior genital part and its posterior branchial part.

Further, the nerve which innervates the hyo-branchial portion of this segment is the VIIth nerve, and Miss Alcock[1] has shown that a branch of the VIIth passes along near the ciliated groove to extend ventrally tailwards as far as the thyroid itself extends; so that looking upon the whole of this segment as the opercular segment we see that the VIIth nerve was originally the opercular nerve just as the IXth was the nerve of the first branchial appendage and the Xth of the remaining branchial appendages.

The comparison of the thyroid segment of Ammocœtes with the anterior genital half of the operculum in Eurypterus and Thelyphonus means, that the thyroid gland must resemble closely the median part of the genital apparatus lying under the operculum of these latter animals; and indeed the resemblance both in position and in structure is extraordinarily close.

The thyroid of Ammocœtes consists of a long tube curled up at its posterior end, that tube containing along the whole of its length a very peculiar glandular structure which is confined to a small portion of its wall.

This glandular structure has no alveoli, no ducts, but consists simply of a column of elongated cells arranged in a wedge-shaped manner, the apex of the wedge being in the lumen of the tube; each cell contains a spherical nucleus situated at the very extreme end of the cell furthest away from the lumen of the tube. Such a structure is different to any other vertebrate gland, its secretion is not in any way evident, it certainly does not secrete mucus or take part in digestion, and it seemed impossible to compare it with any other known glandular structure.

Looking however at the pictures given by Blanchard of the genital apparatus of Thelyphonus and Phrynus it is striking to see how closely in appearance the median part of the genital mechanism resembles the thyroid gland of Ammocœtes, and upon cutting sections of the Scorpion (the only material available) I found under the operculum in the middle line a tube, the walls of which in one part were thickened by the formation of a gland with long cells of the same kind as those of the thyroid, the nucleus being spherical and situated at the very further end of the cell, and the cells being arranged in wedges so that the extremities of each group of cells came to a point on the surface of the inner lining of the tube. This point is marked by a small round opening in the internal chitinous lining of the tube; these cells form a column along the whole length of the tube just as in the thyroid gland, so that the chitinous lining along that column is perforated by numbers of small round holes.

[1] Miss Alcock's paper "On the cranial nerves of Ammocœtes" is not yet published but will appear shortly.

So characteristic is the structure, so different to anything else, that I have no hesitation in saying that the thyroid of Ammocœtes is the same structurally as the thyroid of Scorpio, and therefore in all probability of Thelyphonus and of Eurypterus. It follows consequently that the respiratory chamber in Ammocœtes just as in Scorpio, Eurypterus, Limulus &c. is formed by a series of branchial lamellar appendages, the most anterior of which is known as the operculum and was connected with the genital apparatus as well as with branchiæ.

Whatever the function of the thyroid may be in Scorpio, in Thelyphonus &c., it is of so great importance to the Ammocœtes that it persists until transformation takes place, i.e. until the time of puberty; then when the animal becomes sexually ripe, it disappears, being eaten up by phagocytes, the branchial segments come together in the middle line and nothing remains of the opercular segment, except the remnants of the thyroid, the hyo-branchial part of the segment and the embryonic puzzle whether the VIIth nerve supplies two segments or one. Still throughout the Vertebrate kingdom the modified thyroid remains to show by the serious effects following its removal what an important part in the economy of the body is played by this modified genital gland.

The history of the motor part of the Vth nerve.

This nerve through the whole of the vertebrate class belongs to the same series as the VIIth, IXth and Xth, and as we pass downwards to the fishes, we find that just as the facial nerve was the nerve belonging to the hyoid segment, so the motor part of the Vth supplies the muscles of the mandibular segment, and even as low down as Petromyzon it is still possible to speak of a mandibular segment. When however we pass still lower to the Ammocœtes we find that great changes have taken place at transformation in the muscles supplied by the Vth nerve, and that now the Vth nerve supplies a series of structures which are directly comparable with the appendages of Limulus and Eurypterus. These structures are the velar and tentacular appendages of Ammocœtes, and they correspond strictly with the locomotor and tentacular appendages of Eurypterus and so with the locomotor appendages of Limulus.

The large velar appendage is the least modified, possessing as it does the arthropod tubular muscles, a blood system of lacunar blood spaces, a surface covered with a regular scale-like pattern formed by cuticular nodosities just as in Eurypterus and in Scorpio, and at its base a serrated gnathite edge guarding on each side the place where the old mouth came to the surface, and still acting as a grinder according to Langerhans, or more probably a

strainer for the mixed food and mud which the Ammocœtes takes in to its pharyngeal chamber. The velar appendages have been further utilized in line with the respiratory appendages to assist in respiration, their movements being synchronous with the respiratory movements.

The separate part of the Vth nerve which supplies the velum passes within it from the dorsal to the ventral surface of the animal, and then, as shown by Miss Alcock, turns abruptly forwards to supply the large median tentacle, so that this extraordinary course leads to the direct conclusion that this median tentacle which is really double constitutes with the velum of each side the true velar appendages, and that that separate part of the Vth nerve corresponds to the nerve of the large oar appendage of Eurypterus. Again on each side of the middle line there are in Ammocœtes four large tentacles each of which possesses a system of muscles, mucocartilage and blood spaces precisely similar to the median ventral tentacle already mentioned. Each of these is supplied, as Miss Alcock has shown, by a separate branch of the motor part of the Vth, and each branch is comparable to the branch supplying the large velar appendage with its median ventral tentacular part.

As might be supposed from what has already been said, all these tentacles and the velar appendages vanish upon transformation, and in their place is found the remarkable tongue and suction apparatus of the Petromyzon.

The very process of transformation of the locomotor appendages of the Limulus into the tactile and velar appendages of Ammocœtes is shown to us by the condition of affairs in Eurypterus, where these appendages are no longer chelate, but have been changed into short tactile appendages with the exception of the last one which acted as an oar appendage. We see how their basal parts are all hidden by the lower lip or metastoma which is strictly comparable to the lower lip of Ammocœtes, and how all the foremost ones are capable of being retracted into the large hood which is strictly comparable with the large hood or upper lip of Ammocœtes; let the lower lip grow larger and let the last appendage follow the example of the others, and we have immediately no longer Eurypterus but Ammocœtes, with the separate branches of the motor part of the Vth nerve representing the nerves of the locomotor appendages of Limulus, and so bringing this nerve into its true position as an appendage nerve with the VIIth, IXth and Xth.

The history of the VIIIth nerve.

Throughout the vertebrate kingdom the auditory apparatus remains constant in position, varying in structure to the extent that the cochlea becomes more developed as we pass upwards, yet throughout a gradual development can be traced from the simple auditory capsule of the Cyclostomi to the complicated bony and membranous labyrinths of man; the changes which took place owing to the passage from water breathing to air breathing animals, the formation of a *membrana tympani* with the remarkable history of the ossicles does not concern the history of the VIIIth nerve. We see then that the auditory apparatus and the VIIIth nerve arises in connection with a simple auditory capsule and a special sense nerve which is situated between the trigeminal and vagus group of nerves. Going one step further this means that an auditory apparatus and nerve similar to that of Ammocœtes existed between the locomotor or thoracic appendages and the branchial or abdominal appendages of the Eurypterus-like animal from which the Ammocœtes arose, and when we turn to Limulus we find exactly in the same position an organ which is the counterpart of the auditory organ of Ammocœtes. For, situated at the base of the last large digging appendage—between therefore the locomotor appendages and the operculum—is found a large hemispherical organ to which a moveable spatula-like process is attached, known as the *flabellum* and called by Ray Lankester an "epipodite." This organ is confined to the base of this limb; it is undoubtedly a sense organ, being composed mainly of nerves, and an elaborate arrangement of nerve cells in connection with innumerable fine hairs, which are thickly embedded in the chitin of the upper surface of the spatula. By the varying thickness of the chitin these sense hairs vary with considerable regularity in length, and when the animal is at rest this sensory surface projects upwards and backwards into the crack between the thoracic and abdominal carapaces, so that while the eyes only permit a look out forwards and sidewards, and the whole animal is lying half buried in the sand, any vibrations in the water around can still pass through this open crevice and so reach the sensory surface of this organ. The structure of this sense organ is of a somewhat similar character to that of the lyriform organs described by Gaubert and supposed to be auditory.

Further, the most striking and complete proof that this sense organ of Limulus is the same as the auditory capsule of Ammocœtes is found in the fact that in each case the nerve is accom-

panied into the capsule by a diverticulum of the liver and generative organ.

In the Limulus the liver and generative organ, which surround the central nervous system from one end of the body to the other, do not penetrate into any of the appendages, whether thoracic or abdominal, with the single exception of the *flabellum*.

In the Ammocœtes the peculiar glandular and pigmented tissue which we have already recognised as the remains of the liver and generative organ, which surrounds the brain and spinal cord, does not penetrate into the velar or other appendages with the single exception of the auditory capsule where it enters with and surrounds the auditory nerve just as in Limulus.

Although this organ is not apparent in Eurypterus, yet we can see how the VIIIth nerve came into closer relations with the VIIth or opercular nerve than with the Vth as in Limulus, when the scorpions arose from a Limulus-like form; for in the scorpion we see that a large organ similar in structure to the lyriform organs, evidently a special sense organ with a nerve of its own, viz. the pecten, is situated in closer relationship to the operculum than to the thoracic appendages; so that looking at it superficially Ray Lankester considers it as the second abdominal appendage. In reality its nerve leaves the nervous system anteriorly to the nerve of the operculum, and the pecten therefore belongs to a segment immediately anterior to the operculum, and is in all probability simply the *flabellum* of Limulus enlarged and pushed downwards to gain room, its proper metameric position being shown by the origin of its nerve from the central nervous system between the opercular and last locomotor appendage nerves.

In Limulus the auditory apparatus is enclosed in a chitinous envelope, not yet become cartilaginous, and as already mentioned the cartilaginous auditory capsule and the trabeculæ of Ammocœtes are more recent formations than the branchial cartilages, which latter are already found in Limulus.

The history of the cranial segments and nerves.

From what has been said it follows that the mystery of the vertebrate cranium and cranial nerves is now solved, for we see that the question of the bony segments can be traced back to Petromyzon and so to Ammocœtes, with its direct genetic relationship to Eurypterus, and that the cranial nerves are in their origin the nerves of the prosomatic and mesosomatic appendages of Eurypterus, and so of Limulus.

We may tabulate them as follows:

	Cranial nerves	*Ammocœtes*	*Limulus*
Supra œsophageal ganglia	I	olfactory organ	olfactory organ (well seen in Thelyphonus)
	II	optic segment, lateral eyes	lateral eyes
	II′	pineal eyes	median eyes
Infra œsophageal and abdominal ganglia	III, IV	comparison not yet worked out	
	V_{m1}, V_{m2}, V_{m3}, V_{m4}	tentacles	locomotor appendages
	V_{m5}	velar appendage	last locomotor appendage
	V_s	corresponding epimeral sensory nerves	
	VIII	auditory organ	flabellum
	VII_1	thyroid	genital operculum
	VII_2	hyo-branchial	branchial operculum (found in Thelyphonus, &c., not in Limulus)
	IX	1st branchial	1st branchial
	X_1	2nd „	2nd „
	X_2	3rd „	3rd „
	⋮	⋮	⋮
	X_6	7th „	5th „

The comparison of the sensory part of the Vth with the epimeral nerves of Limulus, the meaning of the IIIrd and IVth nerves, of the sensory branches of the vagus group &c., must be left until the completion of Miss Alcock's investigation of the course and distribution of all the branches of the cranial nerves in Ammocœtes, and their comparison with the homologous nerves of Limulus and Scorpio.

Two points of interest may be mentioned here, viz. the genesis of the medulla oblongata and the reason for the invariable position of the glossopharyngeal as the separate nerve of the first true branchial segment.

The comparison between the medulla oblongata of Ammocœtes and of Limulus shows that in the latter, the only nerve belonging to the lamellar appendages which arises from the medulla oblongata is the nerve of the operculum, i.e. the VIIth nerve of Ammocœtes, the IXth and Xth nerves arising from the separate abdominal ganglia; in other words, only one ganglion and nerve belonging to the abdominal or mesosomatic appendages has

travelled up into the thoracic or prosomatic fused mass of ganglia. In the Scorpions Lankester has shown a further amalgamation in the Scorpionini where the 1st branchial appendage as well as the genital operculum is innervated from the medulla oblongata; and yet again a still further amalgamation in the Androctonini where the two foremost branchial appendages are innervated from the medulla oblongata.

In fact, we see in the passage of the Crustacean into Limulus, and Limulus into Scorpio and Androctonus, the same law of gradual fusion of more and more of the ganglia of the ventral chain to form the medulla oblongata, until at last all the nerves of the branchial appendages arise from these fused ganglia, and the vagus nerves of Ammocœtes result. The same process has continued onwards into the Vertebrates higher than Ammocœtes with the result of the inclusion of vertebræ and spinal nerves into the occipital region of the skull as seen in various Elasmobranchs and Ganoids.

Further, the solitary position of the glossopharyngeal, separated as it always is from the vagus, is suggestive when we find that in all the scorpions the nerve to the 2nd gill appendage is taken up and no more, and that in Thelyphonus and the Pedipalpi generally there exists only one gill-bearing appendage in addition to the gill-bearing part of the operculum. In other words, the nerves which correspond to the glossopharyngeal and facial nerves are invariably found in connection with the operculum and a gill-bearing appendage even when the appendages homologous to those supplied by the vagus nerve did not carry gills.

The history of the heart and ventral aortæ.

Without tracing the modifications of the vascular system throughout the vertebrate kingdom, it will be sufficient to say that the simplification which takes place as we pass down to the Elasmobranchs shows that the heart and conus arteriosus arose from the subintestinal vein which by dividing into two formed the two ventral aortæ, the main function of which was to send the blood into the gills for purposes of aeration.

Further, Paul Mayer in his paper on the origin of the heart and blood vessels in Elasmobranchs, has shown that this apparently single subintestinal vein is in its origin double, and that therefore in all Vertebrates the heart and ventral aorta arises from two long veins which are originally situated on each side of the middle line; by the formation of the head-fold these come together ventrally, coalesce into a single tube which forms the subintestinal vein and heart, still remaining double as the two ventral aortæ with their branchial branches into each gill, as is well shown in

the case of Ammocœtes. The heart, then, of all Vertebrates, unlike the invertebrate heart, is formed from the coalescence of two main venous trunks or sinuses situated ventrally, its original purpose being to send blood into the gills for the purpose of aeration. One step further leads to the conclusion that these two longitudinal venous sinuses are the two longitudinal venous collecting sinuses of Limulus and Scorpio. In both cases they form two large ventrally situated blood vessels from which the blood passes to the gills, and, further, in both cases the walls of these two sinuses are connected with the pericardium by a system of transparent muscles described by Milne Edwards and named by Lankester veno-pericardiac muscles.

These muscles are hollow, both near the vein and near the pericardium, so that the blood in each case fills the cavity, and as they contract with the heart, that part of them in connection with the venous collecting sinus already functions, as pointed out by Milne Edwards and Blanchard, as a branchial heart.

The ventral aortæ of Ammocœtes correspond exactly in position and in function with these two venous collecting sinuses in Scorpion, and therefore in Limulus; and we see in Ammocœtes the position of the original dorsal heart represented by that extraordinary column of peculiar tissue containing fat which is situated in the middle line between the spinal canal and the surface.

The further relation between the vascular and lymphatic systems of Ammocœtes and the vascular system of Scorpio together with the consideration of the pronephros, will be considered later.

The history of the Alimentary Canal.

The history of the vertebrate alimentary canal is the history of nature's great mistake and how she rectified it; for at the time when Vertebrates were formed, the lines upon which evolution was working led straight to an increasing antagonism between the increase of brain power and the sufficient feeding of the organism; for the increase of brain power meant the increase of connections between the supra-œsophageal ganglia and the lower nerve centres, meant therefore an increase of nervous matter around the œsophagus with the result of diminishing the size of the œsophagus to such a degree that at first only liquid nourishment could be taken in, and finally the choice lay between starvation and cessation of growth of brain. In Limulus we already see the commencement of the struggle, and in the formation of the Scorpions and Arachnids we see not only how narrow the œsophagus has become so that the whole class are able only to take in liquid food—, and indeed in certain Arachnids a special suction pump arrangement has been devised in their stomachs

to get through even liquid food—, but also how with the increasing brain development the great mass of liver and generative organ was shifted from the cephalic to the abdominal region, thus bulging out the ventral surface of the body until it came into contact with and fused with the lamellar branchial appendages on each side. In this way the great class of Arachnids was formed, but in vain as far as further evolution went, for no amount of room around the brain could prevent the constricting action of the enlarged œsophageal commissures on the œsophagus. Still among those Limulus and Scorpion-like forms which were in those days lords of creation and amongst whom the struggle for existence was very keen, there were some in which the ventral surface did not reach up to and fuse with the lamellar appendages, although by the coming together of these latter a body surface like that of Eurypterus was formed. In these cases then a respiratory chamber was formed with gills and gill slits between the fused appendages, then came the piercing of the operculum so as to put this chamber into communication with the oral cavity. Such opening of communication is preserved for us embryologically as the breaking through of the septum of the stomatodæum, for this septum is in the exact position of the operculum and has nothing to do with the velar appendages. In this way then these animals took in water of respiration through the mouth or gill slits into a chamber formed by the branchial appendages and with this respiratory water food of various kinds was naturally taken. Such food could be digested because as shewn by Miss Alcock the skin of Ammocœtes pours out a secretion for the purpose of keeping its skin clean, which is easily capable of digesting fibrin. It appears from Hardy's researches that a similar secretion is poured out on the carapace of Daphnia and other Crustaceans. With the growth of the lower lip or metastoma and the modification of the basal portion of the last locomotor appendage, which basal part was inside the lower lip, into a valvular arrangement like the velum, the animal was able to close the opening into the respiratory chamber and feed as blood sucker in the way of the rest of its kind, or when food of the kind was scarce, keep itself alive by the organic material taken into its respiratory chamber with the muddy water in which it lived.

In this way then we see how little by little a chamber was formed by the lamellar branchial appendages which functioned also as a digestive chamber.

Such I imagine to have been the origin of the respiratory part of the vertebrate alimentary canal, and the manner of formation of the rest of the tube is shown to us by the evidence of phylogeny, embryology and palæontology.

Tracing back the alimentary canal along the Vertebrates we

find in the highest forms a great length of intestine innervated by the vagus nerve, between the hind-gut with its innervation from the sacral region and the fore-gut innervated by the glosso-pharyngeal and facial as well as by the vagus.

Passing downwards we see the fore-gut becoming branchial still connected with the hind-gut by a long intestine innervated by the vagus nerve, but the length of this latter in proportion to the length of the whole is much less. In fishes the length becomes smaller and smaller, the tube is straight instead of thrown into innumerable folds and at last in the Ammocœtes we find a small intestine innervated by the vagus between the large branchial fore-gut and smaller hind-gut. In other words, the phylogenetic history of the vertebrate gut shows that the line of tendency as we descend the phylum is steadily towards an approximation of the cranial and sacral portions of the gut: pass one step further and we should find as in Limulus the end of the branchial chamber close to the cloacal termination of the gut, i.e. close to the cloacal termination of the old gut.

In other words, the branchial chamber was continued on as a short tube which broke through into the old existing cloaca and the elongation of that tube is the striking fact in the phylogeny of the vertebrate gut. The formation of this tube from the respiratory region by the coming together of the ciliated hyper- and hypo-pharyngeal ridges is well seen in Ammocœtes, and embryologically the travelling down of the vagus and the fact of this part of the gut being always supplied by the great branchial nerve, the vagus, is strong confirmatory evidence that the middle part of the intestine has arisen in the vertebrate phylum as the gradual elongation of an originally small junction between the respiratory chamber and the cloacal region. The evidence of this junction taking place is preserved for us embryologically by the breaking through of the septum of the proctodæum just as the passage from the mouth region to the respiratory chamber through the operculum is represented by the breaking through of the septum of the stomatodæum.

Again, the embryological development of the Vertebrate illustrates this history in the most striking way, for not only does it show that the embryo grows in length by additions made between the head end and the tail end, but also that at a time when the fore-gut and hind-gut are well formed the mid-gut is still a mass of yolk; this is especially well shown in Shipley's specimens and is a clear indication that the formation of the mid-gut takes place embryologically and therefore took place phylogenetically much later than the formation of the fore-gut or hind-gut, for the argument often used, that yolk retards development and for that reason the mid-gut is formed latest, is to my mind evasive rather than real,

and it seems to me much more probable that the yolk is there because that part is last formed, than that it is last formed because the yolk is there.

The palæontological evidence.

It is clear from what has been said that the evidence of palæontology ought to show, firstly, that the Vertebrates appeared when the waters of the ocean were peopled with the forefathers of the Crustacea and Arachnida, and secondly, the earliest fish-like forms ought to be characterised by the presence of a large cephalic part to which is attached an insignificant body and tail. Such was clearly the case, for the earliest fish-like forms appear in the midst of and succeeding to the great era of strange proto-crustacean animals, when the sea swarmed with Trilobites, Eurypterus, Slimonia, Limulus, Pterygotus, Ceratioceras and a number of other semi-Crustacean and semi-Arachnid creatures; and when we examine these ancient fishes we find such forms as Pteraspis, Pterichthys, Astrolepis, Bothriolepis, Cephalaspis, all characterised by the enormous disproportion between the extent of the head region and that of the body. Such forms would have but small power of locomotion, and further evolution consisted in gaining greater rapidity and freedom of movement by the elongation of the abdominal and tail regions, with the result that the head region became less and less prominent, until finally the ordinary fish-like form was evolved, in which the head and gills represent the original head and branchial chamber, and the flexible body with its lateral line nerve and intestine innervated by the vagus represent the original small tail-like body of such a form as Pterichthys.

Nay more, the very form of Pterichthys and the nature of its two large oar-like appendages, which according to Traquair are hollow like the legs of insects, irresistibly remind one of a form like Eurypterus in which the remaining appendages had shrunk to tentacles as in Ammocœtes, but the large oar-like appendage still remained, coming out between the lower and upper lips and assisting locomotion. The Ammocœtes-like forms which existed between the time of Eurypterus and the time of Pterichthys not having developed bony plates have not yet been found, although the recent find of Traquair of Palæospondylus Gunni is exceedingly interesting in this respect. The evidence of palæontology as far as it goes confirms absolutely the evidence of anatomy, physiology, phylogeny and embryology and assists in forming a perfectly consistent and harmonious account of the origin of Vertebrates, the whole evidence showing how nature made a great mistake, how excellently she rectified it, and thereby formed the new and mighty kingdom of the Vertebrata.

In the discussion which followed, Mr A. Sedgwick said that:

The arguments used by Dr Gaskell to prove that Vertebrates are descended from Arthropods might with equal cogency be used to prove the reverse, *viz.* that Arthropods are descended from Vertebrates; Arthropods being quite as highly specialised as Vertebrates. Further the facts alleged in support of the contention are of a superficial and unimportant character, and not of a kind to be brought forward or considered until the more important and deep-reaching anatomical differences between the two groups have been reconciled. For instance such important anatomical matters as the condition of the cœlom, of the nephridia, of the relation of the blastopore to the neural surface are not even referred to. Nor is a single embryological fact brought forward in support of the astounding contention that the alimentary canal of the Arthropod has been changed into the central nerve-tube of the Vertebrate.

It was further pointed out that Dr Gaskell's paper was wanting in that precision and accuracy of statement required in a speculative investigation of this kind; that it showed internal evidence suggesting that he did not fully appreciate the relative importance of anatomical facts, and finally by the use of certain words and expressions in an erroneous and inaccurate sense it seemed to show that Dr Gaskell was ignorant of some of the most important principles which must underlie all sound and useful phylogenetic speculation.

Mr MacBride said that Dr Gaskell had brought forward some interesting points in the anatomy of Ammocœtes but that no step in advance had been made in the direction of solving the question of Vertebrate ancestry. The objections to be urged against Dr Gaskell's theory on this subject were mainly two.

1. The Vertebrates in the fundamental constitution of some of their most important organs—*viz.* cœlom excretory organs—belonged to a much more primitive grade of evolution than the Arthropods, inasmuch as the same general arrangement was found in the less differentiated members of many quite distinct phyla, *viz.* Mollusca, Brachiopoda, Polyzoa, Annelida, &c., whereas the arrangement found in Arthropods was highly specialized.

2. The reasoning Dr Gaskell employed to prove the descent of Vertebrates from Arthropods was illegitimate. In postulating changes which they suppose to have occurred in the history of the race, zoologists confined themselves to those which could be inferred by comparing together animals of undoubted affinity with each other.

There was no evidence that fundamentally important organs such as the alimentary canal could be lost and redeveloped in another place.

If such assumptions were allowed the descent of any animal from any other could be proved.

Mr S. F. Harmer pointed out that Dr Gaskell had based his conclusions on two main lines of argument; the first being the assumption that the comparative study of any particular organ within the limits of a single group will not only enable us to retrace the steps which evolution has taken, but will give some clue to the conditions which existed prior to the evolution of that organ as such; and the second being derived from a detailed comparison of *Limulus* with Vertebrates.

Instances were given to show that the first assumption is fallacious. The evolution of the lung may have been retraced to the simplest form in which that organ exists in Vertebrates, but it does not follow that the line can be produced backwards so as to give any indication of the conditions antecedent to the evolution of the air-bladder. Nor does the retracing of the line of evolution of the Perissodactyle hand or foot give any information with regard to what preceded the pentadactyle condition.

It was admitted that the resemblances between *Limulus* and Vertebrates were, on Dr Gaskell's showing, of a remarkable nature; but many of the details could be satisfactorily criticized only by one who has made a special study of both *Limulus* and the Ammocœt. It was, however, suggested that since the resemblances extended to such minute points of detail, it would be remarkable, if Dr Gaskell's theory were true, that differences should exist in what would appear to be fundamental points; as, for instance, the numerical difference between the branchial appendages of *Limulus* and the branchial bars of *Petromyzon*.

Dr Gaskell concludes that the fact that the intestine is elongated in the higher Vertebrata and straight in the Ammocœt points to an original "approximation of the cranial and sacral portions of the gut"; and that therefore the branchial chamber formerly terminated close to the cloaca. It would equally well follow from this line of argument that other groups of animals in which the intestine of the higher members is convoluted and of the lower forms straight were also descended from ancestors in which a secondary alimentary canal had been evolved.

The argument from the position of the yolk in Vertebrate embryos has no real weight. Dr Gaskell, in dealing with the Vertebrata, has put a special interpretation on a very general embryological phenomenon. He has entirely ignored other groups of animals, in this respect; and he is no more justified in his conclusion that the embryological evidence indicates a late evolution of the mid-gut in Vertebrates than he would have been in drawing a similar inference from the developmental history of Mollusca.

On the motion of Mr H. Gadow the discussion was adjourned.

Monday, *December* 2, 1895.

Professor J. J. Thomson, President, in the Chair.

The discussion of Dr Gaskell's paper "*On the Origin of Vertebrates*" was resumed.

Mr H. Gadow said that Dr Gaskell combines the study of physiology with that of morphology, the latter not being restricted to the small branch of ontogenetic research which can be guided only by the method of comparative anatomy.

Much of the ordnance store of embryologists consists of weapons which are not those of precision, because at present our knowledge of their nature and meaning is far from satisfactory; for instance notochord, neurenteric canal, nervous system, cœlom, metamerism. Objections to Dr Gaskell's hypothesis, which are based upon such features, are very precarious.

The incredulity, which Dr Gaskell's hypothesis has met with, is referable to two reasons. First, we were ignorant of the astonishing number of important anatomical and physiological features to which Dr Gaskell has drawn our attention in Ammocœtes and in Limulus, especially the larval Ammocœtes being full of revelations. Secondly, we are prejudiced in favour of one or other hypothetical pedigree of vertebrate descent, although it is not obvious which are at present the favourites, namely Worms, Sea Squirts, Sea Anemones or even Sea Urchins.

Anyhow, be Dr Gaskell's hypothesis right or wrong, he has stirred up so many morphological questions, which can no longer be neglected, that work has been cut out for us for many years to come.

Mr Bateson said that it was difficult to make any criticism which should adequately express to the minds of those who were not acquainted with the leading facts of the structure and development of animals how impossible it was to entertain Dr Gaskell's hypothesis of Vertebrate descent. Several difficulties had already been spoken of, regarding Amphioxus, the Vertebrate Cœlom, the Notochord, &c., any of which seemed sufficient disproof. Two points might perhaps be taken which would make the objections clearer to those who were not professed zoologists. It was natural however that these difficulties should appeal most to those whose zoological studies had taken a wider range.

First regarding the relation of the nervous systems of Arthropods and Vertebrates. Setting apart all phylogenetic speculation it is known by observation that the central nervous system of a Vertebrate separates from ectoderm in the middle line of the dorsal side [of a Vertebrate], that is to say that that nervous system

is a piece of the *dorsal* ectoderm. In the Arthropod it is equally certain that the nervous system separates from the ectoderm in the ventral middle line [of the Arthropod], and is therefore a piece of the *ventral* ectoderm. Nevertheless Dr Gaskell held both that the nervous system of the Vertebrate was homologous with that of the Arthropod, and that it was the *dorsal* surface of the Arthropod which corresponded with the *dorsal* surface of the Vertebrate. Dr Gaskell had made no attempt to reconcile the different modes of development in the two cases.

Extraordinary changes of structure and function were postulated by Dr Gaskell. These were supposed to have occurred in evolution, in the course of lineal descent, by the process of variation. If anyone would try to conceive these variations taking place before his eyes, he would realize the difficulties involved. For example if it was alleged that a specimen of *Limulus* had been found which had developed a mouth in its operculum, or a perforation from the space occupied by the branchiæ leading into the intestine, or feeding or digesting by means of that space, or with its nervous system enveloping its alimentary canal, no one would credit the observation without overwhelming testimony. But Dr Gaskell assumed that all these things and many others no less astonishing had occurred as variations. No evidence had been offered that any such variations or any approach to them had ever occurred, or even that they were possible.

In the absence of such evidence there was no reason why Dr Gaskell's suggestion should be seriously considered at all.

Dr Gaskell had mentioned *Pterichthys*, *Coccosteus*, and other fossil Fishes, hinting that he regarded them as possible links between Arthropods and Vertebrates: had Dr Gaskell any reason for supposing that these animals were anything but Fishes in the strictest sense?

Mr Shipley said he thought that in attempting to derive one group of animals from another, certain principles were usually observed, (i) one of these was that in assuming variations to arise in an ancestor the amount of change assumed and its direction should be checked by the kind of variation which is already known to exist in the various members of the group to which the ancestor belongs, but although no one could deny that the members of the Arthropoda varied in almost every possible direction, none of them varied along the lines laid down by Dr Gaskell; (ii) the amount of variation assumed should be limited: there was no fact known amongst animals which led us to believe that they ever lost and redeveloped such fundamental organs as the entire alimentary canal, the renal organs and the reproductive organs, yet this was assumed in Dr Gaskell's hypothesis; (iii) changes of function can arise when the new

function coexists, perhaps subordinately, with the primary function, or coexists in another member of the same group or in one of its larval forms: the remarkable changes of function assumed by Dr Gaskell had no such origin; (iv) in any attempt at Phylogeny we must observe the "embryological unities": Dr Gaskell derives the epiblastic lining of the nervous system from the—in the main—endodermic lining of the alimentary canal and the mesoblastic "packing" cells around the vertebrate nerve-tube —in part—from the endodermic "liver" of the Arthropod: (v) in Phylogeny we must look at the most elementary of the animals whose ancestor we seek, yet Dr Gaskell started some way up the Vertebrate scale and omitted from his consideration the Amphioxus and the larval Ascidian possibly because he regarded them as degenerate but the same reasoning which led to a belief in the degeneracy of these groups has been applied to shew that the Lamprey is itself degenerate.

The Arthropod characters of which we should expect to find some trace in a Vertebrate descended from a Limulus-like ancestor are (i) the *cœlom*, which in the Arthropod is rudimentary and replaced by a body cavity of a highly specialized character and yet the cœlom reappears in the Vertebrate in the most primitive form, just as it exists in the ancestor of the Arthropods themselves; (ii) the *chitinous skeleton* which dominates the structure of the Arthropods from one end of the group to the other, is unrepresented in their supposed descendants: in connection with this it may be pointed out that cilia which with the exception of the Nematodes, are universally present in all animals but the Arthropods, have re-appeared in the Vertebrata; (iii) the *paired, segmented limbs* of Limulus, which are only represented by sensory papillæ round the mouth or by the branchiæ.

The result of the whole hypothesis is that we are asked to believe (i) that the functional alimentary canal of the Limulus, with its—in the main—endodermic lining, has become the lining of the Vertebrate nervous system which is epiblastic in origin; (ii) that the endodermic "liver" and the testes of the Arachnoid ancestor have become the mesoblastic tissue surrounding the nerve-cord of the Ammocœte; (iii) that the Arachnoid ancestor has developed an entirely new alimentary canal along its ventral median line, and that this chitinous-lined tube is completed ventrally by the fusion of the limbs; (iv) that such of the segmented limbs as are not required for this purpose become minute sensory papillæ surrounding the mouth; (v) that the renal organs of the Arachnoid ancestor degenerate into an appendage of the brain, the pituitary body; (vi) that part of the tissue representing the testis of the ancestor is to be found inside the internal ear; (vii) that the hæmocœl of the Arthropod is again replaced by

the primitive cœlom which is reduced almost to extinction in the adult ancestor; (viii) that cilia which are primitive have been lost in the ancestor and re-acquired in the descendant, whilst the chitinous coating of the former leaves no trace; (ix) that the heart has disappeared; (x) that with the exception of a part of the branchial skeleton, the entire skeleton including the notochord is a new formation, and so are the kidneys and the generative organs; the paper before us throws no light upon the question as to their origin.

Mr E. J. Bles said that the gap between an Arthropod ancestor and the vertebrates is as great as the gap between the vertebrates and any other ancestral form hitherto put forward. In considering Dr Gaskell's theory it is therefore necessary to make sure that besides deriving support from resemblances in anatomy and histology, the theory takes into account the main distinctions between the hypothetical ancestor and the vertebrates. Using a table from the most recent authority (Haeckel, *Systematische Phylogenie der Wirbelthiere*, p. 12), it appears that all the valid distinctions drawn between Articulata and Vertebrata have been dealt with in Dr Gaskell's paper. As for the remaining points: (1) the presence of a cuticular layer in vertebrates has been recognised by Leydig, F. E. Schultze, etc., and demonstrated to be chitinous by G. Wolff; (2) the structure and mode of development of the notochord throughout the chordates is so uniform that no distinct clue is given to its origin from any pre-existing organ in an ancestral form.

Dr Gaskell said in reply—I have listened most carefully to the arguments brought forward against my theory, and it seems to me that the most important of them may be summed up as follows.

"The resemblances and coincidences between the structure of Ammocœtes on the one hand and of Limulus and Scorpions on the other which I have brought forward are mere trivialities in comparison with the true criteria of what constitutes a Vertebrate, viz. the presence of a cœlom of a more primitive type than that of an Arthropod, and the existence of a notochord. These are the two most important points, and as I have given no explanation of them, the coincidences which I have pointed out, although interesting, are of no real importance."

I quite understand that these close resemblances between similar parts in Limulus and Scorpions and in Ammocœtes are very unorthodox, but they are certainly not trivial. The argument reminds me of the commotion made in the orthodox world when it was first suggested that man and the ape were closely akin;

how it was then said by an eminent comparative anatomist—These coincidences and resemblances between the structure of man and the ape are merely trivial and have no weight in comparison to the great fact that man has a *hippocampus minor* in his brain and the ape has none—to which was afterwards added—man has an *opponens pollicis* and the ape has none.

I venture to compare the argument about the cœlom with that of the *hippocampus minor* in the earlier controversy, and just as in the latter it was shown that the ape did possess a *hippocampus minor* so I propose to show, not only that there is no vital difference between the cœlom of the Vertebrate and that of Limulus or Scorpio, but also that no stronger evidence could be adduced of the correctness of my theory than is afforded by this very appeal to the cœlom.

It is clear that the cœlomic cavities which must be compared in order to test the truth of my theory are in the case of the Vertebrate the head cavities of Balfour and in the case of Limulus the prosomatic and mesosomatic cœlomic cavities, for the whole strength of my argument is based upon the comparison of the prosomatic and mesosomatic segments of Limulus with the cranial segments of Ammocœtes.

Fortunately a recent paper by Kishinouyi on the development of Limulus allows such a comparison to be made; he not only shows that the cœlomic cavities in each segment of Limulus are formed by the splitting of the splanchnopleur and somatopleur in the same way as in Vertebrates, but also that although each one of the mesosomatic segments possesses a cœlomic cavity, this is not the case for the prosomatic segments. In these latter the 1st cœlomic cavity is a large præoral one, common to the segment of the 1st appendage and all the segments in front of it; the segments belonging to the 2nd, 3rd and 4th appendages have no cœlomic cavities formed in them; the 2nd cœlomic cavity belongs to the segment of the 5th appendage and then each subsequent segment possesses a cœlomic cavity. Clearly on my theory the 1st large præoral cœlomic cavity common to the most anterior segments of Limulus corresponds to the 1st large præoral head cavity of Balfour and of v. Wijhe. Balfour's 2nd head cavity is the mandibular associated especially with the Vth nerve. According to my view the motor part of the Vth represents the five locomotor appendages of Limulus, and we see already in Limulus that the segments corresponding to the three foremost of these appendages do not form cœlomic cavities. Posteriorly to the segments belonging to the Vth nerve a head cavity is found in each segment in Elasmobranchs and Ammocœtes just as in Limulus, the main difference between v. Wijhe and Balfour being that according to the former Balfour's hyoid segment is in reality double.

It is clear that the study of the cœlomic cavities in the head not only gives no support to the opponents of my theory but like the corresponding argument of the *hippocampus minor* affords yet another extraordinary coincidence, so that I presume that the resemblances between the head cavities of the Vertebrate and of Limulus must now be included among the rest of the trivial coincidences.

Before leaving the consideration of the cœlom, I must say that I heard to my utter astonishment the statement of both Mr Macbride and Mr Shipley that my theory was preposterous, because I made the generative and excretory organs of the Vertebrate arise *de novo*. Seeing that I have not said a word about the origin of the excretory and generative organs of Vertebrates and have never even dreamt of making them arise *de novo*, I cannot understand how such an argument can be used against me. It is true that I believe that the pituitary gland is simply the coxal glands of Limulus and Scorpions, but that does not imply, as Mr Shipley asserts, that the renal organs of the Arachnoid ancestor degenerate into the pituitary body; for undoubtedly Scorpions and other Arachnids possess a well-defined renal system known by the name of the Malpighian tubes.

Then I am told that the Arthropod does not possess a notochord, while the notochord is the great characteristic of the Vertebrate; and I am asked, Where is the notochord in Limulus? Similarly in the old controversy it was asked, Where is the *opponens pollicis* in the ape? Just as in the latter case the *opponens pollicis* is an attribute of man because he has discarded the quadripedal method of progression and become a biped, so the notochord is an attribute of the Vertebrate because it has discarded the old alimentary canal and formed a new one. I had not intended to discuss the origin of the notochord at present, but as stress has been laid upon its presence and I have been challenged, I will simply ask the following questions:—"Do embryologists consider that the notochord arose as a simple tube of hypoblast which terminated in the region of the infundibulum and extended the whole length of the body?" If so, "Is it possible to conceive of more than one interpretation of the original purpose of an unsegmented hypoblastic tube in a segmented animal which extends the length of the body from mouth to anus?" I will also point out that just as the notochord is formed by the proliferation of cells in the median line ventral to the nervous system, so later on in the history of the animal a similar proliferation of cells takes place in the same region to form the sub-notochordal rod; and again when the Ammocœtes transforms into the Petromyzon, yet another similar proliferation takes place in exactly the same region, but now confined to the ancient part of the animal, viz. the head

region; a proliferation which forms a solid rod of cells entirely occluding the opening from the branchial chamber to the anterior intestine, and then, by a hollowing out process beginning at the posterior end, forms ultimately the new gut of the Petromyzon.

Mr Bateson objected that the neural plate of the Arthropod is derived from the ventral epiblast and the gut is dorsal to it; on the contrary, in the Vertebrate the central nervous system is derived from the dorsal epiblast,—including in the term central nervous system both the nervous layers and the layer of the central canal,—and ventral to it is the notochord and vertebrate gut. This difficulty seems to me more apparent than real; the nerve layer in the Vertebrate as soon as it can be distinguished is always found to lie ventrally to the layer of epiblast which forms the central canal. In the middle line of the body, owing to the absence of the mesoblast layer, the cells which will ultimately form the notochord and those which form the central nervous system form a mass of cells which cannot be separated in the earlier stages. The nerve layer in the Arthropod lies between the ventral epiblast and the gut, the nerve layer in the Vertebrate lies between the so-called hypoblast (i.e. the ventral epiblast of the Arthropod) and the neural canal (i.e. the old gut of the Arthropod). The new ventral surface of the Vertebrate in the head region is not formed until the head fold is completed; before this time when we watch the vertebrate embryo lying on the yolk, with its nervous system and central canal formed and its lateral plates of mesoblast, we are watching the embryonic representation of the original Limulus-like animal; then when the lateral plates of mesoblast have grown round and met in the middle line to form the new ventral surface and the head fold is completed, we are watching the embryonic representation of the transformation of the Limulus-like animal into the Scorpion-like ancestor of the Vertebrates.

Mr Bateson and Mr Shipley object that my theory necessitates enormous jumps, both in conformation and function; I feel sure upon further consideration they will see that this is not so, and that in fact the main jump is not, as Mr Bateson humorously put it, the sudden transformation of a Limulus into an Ammocœtes, but rather the transformation of a Limulus into a Scorpion-like animal by the meeting of the branchial appendages in the middle ventral line. My whole argument and the whole of my work show that Ammocœtes is closely allied to a Scorpion-like animal such as Thelyphonus and Eurypterus, the Limulus-like animal being the more remote ancestor of all three. Messrs Bateson and Shipley's objections therefore are not directed against my theory of the origin of Vertebrates, but against the current view of the manner in which Scorpions arose from Limulus-like forms; a view which

is based on Macleod's observations, and is considered reasonable and indeed highly probable by all comparative anatomists who know anything about the subject.

There is on my theory no need to assume any sudden change of function or of anatomical arrangement; I do not presuppose any transformation anything like as sudden or as marvellous as that which takes place under our eyes at the time of transformation, when the Ammocœtes becomes a Petromyzon.

Mr Shipley's difficulties about the absence of cilia and the presence of chitin in Arthropods do not appear serious, seeing that the alimentary canal of Daphnia appears to be ciliated according to Hardy's paper and also the laminated layer of the skin of Ammocœtes is histologically exactly the same as an internal chitinous layer. Finally, in answer to the argument of Mr Sedgwick and Mr Shipley that Ammocœtes is not the lowest Vertebrate, that therefore I ought to have compared the Limulus and Scorpion forms with Amphioxus and the Tunicates, to which Mr Shipley was careful to add that, if I answered that these were degenerate vertebrate forms, then he had as good right to call the lamprey degenerate too, I wish to point out that the whole transformation of the Ammocœtes to the Petromyzon is characterised by the discarding and throwing away of invertebrate characteristics and the putting on of a more distinctly vertebrate appearance.

We see how by a process of histolysis, the tubular muscles, the muco-cartilage, the thyroid gland, the velar, tentacular and branchial appendages are all destroyed, or utterly changed and new tissues are formed, the net result of which is not degeneration but the formation of an animal of a distinctly higher grade, distinctly more comparable with other vertebrates, as instanced by the possibility of a direct comparison between the skull of Petromyzon and that of a tadpole, by the growth of the brain and the appearance of new nerves such as the hypoglossal, the coming out of the eyes, and the free swimming fish-like form. In fact just as the comparison between the Scorpion-like ancestor and the Vertebrate is shown in the organs of the larval stage, viz. in Ammocœtes, so the comparison between the higher Vertebrates and Ammocœtes is shown by the organs of the adult stage, viz. Petromyzon. I utterly fail to see any reason for considering the lamprey as degenerate, and suppose that Mr Shipley is referring to Dohrn's overstrained and untenable hypothesis that the lamprey is a degenerate Elasmobranch.

On the other hand the nearest allies to the Amphioxus and the Tunicates are the very Ammocœtes themselves, but how transformed! in the case of the former, no eyes, no cerebral hemispheres, no special organs of sense; while in the latter, with

the degeneration of the central nervous system, the adult animal has degenerated into an alimentary bag with almost all its vertebrate characteristics gone. It would almost seem as though in the minds of my opponents the *ego* of each individual is to be looked for in his alimentary canal and not in his brain, whereas the one certain rule in Nature's law of Evolution may be summed up as follows—If you want to rise in this world you must get more brains.

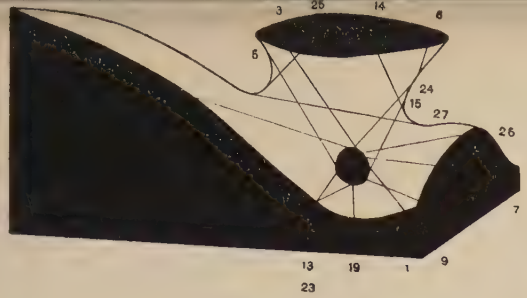


Fig. II.

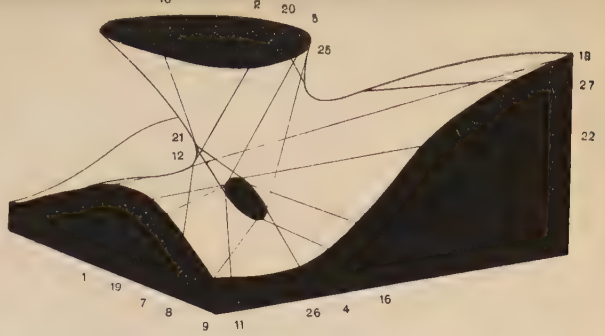


Diagram (1).

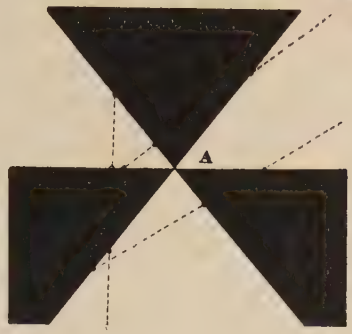


Fig. III.

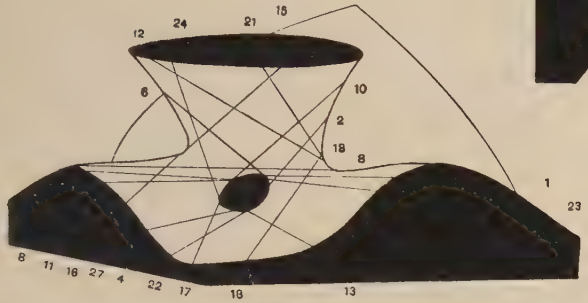


Diagram (2). Diagram (3).

Fig. IV.

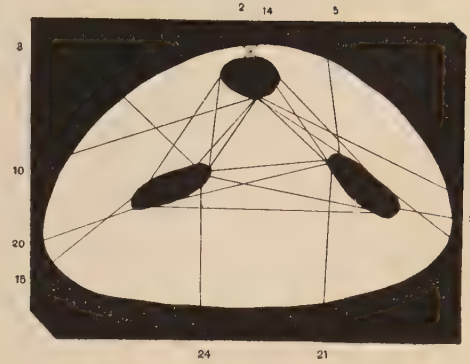
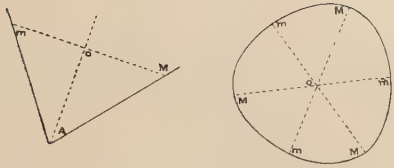


Plate I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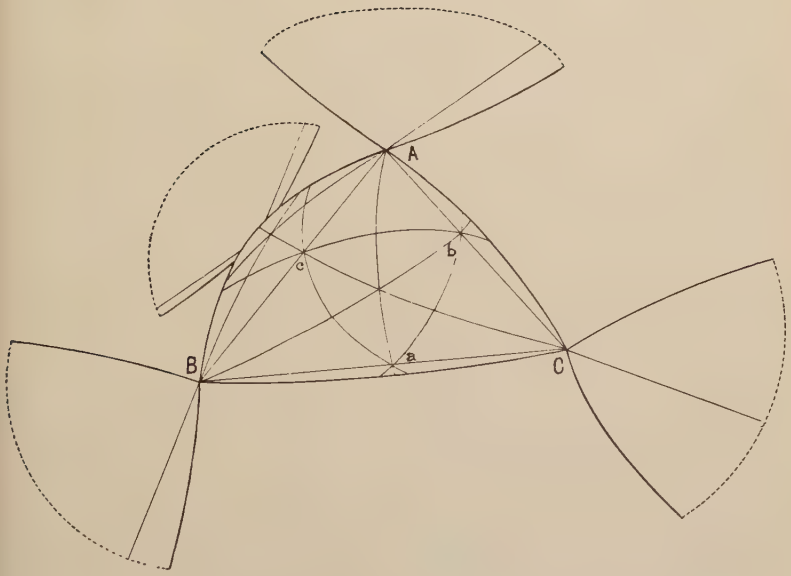
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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Monday, 27 *January* 1896.

PROFESSOR G. D. LIVEING, VICE-PRESIDENT, IN THE CHAIR.

The following communications were made to the Society:

(1) *Longitudinal Electric Waves, and Röntgen's X Rays.* By J. J. THOMSON, M.A., F.R.S.

In the theory of Electric Waves given by Maxwell the only currents considered are (1) displacement or dielectric currents, and (2) conduction currents; the first of these is proportional to the differential coefficient of the electric intensity with respect to the time, while the second is directly proportional to the electric intensity itself. When we confine our attention to these currents Maxwell's investigation shows that both the electric intensity and the magnetic force are in the wave front. There is thus no longitudinal wave, and X, Y, Z the components of the electric intensity satisfy the condition

$$\frac{d}{dt}\left\{\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}\right\}=0;$$

this, which expresses that the volume density of the electrification remains constant, is the condition that there should be no longitudinal wave. Now though the dielectric and conduction currents leave the volume density of the electrification unaltered, there is a third class of currents, not considered by Maxwell, which may alter the volume density of the electrification; these are convection currents, i.e. currents due to the motion of electrified bodies through the ether or of the ether past electrified bodies. When these currents exist the condition for the vanishing of the longitudinal wave is not satisfied, and in fact in this case we may evidently have longitudinal waves. Thus we know that in a vacuum tube the electrification has a finite value in certain parts

of the tube, and that the charges producing this electrification can move about, and by so doing produce a convection current. Let us suppose, for example, that we have a vacuum tube, and that in this tube all the atoms of the gas which are charged at all are charged with electricity of the same sign and move with a velocity p parallel to the axis of the tube, let the electric intensity be parallel to this axis and equal to X, let K be the specific inductive capacity of the medium in the tube, ρ the volume density of the electrification. Thus in this tube there is a dielectric current equal to

$$\frac{d}{dt}\left\{\frac{K}{4\pi}X\right\},$$

and a convection current equal to

$$p\rho = p\,\frac{1}{4\pi}\,\frac{d}{dx}(KX);$$

the total current parallel to the tube is thus

$$\frac{d}{dt}\left\{\frac{K}{4\pi}X\right\} + p\,\frac{1}{4\pi}\,\frac{d}{dx}(KX),$$

and this must be the same at all parts of the tube. For if u, v, w are components of the total current

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0,$$

and in this case $\qquad v = 0,\ w = 0.$

Hence we have

$$\frac{d}{dt}\{KX\} + p\,\frac{d}{dx}\{KX\} = f(t),$$

where $f(t)$ represents a function of the time.

The solution of this is

$$KX = \int f(t)\,dt + F(x - pt),$$

where F denotes an arbitrary function of $x - pt$. This represents a longitudinal wave of electric intensity, travelling with the velocity p: the velocity of translation of the electrified atoms.

When, as in the case of a vacuum tube, we have free charged atoms present the existence of convection currents is quite obvious. The following considerations show that such currents may, however, occur in a solid dielectric, with all its molecules intact, provided each molecule consists of a positively electrified atom paired with a negatively electrified one. For though in this case ρ will vanish if we take its value over a space enclosing a large number of molecules, yet if we are dealing with phenomena

involving lengths comparable with the size of a molecule, if we are, for example, considering waves whose wave lengths are comparable with molecular distances, it is evident that we cannot assume that ρ is zero; we must regard ρ as having at each point of space a finite value which will be given by the equation

$$4\pi\rho = \frac{d}{dx}(KX) + \frac{d}{dy}(KY) + \frac{d}{dz}(KZ).$$

Suppose now that the ether is moving past the molecules with a velocity whose components are p, q, r; or more generally let p, q, r be the components of the ether relative to the molecule, then this will produce convection currents whose components are $p\rho$, $q\rho$, $r\rho$; these convection currents will give rise to longitudinal waves.

Thus in this case the conditions for the production of such waves are (1) that we should have some means of producing waves whose length is comparable with molecular distances, (2) that we should be able to set the ether in motion.

Now it follows from the laws of electrodynamics that if we regard the ether as a perfect fluid the pressure due to which is equal in all directions, then it must in general be set in motion when changes are occurring in an electric field. For each unit volume in the electric field possesses a momentum whose components are

$$\frac{K}{4\pi}\{Zb - Yc\}, \quad \frac{K}{4\pi}\{Xc - Za\}, \quad \frac{K}{4\pi}\{Yb - Xa\},$$

where K is the specific inductive capacity, X, Y, Z the components of the electric intensity, a, b, c the components of the magnetic induction. Let U, V, W denote the differential coefficients of these three quantities with respect to the time. Then a force having the components U, V, W must act on each unit volume of the field. This force will set the ether in motion unless it can be balanced by the pressure due to the ether, the conditions for this are, if p is the pressure of the ether,

$$\frac{dp}{dx} = U; \quad \frac{dp}{dy} = V; \quad \frac{dp}{dz} = W,$$

or

$$\frac{dU}{dy} - \frac{dV}{dx} = \frac{dV}{dz} - \frac{dW}{dy} = \frac{dW}{dx} - \frac{dU}{dz} = 0 \text{ .....(a)},$$

unless conditions (a) are satisfied the ether will be set in motion. These conditions are not, however, satisfied in general when the electric field is changing; they are not, for example, satisfied in the field produced by the ordinary dumb-bell vibrator. Thus a varying electric field will in general set the ether in motion, and thus fulfil requirement (2) for the production of longitudinal electric

waves; thus if there were any centre of disturbance to generate waves whose length is comparable with molecular dimensions longitudinal waves will spread through the dielectrics in the variable field.

Before proceeding to discuss mathematically the laws of propagation of these waves it may be of advantage to state some of their peculiarities. We shall call the longitudinal waves due to the motion of the charged atoms in a vacuum tube convective waves, those due to the motion of the ether through a dielectric, longitudinal dielectric waves; these second waves might exist in a vacuum tube along with the convective waves. In the first place we notice that each of these classes of waves requires for its propagation the presence of matter carrying electric charges as well as ether; longitudinal waves could not on Maxwell's theory be propagated through pure ether. The wave length of the convective waves is not limited, while longitudinal dielectric waves can only be transmitted when their wave lengths are comparable with molecular dimension, and therefore are exceedingly small compared with the lengths of the visible and ultra violet rays; the finer the structure of the dielectric the more limited the range of waves that can get through. The velocity of propagation of the convective waves is equal to the velocity of translation of the charged atoms, while that of the longitudinal dielectric waves is equal to the velocity of the ether through the dielectric, this velocity will depend upon the strength of the varying electric field being larger in strong fields than in weak. If the ether moves freely through the dielectric then its velocity will not change abruptly in passing from one medium to another. Neither of these waves produce any magnetic effects if they travel in the one case in the direction of the moving atoms, in the second case in that of the moving ether.

We shall now proceed to find the equations which hold when convective currents are present in the medium.

We shall begin with the case of convective waves where the convective currents are carried by moving charged atoms, and we shall begin with the case when all the charged atoms, whatever the sign of their charges, are moving with the same velocity: let p, q, r denote the components of this velocity, X, Y, Z the components of the electric intensity, α, β, γ the components of the magnetic force; u, v, w the components of the total current, K the specific inductive capacity of the medium, and μ its magnetic permeability.

Then ρ, the volume density of the electrification, is given by the equation

$$4\pi\rho = \frac{d}{dx}\{KX\} + \frac{d}{dy}\{KY\} + \frac{d}{dz}\{KZ\}.$$

The convection current has for components

$$p\rho, \quad q\rho, \quad r\rho.$$

Hence

$$u = \frac{d}{dt}\left(\frac{K}{4\pi}X\right) + p\rho, \quad v = \frac{d}{dt}\left(\frac{K}{4\pi}Y\right) + q\rho, \quad w = \frac{d}{dt}\left(\frac{K}{4\pi}Z\right) + r\rho.$$

But

$$4\pi u = \frac{d\gamma}{dy} - \frac{d\beta}{dz},$$

and

$$\mu\frac{d\gamma}{dt} = \frac{dZ}{dy} - \frac{dY}{dz}; \quad \mu\frac{d\beta}{dt} = \frac{dX}{dz} - \frac{dZ}{dx}.$$

Hence we have

$$\frac{d}{dt}\left\{\frac{d}{dt}\left(\frac{K}{4\pi}X\right) + p\rho\right\} = \frac{1}{4\pi\mu}\left\{\nabla^2 X - \frac{d}{dx}\left(\frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz}\right)\right\} \ldots(1),$$

with similar equations for Y and Z.

By differentiating the equation for X with respect to x, that for Y with respect to y, that for Z with respect to z and adding, we get

$$\frac{d}{dt}\left\{\frac{d\rho}{dt} + p\frac{d\rho}{dx} + q\frac{d\rho}{dy} + r\frac{d\rho}{dz}\right\} = 0.$$

This equation merely represents that the changes in the volume of electrification at any point is due to the motion of the charged atoms into or away from the small volume surrounding the point.

To consider the case of a plane wave put

$$X = A\sin\frac{2\pi}{\lambda}(\omega t - (lx + my + nz)),$$

$$Y = B\sin\frac{2\pi}{\lambda}(\omega t - (lx + my + nz)),$$

$$Z = C\sin\frac{2\pi}{\lambda}(\omega t - (lx + my + nz)).$$

And substitute in equations 1, we get

$$A\left(\omega^2 - \frac{1}{\mu K}\right) = (lA + mB + nC)\left(\omega p - \frac{l}{\mu K}\right),$$

$$B\left(\omega^2 - \frac{1}{\mu K}\right) = (lA + mB + nC)\left(\omega q - \frac{m}{\mu K}\right),$$

$$C\left(\omega^2 - \frac{1}{\mu K}\right) = (lA + mB + nC)\left(\omega r - \frac{n}{\mu K}\right)$$

Eliminating A, B, C we find two values for ω, the first is given by the equation $\omega^2 = 1/\mu K$ which corresponds to the transverse waves as in this case $lA + mB + nC = 0$; the second by the equation

$$\omega = pl + qm + rn.$$

This represents a longitudinal wave propagated with the velocity

$$pl + qm + rn.$$

We see that

$$\frac{X}{\omega p - \frac{l}{\mu K}} = \frac{Y}{\omega q - \frac{m}{\mu K}} = \frac{Z}{\omega r - \frac{n}{\mu K}} \quad \ldots\ldots\ldots\ldots(2),$$

so that unless

$$p : q : r = l : m : n ;$$

i.e. unless the direction of propagation of the wave is parallel to the direction in which the particles are moving the electric intensity is not at right angles to the wave front.

We see from these equations that a longitudinal wave of electric intensity parallel to a fixed direction, say x, can be propagated by a medium containing moving charged atoms even though the velocity of these atoms is not parallel to x. Thus suppose the wave of electric intensity parallel to x is being propagated by the atoms moving with a velocity whose components parallel to x, y, z are p, q, o, then since $Y = 0$, $Z = 0$ we have $n = 0$, $m = \frac{\omega q}{V^2}$, where V is the velocity of the transverse electrical waves through the ether. Thus the wave front is not at right angles either to the direction of motion of the charged atoms or to that of the electric intensity. If the velocity of the charged atoms is small compared with the velocity of light m will be small, l nearly unity, hence $m = \frac{pq}{V^2}$ approximately. Now m is the angle which the normal to wave front when q is finite makes with the direction of the electric intensity. Hence this angle varies as the square of the ratio of the velocity of the atoms to the velocity of light. The angular deflection of the front of the waves produced by a transverse velocity imparted to the atoms would be almost infinitesimal compared with the deflection of the atoms themselves.

When the charged atoms move in a magnetic field they are deflected from their rectilinear course, so that as the preceding investigation shows a wave in which the electric intensity is parallel to a fixed direction would be deflected, but the deflection would be infinitesimal compared with those produced by the magnetic field on the cathode rays.

We see from equation (2) that if we neglect the square of the ratio of the velocity of the atoms to that of light we have

$$\frac{X}{l}=\frac{Y}{m}=\frac{Z}{n},$$

so that to this approximation the direction of propagation of the wave coincides with the direction of the electric intensity; in this case there is no magnetic force propagated with the wave.

An interesting case of these convective waves is afforded by the problem of a column of air moving like a wind and carrying along with it both positively and negatively electrified atoms. Some experiments by Hertz (*Wied. Ann.* 19, p. 78, 1883) seem to indicate that something of this nature exists in the neighbourhood of the positive electrode.

Let us consider the simple case when the velocity (u) of the column of gas as well as the electric intensity X is parallel to the axis of x.

In this case the dielectric current is equal to

$$\frac{K}{4\pi}\frac{dX}{dt}.$$

The convection current is equal to $u\rho$ where ρ is the density of the electrification; we have however

$$4\pi\rho-\frac{d}{dx}(KX)=0.$$

So that the total current, i.e. the sum of the dielectric and convection currents is equal to

$$\frac{K}{4\pi}\left\{\frac{dX}{at}+u\frac{dX}{dx}\right\},$$

if K is constant. The total current however in this case vanishes, so that

$$\frac{dX}{dt}+u\frac{dX}{dx}=0 \quad \text{.....(1).}$$

Now let us consider the case when u depends to some extent on the intensity of the electric field, let for example

$$u=p+\alpha X,$$

where p is independent of X.

Then equation (1) becomes

$$\frac{dX}{dt}+(p+\alpha X)\frac{dX}{dx}=0.$$

The solution of which is

$$X = f\{x - (p + \alpha X)\, t\} \ldots\ldots\ldots\ldots\ldots\ldots (2),$$

where $f(x)$ denotes an arbitrary function of X.

Let us take a case where initially the volume density of the electrification is represented by a harmonic function of x so that

$$\rho = \frac{K}{4\pi}\frac{dX}{dx} = -\frac{AK}{4\pi}\sin mx,$$

so that when $t = 0$,

$$X = C + \frac{A}{m}\cos mx,$$

and hence by (2)

$$X = C + \frac{A}{m}\cos m\,(x - (p + \alpha X)\, t),$$

thus

$$\frac{dX}{dx} = -\frac{A\sin m\,\{x - (p + \alpha X)\, t\}}{1 - A\alpha t \sin m\,\{x - (p + \alpha X)\, t\}}.$$

We see from this equation that the denominator of this expression can vanish and thus ρ become infinite. The earliest time at which this takes place is given by

$$t = \frac{1}{A\alpha},$$

$$x = \frac{p + \alpha C}{A\alpha} + (2n + \tfrac{1}{2})\frac{\pi}{m}.$$

The value of dX/dx, which is proportional to the volume density of the electrification, at the time $t = 1/A\alpha$ is given by the equation

$$\frac{dX}{dx} = -\frac{A\sin\left\{m\left(x - \dfrac{p + \alpha X}{A\alpha}\right)\right\}}{1 - \sin\left\{m\left(x - \dfrac{p + \alpha X}{A\alpha}\right)\right\}},$$

comparing this with the initial value $-A\sin mX$, we see that the effect of the motion of the air in the neighbourhood of the positive electrode has been to enormously increase the maximum value of the density of the negative electrification, while it has diminished the maximum value of the positive electrification. It has thus accentuated the difference in the initial state of electrification in the tube, for all the negative electrification is practically concentrated in a series of equidistant spots on the tube, while the positive electricity is more evenly distributed than it was initially. This effect, which is represented by equations similar to those

which represent the breaking of a sea wave as it rushes into shallow water, would tend, if there were any inequalities in the distribution of electrification in the tube to begin with, to accentuate these inequalities, and may possibly show itself in the sharply defined striations in the positive column.

Normal Waves in a Moving Ether.

We have seen that the ether in a varying electromagnetic field must in general be moving. Let us consider the differences between this case and the ether at rest. In the case of insulators with a quiescent ether the dielectric current is proportional to the rate of increase of the polarization with the time, in this case there is no ambiguity,—but when the ether is in motion the rate of increase to which the current is proportional may either be estimated at a point fixed in space or at a point moving along with the ether. The phenomena connected with the effect of the motion of a refracting substance on the velocity of light passing through it seem on the whole to be in favour of the latter view. But if this is the case then the motion of the ether through an electric field in which the electric intensity varies from point must produce forces which tend to keep the intensity constant at a point moving with the ether; for example, when the ether is moving through an electrified plate, forces must be generated tending to make the electricity leave the plate and travel along with the ether.

When the ether is at rest the variation of the electric intensity at any point is opposed by all the inertia of the electromagnetic field, while when the ether is moving all the influence of the inertia is on the side of making the electric intensity at a fixed point vary, the variation being such that the intensity remains constant in a given portion of the moving ether.

Let us take the simple case when the electric intensity, X, and the velocity of the ether, p, are each parallel to the axis of X. Then the dielectric current parallel to x is

$$\frac{K}{4\pi}\left\{\frac{dX}{dt}+p\frac{dX}{dx}\right\}.$$

Thus if the ether were moving in a field in which the electric intensity was constant with respect to the time, but which varied from place to place, i.e. a field in which dX/dt vanishes, but where dX/dx is finite, the motion of the ether would produce a system of

currents, and a magnetic field; the establishment of this field would have to contend against all the inertia of the system. If we have a part of field where initially dX/dx is finite but varies rapidly from point to point, regarding dX/dx for simplicity as a harmonic function of x, let us suppose that the wave length is comparable with molecular distances: then if the molecule of a substance consists of a positively and a negatively electrified atom, the molecules could arrange themselves so as to produce a distribution of dX/dx of the kind considered.

Now if the molecules group themselves so that X keeps constant at a point moving along with the ether, there will be no current, no magnetic field, and no electrokinetic energy. There consequently will be a tendency for the molecules in the tract of the moving ether to arrange themselves so as to produce this result. This alteration in the disposition of the molecules implies a longitudinal wave of the electric intensity. If the wave length of the original distribution of x exceeded molecular distances the molecules would have to split up into atoms to reproduce this distribution of X, and there will, in consequence of the inertia of the system, be a tendency for them to do so.

The equations representing the transmission of wave in the moving ether are as follows:

Using the same notation as on page 52, p, q, r now denoting the velocity of the moving ether, we have

$$u = \left(\frac{d}{dt} + p\frac{d}{dx} + q\frac{d}{dy} + r\frac{d}{dz}\right)\frac{K}{4\pi}X;$$

with similar equations for v and w, we have also

$$4\pi u = \frac{d\gamma}{dy} - \frac{d\beta}{dz},$$

and

$$\mu\left(\frac{d}{dt} + p\frac{d}{dx} + q\frac{d}{dy} + r\frac{d}{dz}\right)\gamma = \frac{dZ}{dy} - \frac{dY}{dz},$$

$$\mu\left(\frac{d}{dt} + p\frac{d}{dx} + q\frac{d}{dy} + r\frac{d}{dz}\right)\beta = \frac{dX}{dz} - \frac{dZ}{dx}.$$

Hence when p, q, r are independent of x, y, z we have

$$\mu K\left\{\frac{d}{dt} + p\frac{d}{dx} + q\frac{d}{dy} + r\frac{d}{dz}\right\}^2 X = \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)X - \frac{d\theta}{dx}, \quad (1)$$

where

$$\theta = \frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz},$$

with similar equations for Y and Z.

Take the case of a plane wave, when we may put

$$X = A \sin \frac{2\pi}{\lambda}(\omega - (lx + my + nz)),$$

$$Y = B \sin \frac{2\pi}{\lambda}(\omega - (lx + my + nz)),$$

$$Z = C \sin \frac{2\pi}{\lambda}(\omega - (lx + my + nz)),$$

substituting these values in equation (1) we get

$$\mu K \{\omega - (pl + qm + rn)\}^2 A = A - l \{lA + mB + nC\},$$
$$\mu K \{\omega - (pl + qm + rn)\}^2 B = B - m \{lA + mB + nC\},$$
$$\mu K \{\omega - (pl + qm + rn)\}^2 C = C - n \{lA + mB + nC\}.$$

Eliminating A, B, C we find that either

$$\sqrt{\mu K}(\omega - (pl + qm + rn) = 1,$$

or

$$\omega - (pl + qm + rn) = 0.$$

If we take the first solution, we have

$$lA + mB + nC = 0,$$

i.e. the wave is a transverse wave; while if we take the second solution, we have

$$\frac{l}{A} = \frac{m}{B} = \frac{n}{C},$$

or

$$\frac{l}{X} = \frac{m}{Y} = \frac{n}{Z}.$$

This represents a normal wave, the direction of propagation coinciding with the direction of the electric intensity, the velocity of wave propagation is equal to the component of the velocity of the ether in the direction of the electric intensity. If there is only the normal wave, then since

$$\frac{dY}{dx} = \frac{dX}{dy}, \quad \frac{dZ}{dy} = \frac{dY}{dz}, \quad \frac{dX}{dz} = \frac{dZ}{dx}$$

the magnetic force will vanish. Since, however,

$$\frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz}$$

is not zero throughout the wave, the volume density of the electrification is not zero; thus the wave could not be propagated through space devoid of matter, nor could the wave be propagated without dissociation through an insulator unless the wave length were comparable with molecular distances.

We thus see that on the electromagnetic theory normal waves of very short wave lengths can be propagated if the ether is moving.

A normal wave in passing from one medium to another would be deflected if the lines of electric intensity were bent in passing from one medium to another. At the junction of two media of different specific inductive capacity the lines of electric intensity are refracted, being bent away from the normal as they pass from a medium of small specific inductive capacity to one of large. In the case of these very short waves there seems, however, no reason to suppose that the specific inductive capacity of a medium would differ from unity. For if we regard a molecule as made up of a positively electrified atom joined to a negatively electrified one, then, just as the magnetic permeability may be regarded as due to the setting of molecular magnets under the influence of the magnetic force the specific inductive capacity will be due to the setting of these molecules under the influence of the electric intensity. If now the wave length of the electric intensity is comparable with the size of a molecule, a molecule at one part of its length will be acted on by a force tending to make it set in one way, and at another part of its length by a force tending to make it set in the opposite way; the resultant effect therefore will be little or nothing, and the specific inductive capacity will practically be the same as that of a vacuum.

Quite recently Professor Röntgen has discovered some remarkable effects which he is inclined to attribute to normal waves. We do not know enough about the laws governing those new rays to be able to say whether they could or could not belong to the type of normal waves we are investigating; we do not even know at present that they are normal waves at all.

A series of photographs taken by Professor Röntgen's method was shown, and a photograph was taken during the meeting of the Society.

A very noticeable feature in the bulb producing these Röntgen rays is the phosphorescence of the glass of the bulb. I thought it therefore of interest to try if these rays were generated when the phosphorescence of the glass was produced by other means than the discharge from a negative electrode. To do this, I produced a ring discharge in an electrodeless bulb; this when the pressure of the gas is very low is accompanied by intense phosphorescence of the glass. I exposed a photographic plate protected by thick cardboard for an hour to such a bulb, but without the slightest effect. I next tried filling the bulb with oxygen, a gas which is itself made phosphorescent so as to have both the glass and the gas phosphorescent, but again a photographic plate was not affected after an hour's exposure.

I also tried without success to photograph in this way by the phosphorescence excited in a screen powdered over with luminous paint, by the sparks passing between the terminals of a Ruhmkorff coil placed close to the screen.

It would thus appear that we can have vivid phosphorescence without any production of these rays.

I next tried whether the negative electrode could excite these rays without the aid of the walls of the tube. To test this, I cut a piece off a photographic plate and enclosed it in a small ebonite box in the tube in such a way that it came between the negative electrode and the glass; in this case the plate was not affected.

The next experiment I tried was designed to try if any evidence as to the transverse or longitudinal nature of these rays could be obtained by comparing the depth of the shadows cast by two plates of tourmaline, (1) when the axes were crossed, (2) when the axes were parallel. Two pairs of plates A and B were used and a series of photographs were taken; in some the axes of A crossed, and those of B parallel, while in others the axes of A were parallel and those of B were crossed. A very considerable number of photographs were taken, but there was no evidence of any effect being produced by the relative position of the axes.

A series of experiments (which were not completed until Jan. 29, the day after the paper was read) were made to test whether a charged electrified plate would retain its charge of electricity when exposed to these rays: it was found that the incidence of these rays produced a rapid discharge of the electricity from the plate, whether this was charged with positive or negative electricity; the rate of leak was independent of the sign of the charge. The potential difference used in these experiments was about 200 volts. An uncharged metallic plate when exposed to these rays was not sensibly electrified.

I have much pleasure in thanking Mr Everett and Mr Hayles for the assistance they have given me in these experiments.

(2) *The Equilibrium of Isotropic Elastic Solid Shells of nearly Spherical form.* By C. Chree, Sc.D.

The equations of equilibrium for isotropic elasticity can be solved approximately in cases where the surface is not exactly but only nearly spherical by a method which I have applied in the *American Journal of Mathematics*, Vol. 16, pp. 299 *et seq.* In all the problems solved in the paper referred to, the material was bounded by a single surface, but it was indicated *l. c.* p. 382, how the method could be applied to a shell. The present paper deals

in detail only with a simple case of the shell problem, viz. that in which the forces consist exclusively of uniform normal pressures, different over the two surfaces.

The equations to be solved prove to be ultimately identical with those treated by me in the Society's *Transactions*, Vol. xv., p. 351. Attention is mainly directed to the case of a thin shell whose surfaces are concentric, similar and similarly situated, as being that of most physical interest.

Let p, p' denote the uniform pressures, and let the equations to the two surfaces be

$$r = a(1 + \epsilon\sigma_i) \quad \ldots\ldots(1),$$
$$r = b(1 + \epsilon'\sigma_i) \quad \ldots\ldots(2),$$

where ϵ and ϵ' are small quantities whose squares are neglected, and σ_i is a surface harmonic of degree i. For simplicity there is supposed to be only one harmonic term, but the method obviously applies, however numerous these terms may be, supposing the sum of their numerical values small compared to unity.

Referred to the fundamental polar directions r, θ, ϕ at any point on the surface (1), the direction cosines of the normal, since ϵ^2 is neglected, are

$$1, \quad -\epsilon\frac{d\sigma_i}{d\theta}, \quad -\epsilon\frac{1}{\sin\theta}\frac{d\sigma_i}{d\phi};$$

similar results with ϵ' written for ϵ apply over (2).

If the shell were truly spherical the complete solution would consist in the radial displacement

$$u = \frac{1}{a^3 - b^3}\left\{\frac{b^3p' - a^3p}{3m - n}r + \frac{(ab)^3}{r^2}\frac{p' - p}{4n}\right\} \quad \ldots\ldots(3)^*,$$

where m and n are the elastic constants in the notation of Thomson and Tait's *Natural Philosophy*.

Answering to (3), the dilatation Δ and the stresses, in the notation of Todhunter and Pearson's *History*, are given by

$$\left.\begin{aligned}
\Delta &= (b^3p' - a^3p) \div \left\{\left(m - \frac{n}{3}\right)(a^3 - b^3)\right\}, \\
\widehat{rr} &= \frac{1}{a^3 - b^3}\left\{b^3p' - a^3p + \left(\frac{ab}{r}\right)^3(p - p')\right\}, \\
\widehat{\theta\theta} = \widehat{\phi\phi} &= \frac{1}{a^3 - b^3}\left\{b^3p' - a^3p + \tfrac{1}{2}\left(\frac{ab}{r}\right)^3(p' - p)\right\}, \\
\widehat{r\theta} = \widehat{r\phi} &= \widehat{\theta\phi} = 0
\end{aligned}\right\} \quad \ldots(4)^*.$$

* Cf. *Transactions*, Vol. xv., p. 343.

When the surfaces are no longer spherical, but are given by (1) and (2), the solution (3) is obviously incomplete, and we require the addition of some terms from the general solution contained in my paper above referred to in Vol. XV. of the *Transactions*. For brevity I shall simply refer by number to the equations of that paper, distinguishing them by the letter *A*. The terms required are the "mixed radial and transverse" of degree i containing the harmonic σ_i. For these from (30_A), (31_A), (32_A) we have

$$u = \sigma_i \left[-r^{i+1} \frac{im - 2n}{2(2i+3)n} Y_i + r^{i-1} Z_i - r^{-i} \frac{(i+1)m + 2n}{2(2i-1)n} Y_{-i-1} + r^{-i-2} Z_{-i-1} \right] \quad \ldots\ldots(5),$$

$$v = \frac{d\sigma_i}{d\theta} \left[-r^{i+1} \frac{(i+3)m + 2n}{2(2i+3)(i+1)n} Y_i + \frac{1}{i} r^{i-1} Z_i + r^{-i} \frac{(i-2)m - 2n}{2(2i-1)in} Y_{-i-1} - \frac{1}{i+1} r^{-i-2} Z_{-i-1} \right] \ldots(6),$$

$$w = \frac{1}{\sin\theta} \frac{d\sigma_i}{d\phi} [\text{same expression as inside square bracket in (6)}] \quad \ldots\ldots\ldots(7).$$

Here it should be noticed Y_i, Z_i, Y_{-i-1}, Z_{-i-1} are simply arbitrary constant multipliers of the harmonic σ_i, and not themselves surface harmonics as in the paper from which the results are quoted. They are determined presently from the surface conditions. These constants are all of the order ϵ of small quantities; the terms containing them will thus be called "subsidiary terms" when it is desired to distinguish them from the "principal terms" as given by (3).

The contributions of the subsidiary terms to the stresses $\widehat{rr}$, $\widehat{r\theta}$, $\widehat{r\phi}$ are given in (33_A), (34_A), (35_A); their contributions to the other stresses are not required for our present purpose.

As terms of order ϵ^2 are neglected, the surface conditions over (1) take the form

$$\widehat{rr} - \epsilon \frac{d\sigma_i}{d\theta} \widehat{r\theta} - \epsilon \frac{1}{\sin\theta} \frac{d\sigma_i}{d\phi} \widehat{r\phi} = -p \quad \ldots\ldots\ldots\ldots(8),$$

$$\widehat{r\theta} - \epsilon \frac{d\sigma_i}{d\theta} \widehat{\theta\theta} - \epsilon \frac{1}{\sin\theta} \frac{d\sigma_i}{d\phi} \widehat{\theta\phi} = \epsilon \frac{d\sigma_i}{d\theta} p \quad \ldots\ldots\ldots\ldots(9),$$

$$\widehat{r\phi} - \epsilon \frac{d\sigma_i}{d\theta} \widehat{\theta\phi} - \epsilon \frac{1}{\sin\theta} \frac{d\sigma_i}{d\phi} \widehat{\phi\phi} = \epsilon \frac{1}{\sin\theta} \frac{d\sigma_i}{d\phi} p \ldots\ldots(10).$$

The surface equations over (2) are obtained from these by writing ϵ' for ϵ and p' for p.

The two equations (9) and (10) lead to such results as

$$\frac{df(\theta, \phi)}{d\theta} = 0, \quad \frac{1}{\sin\theta}\frac{d}{d\phi} f(\theta, \phi) = 0,$$

and are really tantamount to but a single equation

$$f(\theta, \phi) = 0.$$

To obtain the values of principal terms over the surface (1) we must write $a(1 + \epsilon\sigma_i)$ for r and retain terms containing the first power of ϵ; for instance the contribution to $\widehat{rr}$ of the principal terms is easily found from (4) to be

$$-p + \frac{3b^3}{a^3 - b^3}\epsilon\sigma_i(p' - p).$$

In the subsidiary terms proceeding from (5), (6) and (7) it suffices to write a for r. In such an expression as $-\epsilon\frac{d\sigma_i}{d\theta}\widehat{r\theta}$ the contribution of the subsidiary terms would be of order ϵ^2 and so is negligible.

Bearing these points in mind, and noticing that the terms independent of ϵ cut out, it is easy to verify that the surface conditions take the form:

$$-\frac{(i^2 - i - 3)m + n}{2i + 3} a^i Y_i + 2(i-1) n a^{i-2} Z_i + \frac{(i^2 + 3i - 1)m + n}{2i - 1} a^{-i-1} Y_{-i-1} - 2(i+2) n a^{-i-3} Z_{-i-1} = \frac{3b^3}{a^3 - b^3}\epsilon(p - p') \quad \ldots\ldots\ldots(11),$$

$$-\frac{i(i+2)m - n}{(i+1)(2i+3)} a^i Y_i + \frac{2(i-1)}{i} n a^{i-2} Z_i - \frac{(i^2 - 1)m - n}{i(2i-1)} a^{-i-1} Y_{-i-1} + \frac{2(i+2)}{i+1} n a^{-i-3} Z_{-i-1} = -\frac{3}{2}\frac{b^3}{a^3 - b^3}\epsilon(p - p') \ldots\ldots(12),$$

$$-\frac{(i^2 - i - 3)m + n}{2i + 3} b^i Y_i + 2(i-1) n b^{i-2} Z_i + \frac{(i^2 + 3i - 1)m + n}{2i - 1} b^{-i-1} Y_{-i-1} - 2(i+2) n b^{-i-3} Z_{-i-1} = \frac{3a^3}{a^3 - b^3}\epsilon'(p - p') \quad \ldots\ldots\ldots(13),$$

$$-\frac{i(i+2)m-n}{(i+1)(2i+3)}b^i Y_i + \frac{2(i-1)}{i} nb^{i-2} Z_i$$

$$-\frac{(i^2-1)m-n}{i(2i-1)} b^{-i-1} Y_{-i-1}$$

$$+\frac{2(i+2)}{i+1} nb^{-i-3} Z_{-i-1} = -\frac{3}{2}\frac{a^3}{a^3-b^3}\epsilon'(p-p') \ldots\ldots(14).$$

These equations are identical with (38_A) to (41_A) if in the latter we put

$$\left.\begin{aligned} V_i &= V_{-i-1} = 0, \\ R_i &= -2T_i = 3b^3\epsilon(p-p')/(a^3-b^3), \\ R'_i &= -2T'_i = 3a^3\epsilon'(p-p')/(a^3-b^3) \end{aligned}\right\} \ldots\ldots\ldots(15).$$

The values of Y_i, Z_i, Y_{-i-1}, Z_{-i-1} in the present problem are thus given by (68_A) to (71_A) when the substitutions (15) are made. To obtain the subsidiary terms in the displacements for the present problem, all we have to do is to take the results (92_A), (93_A) and the corresponding expression for w, and in them replace

$$\left.\begin{aligned} R_i &\text{ by } 3b^3\epsilon\sigma_i(p-p')/(a^3-b^3), \\ R'_i &\text{ ,, } 3a^3\epsilon'\sigma_i(p-p')/(a^3-b^3), \\ T_i &\text{ ,, } -\frac{3}{2}b^3\epsilon\sigma_i(p-p')/(a^3-b^3), \\ T'_i &\text{ ,, } -\frac{3}{2}a^3\epsilon'\sigma_i(p-p')/(a^3-b^3) \end{aligned}\right\} \ldots\ldots(16).$$

A similar substitution in (94_A) and (95_A) gives the subsidiary terms in the expressions for the stresses $\widehat{rr}$, $\widehat{r\theta}$.

This substitution is perfectly straightforward, but the results of it in general a little cumbrous; I propose giving them explicitly only for the case when

$$\epsilon' = \epsilon,$$

and

$$h/a \equiv (a-b)/a$$

is very small.

In this case we make the substitutions (16) in (96_A), (97_A) and the corresponding expression for w, putting $\epsilon' = \epsilon$. After effecting the substitutions I have written h for $a-b$ in the terms depending on σ_i. Combining the subsidiary terms with the principal term (3), it will be found that

$$\left.\begin{aligned} u &= \frac{1}{a^3-b^3}\left\{\frac{b^3p'-a^3p}{3m-n}\,r+\frac{(ab)^3(p'-p)}{4nr^2}\right\} \\ &\quad+\epsilon\,\frac{a^2}{h}\,\sigma_i\,(p-p')\,\frac{(2i^2+2i-3)\,m+n}{4n\,(3m-n)}, \\ v\Big/\frac{d\sigma_i}{d\theta} &= w\Big/\left(\frac{1}{\sin\theta}\,\frac{d\sigma_i}{d\phi}\right)=\frac{\epsilon a^2}{h}\,\frac{p-p'}{4n} \end{aligned}\right\}\quad\ldots\ldots(17).$$

The subsidiary terms vanish when $p'=p$. It is obvious in fact from (11) to (14) that they must do so in any case, irrespective of the magnitude of $(a-b)/a$. Along the same radius vector they are constant, to the present degree of approximation. The variation of the principal term in u along a radius vector was considered in my previous paper (see (13_A) &c. writing $-p$ for R, $-p'$ for R'). This variation is important only when $(p\sim p')/p$ is small, and in this case the subsidiary terms in ϵ tend to vanish. As our present object is to examine the significance of these terms it will be supposed that $(p\sim p')/p$ is not small. Omitting the small part of the principal term in u which varies throughout the thickness, we may write (17) in the form

$$\left.\begin{aligned} u &= -\frac{a^2\,(p-p')}{h}\left[\frac{1}{4n}-\frac{\eta}{E}-\frac{\epsilon\sigma_i\{(2i^2+2i-3)\,m+n\}}{4n\,(3m-n)}\right], \\ v\Big/\frac{d\sigma_i}{d\theta} &= w\Big/\left(\frac{1}{\sin\theta}\,\frac{d\sigma_i}{d\phi}\right)=\frac{\epsilon a^2\,(p-p')}{4nh} \end{aligned}\right\}\quad\ldots(18),$$

where E is Young's Modulus, η Poisson's ratio.

Remembering the values of the direction cosines of the normal, we see that (18) is equivalent to the following three displacements:

a displacement $-a^2\,(p-p')/4nh$ along the normal,

" " $\eta a^2\,(p-p')/Eh$ " " radius vector,

" " $\dfrac{\epsilon\sigma_i a^2\,(p-p')\,\{(2i^2+2i-3)\,m+n\}}{4n\,(3m-n)\,h}$ along the normal *or* radius vector.

Thus to the present degree of approximation the displacement is along the normal, that is in the line of action of the applied force, when

$$\eta=0.$$

The nature of the change produced by the pressures in the shape of the thin shell is easily found as follows. It will suffice to treat the outer surface (1), as the results for the inner are of the same character. Denote the radius vector by ρ, i.e. let

$$\epsilon\sigma_i=(\rho-a)/a.$$

Then we easily find

$$\left.\begin{aligned} u/\rho &= C + i(i+1)(\rho - a)(p - p')/(2hE), \\ v \Big/ \frac{d\rho}{d\theta} &= w \Big/ \left(\frac{1}{\sin\theta}\frac{d\rho}{d\phi}\right) = \epsilon a (p - p')/(4nh) \end{aligned}\right\} \ldots\ldots(19),$$

where

$$C \equiv -\frac{1}{a^3 - b^3}\left\{\frac{pa^3 - p'b^3}{3m - n} + \frac{b^3(p - p')}{4n}\right\}.$$

A displacement

$$u = C\rho,$$

be it noticed, would simply change the surface into one similar and similarly situated to itself, and so leave the shape unaltered.

If pressure be applied over the outer surface only, we have

$$u/\rho = -C_1 + i(i+1)(\rho - a)\, p/(2hE) \ldots\ldots\ldots\ldots(20),$$

where C_1 is positive and numerically large compared to the other term on the right. The radius vector is thus everywhere reduced, and the proportional reduction is above or below the average according as

$$\rho \text{ is less or greater than } a.$$

Under the same conditions

$$v = \frac{\epsilon a p}{4nh}\frac{d\rho}{d\theta}, \quad w = \frac{\epsilon a p}{4nh}\frac{1}{\sin\theta}\frac{d\rho}{d\phi} \quad \ldots\ldots\ldots\ldots(21).$$

The resultant tangential displacement is thus along the direction in which ρ *increases* most rapidly; e.g. in a spheroid the tangential displacement is along the meridian, being directed away from the axis of symmetry or towards it according as the spheroid is oblate or prolate.

It is thus obvious that pressure on the outer surface tends to exaggerate the departure of the surface from the truly spherical form.

If, on the other hand, pressure be applied over the inner surface only, we have

$$u/\rho = C_2 - i(i+1)(\rho - a)p'/(2hE) \ldots\ldots\ldots\ldots(22),$$

$$v = -\frac{\epsilon a p'}{4nh}\frac{d\rho}{d\theta}, \quad w = -\frac{\epsilon a p'}{4nh}\frac{1}{\sin\theta}\frac{d\rho}{d\phi} \quad \ldots\ldots\ldots\ldots(23),$$

where C_2 is positive.

Here every radius vector tends to lengthen, but the lengthening is proportionally least where the radius vector is initially greatest; also the tangential displacement is along the direction in which the radius vector *shortens* most rapidly. Pressure on the inner surface thus tends to make the shape more nearly spherical.

In all cases it will be noticed that for a given magnitude in the maxima values of $\epsilon\sigma_i$ the importance of the subsidiary term in u increases rapidly with i. It has been tacitly assumed that the subsidiary terms are relatively small, so that the solution should not, without further investigation, be applied to cases in which $\epsilon i^2\sigma_i$ is anywhere comparable to unity.

The effect of the departure from sphericity on the change of density produced by the pressure is deducible at once from the value of the dilatation Δ. For the thin shell we have from (10_A) and (101_A), putting in the latter

$$F_i = R_i - R'_i,$$

where R_i, R_i' are given by (16),

$$\Delta = -\frac{a^3p - b^3p'}{(m - n/3)(a^3 - b^3)} - \frac{3\epsilon a\sigma_i (p - p')}{(3m - n)h} \quad \ldots\ldots\ldots(24).$$

Thus if ρ as before be the surface radius vector,

$$\Delta = -\frac{(a^3p - b^3p')}{(m - n/3)(a^3 - b^3)} - \frac{3(\rho - a)(p - p')}{(3m - n)h} \quad \ldots\ldots(25).$$

Thence in the case either of internal or external pressure, Δ is numerically largest—i.e. the change of density of the material is greatest—in the neighbourhood of the longest radii vectors.

In the case of nearly spherical shells exposed to bodily forces depending on solid harmonics, or surface forces depending on surface harmonics, the equations determining the values of the constant multipliers in the subsidiary terms required are of the types (36_A), (37_A) or (38_A) to (41_A)—the latter including the case $i = 0$. The quantities on the right-hand sides of these equations, answering to R_i, T_i etc., are deduced in the same way as the corresponding quantities for nearly spherical solids were deduced in my paper in the *American Journal*. The only mathematical difficulty consists in representing products of spherical harmonics, and products of their differential coefficients &c., as simple spherical harmonics or differential coefficients of simple spherical harmonics. The necessary results of this kind for the shell problems analogous to the single surface problems I treated in the *American Journal* already exist there. When the quantities answering to R_i, T_i &c. in (36_A) to (41_A) are known, nothing remains but to substitute them for R_i, T_i, &c. in the general formulæ, such as (91_A).

(3) *Distribution of Solar Radiation, and its dependence on Astronomical Elements.* By R. HARGREAVES, M.A., formerly Fellow of St John's College.

(Printed in full in *Transactions*, Vol. XVI.)

(*Abstract.*)

This investigation was undertaken with the object of providing exact data for the discussion of the influence of changes in certain astronomical elements on the distribution of solar radiation on the surface of the earth. It is well known that secular changes of climate, of such extent as to give rise to the terms Glacial and Genial epochs, have been attributed by some authors to this cause. In view therefore of the interest which this question has for geologists and others, who might be repelled by the mathematical analysis, this is preceded by a somewhat full outline, and numerical results are given in connection with each point.

The paper begins with the expression of the amount of heat due to the earth in any latitude, on the assumption of a diathermanous atmosphere, by means of a harmonic series which is carried numerically as far as the term with one quarter of a year for period. Denoting by H/r^2 the amount of radiation on unit-surface in unit-time exposed perpendicularly to the sun's rays at distance r, the element of heat-supply is

$$\frac{Hd\theta}{\pi h}(L_0 + L_1 \sin\theta + L_2 \cos 2\theta + L_4 \cos 4\theta + \ldots),$$

θ being the orbital angle of the sun measured from the spring equinox.

If the orbit of the earth (or any other planet) was circular θ would be proportional to mean time. The actual conversion to mean time is given later, but various conclusions can be drawn from the present form. The constant h is introduced by the relation $r^2 d\theta/dt = h$; when a year is unit of time, its value is $2\pi ab$. As the minor axis is dependent on eccentricity, but only varies within very narrow limits, the effect of this general divisor is very slight. The coefficients L_0, L_2, L_4 ... are functions of latitude, and obliquity of the ecliptic, the same for both hemispheres, expressed in finite terms by complete elliptic integrals of the three kinds, and also in series of zonal harmonics. The coefficient of the annual variation L_1 has the simple form

$$\frac{\pi}{2} \sin\lambda \sin\epsilon,$$

with opposite signs in the two hemispheres.

On L_0 depends the total annual supply; with no obliquity its value is $\cos\lambda$, vanishing at the pole; as the obliquity is increased the value in low latitudes slowly diminishes, that in high latitudes rapidly increases. The character of the resulting change in distribution is first considered generally, and then with special reference to the limits within which the obliquity is supposed to vary, Stockwell's limits 21° 58′ 36″ and 24° 36′ being used. Within these limits the total range of L_0 amounts at the equator to ·93 per cent. of its present value, diminishes to zero in lat. 43° 20′; here the sign of the change is reversed and the total range of L_0 increases up to $10\frac{1}{2}$ per cent. at the pole. This alteration affects both hemispheres alike.

For the normal annual amplitude L_1 the total range is in every latitude $10\frac{1}{2}$ per cent. of its value, but this effect blends with changes due to eccentricity, the influence of which appears on transferring to mean time. This transformation is made for the present position, numerical results being tabulated; and also for those positions in which eccentricity has its most pronounced influence in modifying the annual term, viz. when the line of equinoxes is at right angles to the major axis of the orbit, making the inequality in duration of summer and winter greatest.

If these epochs be described as genial and glacial, the first has for element of heat-supply in north latitude expressed in mean time

$$\frac{2H}{h}Q'dt, \text{ where } Q' = L_0 - (L_1 - 2eL_0)\cos 2\pi t \ldots,$$

while for the second $\quad Q'' = L_0 + (L_1 + 2eL_0)\cos 2\pi t \ldots;$

only the more important terms of the coefficient of annual term $\cos 2\pi t$ being here written. Attending only to these terms, in high latitudes L_1 is large, and a high value of eccentricity is required to produce any considerable modification. But L_1 vanishes on the equator and is small in low latitudes, while L_0 has its greatest value there, so that a moderate value of e makes the difference between the two epochs quite considerable. One of these formulæ is derivable from the other by changing the sign of L_1 which is precisely the change made in passing from north to south latitude.

Hence in so far as change of eccentricity is a *vera causa* for glacial epochs it acts in opposite senses in the two hemispheres, making a genial epoch in the one contemporaneous with a glacial epoch in the other. Also apart from the minute effect mentioned at the outset, the action of eccentricity is through the periodic terms, whereas the alterations due to obliquity affect also the non-periodic term. The effects of changes in obliquity of the

ecliptic are small in low latitudes, and well marked in high latitudes; those of eccentricity, with appropriate position of perihelion, are conspicuous in low latitudes, while producing a fair modification in high latitudes.

In each case the effects are so different in high and in low latitudes that an average taken over so large a range as a hemisphere quite obscures their special character.

The question of drawing inferences as to temperature is rendered extremely complex by a variety of modifying conditions, of which the principal is the action of the ocean and atmosphere as carriers and reservoirs of heat. Apart from these, a solution for temperature may be easily obtained, but it is important to observe that it is necessary to take non-periodic and the various periodic terms separately, these being affected with different multipliers in the integration, and the latter subject each to a different modification of phase. This may render the comparison of terms such as midsummer and midwinter heat with the corresponding temperatures seriously fallacious. It appears to be important in every case to compare non-periodic only with non-periodic under other conditions, amplitude of annual term with amplitude of annual, and so on.

In order to estimate the effect of absorption, the transmitted part is assumed to be equal to

$$e_0 + e_1 \cos I + e_2 \cos^2 I + \dots,$$

where I is angle between the rays and the normal to the surface. This seems suited to express an absorption increasing as the sun's altitude diminishes, with considerable generality, and has the advantage that each term is exactly integrable in the manner of the diathermanous case.

This integration is effected, and some results tabulated, from which it appears that in low latitudes the periodic terms are exaggerated in comparison with non-periodic, the effect tailing off as the pole is approached; the amount of the absorption being of course much less for low than for high latitudes. This relative alteration ought to be taken into account in drawing conclusions as to temperature.

Monday, February 10, 1896.

PROFESSOR J. J. THOMSON, PRESIDENT, IN THE CHAIR.

Professor H. Marshall Ward, Sc. D., Christ's College, and W. H. Duckworth, B.A., Fellow of Jesus College, were elected Fellows of the Society.

The following communications were made:

(1) By Dr LAZARUS BARLOW.

The author exhibited an osmometer designed for the estimation of the initial rate of osmosis and a modification of the same suitable for the examination of small quantities of fluids. The principle of both is the same, viz. that the stem of the osmometer is kept horizontal, and the level of the fluid inside the osmometer is the same as the level of the fluid outside the osmometer. The larger instrument is fully described in the *Journal of Physiology*, vol. XIX. 1896, p. 140 and foll.; the smaller instrument differs from it in that it is immersed in water until the stem of the osmometer just shows above the surface. By these instruments the author found that the initial rates of osmosis are not indicated by their final osmotic pressures, or in other words by their freezing points; thus, to give an example, with a copper-ferrocyanide membrane the initial rate of osmosis of a decinormal solution of urea was half as great again as the rate of osmosis of a decinormal solution of glucose, whereas urea and glucose being indifferent substances, decinormal solutions of each have the same freezing point and the same final osmotic pressure. The object for which the research was performed was to see whether it is justifiable to apply deductions from the freezing point of fluids existing in the animal body to the question of lymph formation. Since the freezing point determinations give no indications as to the relative amounts of osmosis of two solutions of different composition at pressures possible within the animal body, *i.e.* approximately at atmospheric pressure, application of the laws of osmosis which are true at final osmotic pressures are not justifiable in the case of the animal body where such pressures can at best but rarely occur.

(2) *On the communication between peritoneal cavity and renal veins through the nephrostomial tubules in the frog (R. temporaria).* By EDWARD J. BLES, B. Sc.

The discovery of nephrostomes in the kidneys of adult Amphibia was announced by Spengel in his fine paper: *Das Urogenitalsystem der Amphibien*, published in 1876[1]. In this he showed clearly that the nephrostomial tubules in Urodeles and in Cœcilia have a lumen persistent throughout life and always continuous with the perivisceral space at one end and with the lumen of the neck of the Malpighian body at the other end. Semper[2] and Balfour[3] had both described in 1874 a similar state of things in the Wolffian bodies of Elasmobranch embryos, and Semper[4] gave a list of Elasmobranchs in which the nephrostomes remain open in the adult. Thus it was shown that in this respect there is a close resemblance between adults of many species of Elasmobranchs, Urodeles and Cœcilia.

When Spengel, however, examined the adult kidney of Anura, although the nephrostomes were clearly visible, he could not trace a single tubule from them into the neck of a Malpighian body. F. Meyer of Leipzig, who discovered the "stomata," as he called them, independently of Spengel, was equally unable to find the inner opening into a tubule of the kidney[5]. The subject was next investigated by M. Nussbaum who published in 1880 a short paper[6], without illustrations, containing the statement that the nephrostomial tubules in adults of the genera *Rana, Bufo* and *Bombinator* communicate at their inner ends with veins which he identified with renal portal veins. This result was accepted by Wiedersheim both in his *Lehrbuch*[7] and his *Grundriss*[8] *der vergleichenden Anatomie der Wirbelthiere* and in his portion of Ecker's *Anatomie des Frosches*[9]. Still the observations seem to have been met with incredulity and Nussbaum with his pupil Wichmann

[1] *Arb. a. d. zool. zoot. Inst. d. Univ. Würzburg.* Bd. III. (1876), p. 1. A short notice had already appeared early in 1875 in *Centralbl. f. d. med. Wissensch.* No. 23.

[2] "Ueber die Stammverwandtschaft der Wirbelthiere und Anneliden." *Centrbl. f. d. med. Wiss.* 1874. No. 35.

[3] *Q. J. M. S.* 1874. Vol. XIV. N. S. p. 357.

[4] "Segmentalorgane bei ausgewachsenen Haien." *Centrbl. f. d. med. Wiss.* 1874. No. 52.

[5] Fritz Meyer. "Beitrag zur Anatomie des Urogenitalsystems der Selachier und Amphibien." *Sitzber. d. Naturf. Ges. zu Leipzig.* 1875. p. 38.

[6] "Ueber die Endigung der Wimpertrichter in der Niere der Anuren." *Zool. Anz.* III. Jahrg. p. 514.

[7] 2te Auflage. 1886. p. 758.

[8] 2te Auflage. 1888. p. 355. English ed. 1886. p. 307.

[9] 1882. p. 41.

paid further attention to the subject. Then, in 1886, they published a paper[1] accompanied by drawings confirming, extending and in one point correcting the earlier results. The figures show that in *Rana fusca, Rana esculenta, Bufo calamita* and *Alytes obstetricans* the nephrostomial tubules open into veins. Nussbaum correctly identifies the veins as the roots of the *vena cava posterior* and hence renal veins, not renal-portal. Figures are also given showing how the nephrostomial tubules in young *R. fusca* larvæ open into the neck of the Malpighian body or into Bowman's capsule just as they do in adult Urodeles, and showing a stage where the tubules end blindly after losing the above connection, and finally a later stage with the venous communication. In spite of this, C. K. Hoffmann in the same year[2] asserts that in adult *Rana, Bufo* and *Alytes* the tubules in question end blindly, but confirms the communication with the neck of the Malpighian body in larvæ.

In the course of researches which were carried out by the late Prof. A. M. Marshall and myself[3], we were able to confirm Nussbaum's results and extend them to the English *R. temporaria*. The adult kidney has a number of nephrostomial tubules which appear to end blindly, but there are others which have distinct inner openings into the renal veins.

In this form, generally considered as identical with *R. fusca*, the development did not follow the same course. A rod of cells given off from the segmental tubule in its early solid condition grows towards the peritoneum, fuses with it and acquires a lumen from the peritoneum inwards. Before the lumen has reached the inner end of the tubule this end loses its connection with the segmental tubule and the nephrostomial tubule ends blindly. This occurs in tadpoles 17—18 mm. long, but in 21 mm. tadpoles the tubule has acquired its connection with the renal vein.

The course of development is evidently cœnogenetic in this species as compared with *R. fusca*, as it is in other respects in comparison with *R. esculenta*. This divergence, therefore, cannot invalidate Nussbaum's results and Hoffmann's confirmation of them, and Prof. Marshall did not hesitate to incorporate the main facts into his *Vertebrate Embryology* (p. 192).

Nevertheless there are still many who refuse to acknowledge the existence of so direct a communication between body-cavity and the circulatory system, and for that reason I considered

[1] "Zur Kenntniss der Nierenorgane." *Arch. f. mikr. Anat.* XXVII. 1886. pp. 466–9 and plate 23.

[2] "Zur Entwicklungsgeschichte der Urogenitalorgane bei den Anamnia." *Z. W. Z.* Vol. XLIV. p. 594.

[3] The development of the kidneys and fat-bodies in the Frog. *Studies Biol. Lab. Owens Coll.* Vol. II. 1890. p. 133.

it worth while to exhibit before this Society specimens to demonstrate the point.

A kidney of a common frog (not one year old) was hardened in Hermann's fluid, imbedded in paraffin after gradual impregnation in xylol and paraffin, cut in a series of sections and stained on the slides either with aniline blue-black or by the Biondi-Heidenhain (acid) method. The latter gives plasma well stained with fuchsin and blood corpuscles differentiated with methyl green. A bunch of flagella can be seen protruding through the inner opening of the tubule into a space lined with squamous endothelial cells, and either containing blood corpuscles or standing in open communication with similar spaces on the adjoining sections which do contain blood corpuscles. These vascular spaces can easily be traced into the renal veins. On all four sections shown the peritoneal opening of the tubule could also be seen. On one section a nephrostomial tubule could be traced from the peritoneum and running towards the flagellate neck of a Malpighian body, but just short of reaching this opening into a narrow space lined with endothelium and containing a blood corpuscle, the space being continuous with venous spaces on neighbouring sections.

These facts do not stand alone as isolated instances of a channel of communication between cœlom and vascular system. Mr Shipley read a paper before this Society in 1888 (*Proc. Phil. Soc.*, vol. VI. p. 218) in which he cites, with the exception of M. Nussbaum's, the cases occurring in the animal kingdom of such a communication.

Some of these cases still hold good and amongst them certainly the communication through the peritoneal stomata of the diaphragm in mammals from the great abdominal serous cavity through the lymphatic spaces and ducts into the great veins.

Physiologically these lymph channels can be compared with what exists in the above-mentioned five forms of Anura. The perivisceral cavity is a lymph space continuous with the cisterna magna lymphatica and passing lymph into it, and there are reasons for thinking that the peritoneal lymph can also pass into the renal veins through the nephrostomes.

Anatomically considered, the adult *Rana fusca* can be easily brought into line with the other Amphibia with respect to its nephrostomial tubules when it is borne in mind that the larva in its ontogeny passes through stages in which the tubules have successively relations corresponding (1) to the tubules in adult Urodela, (2) to an indifferent condition still persisting in part in other adult Anura, viz. ending blindly, (3) to a condition which occurs in at least four other forms of Anura and easily derived from the indifferent condition.

(3) *On the effect of water currents on the assimilation of aquatic plants.* By FRANCIS DARWIN, M.A., F.R.S., and D. F. M. PERTZ.

The principal fact established by our research is the notable increase in the gas given off by certain aquatic plants in the process of assimilation, when the water in which they are cultivated is disturbed by stirring.

The fact is of interest as bearing on our general knowledge of the gaseous interchange in aquatic plants. It also bears on the nutrition of these plants in still and running water. Moreover it is of interest on the question of method, *i.e.* on the question of how far the yield of gas represents assimilative activity.

This last point is illustrated by the history of our research which began with a chance observation made a few years ago.

If a small quantity (about 5 p.c.) of alcohol is added to the water in which a sprig of *Elodea canadensis* is bubbling under the influence of light, the rate of bubbling at once shows a considerable temporary increase. It becomes about twice as quick, and returns to its normal pace in four or five minutes.

It seemed possible that the plant was stimulated by the alcohol, but it appeared more probable that the result was physical rather than physiological. We therefore fitted up a simple apparatus by means of which air was forced by the pressure of a column of water to escape through a capillary tube (0·2 to 0·3 mm.) into a beaker of water. The air ceased to escape from the tube while the column of water showed a pressure of about 20 cm. When alcohol was added in such a way as to mix with the water in the beaker close to the opening of the tube, air at once escaped from the capillary and the water column fell several cm. This result obviously depends on the fact that the surface-tension of a mixture of alcohol and water is less than that of water.

To discover whether the effect of alcohol on water-plants also depended on diminished surface-tension at the point of escape of the bubbles, it was necessary to eliminate the possible stimulative effect of alcohol on the leaves. This was done by Devaux's method[1]. The tube of a funnel was cut short below the conical part, and in this opening the stalk of a *Hottonia* was held by a plug of cotton-wool, so that the cut end projected into the expanded part of the funnel. Melted gelatine was then poured in from above, which became solid rapidly and thus closed the funnel below with a water-tight plug through which the stalk passed. The main body of the plant could now be immersed in a beaker of water, while the bubbles from the cut end of the stalk escaped into water contained in the funnel. When alcohol (5 p.c.) was

[1] *Annales des Sciences Naturelles*, IX. 1889. p. 67. In our experiments the plant is in a reversed position to that figured by Devaux.

added to the water in the funnel the rate of bubbling became about eight times as quick, returning after two minutes to about its previous rate.

p-m.	Rate	
2.21	14	
2.23	14	5 p.c. spirit added
2.25	111	
2.27	18	

A similar striking effect was obtained by adding 10 p.c. after a few minutes. Later in the day the effect was not so marked, the rate of bubbling was doubled when alcohol was added to the water in the funnel. The result is what might have been expected since the escape of gas varies with the pressure at the cut end of the stalk. This well-known fact may be simply demonstrated by cementing the cut stalk with gelatine into the lower end of a vertical tube which is filled with water to various heights by means of a side opening and rubber tube, and thus exposes the cut end of the stalk to varying pressures.

The following table shows a diminution in, and final cessation of, the bubbling as the pressure rises, and a rise as it is allowed to fall again.

27th June, 1895. *Elodea canadensis.*

Time	Pressure	Rate	Temperature
12 – 12.5	15 cm.	66	
7 – 10	24 cm.	41	21·4 C
12 – 16	30 cm.	50	21·4
19 – 25	51 cm.	33	21·4
29 – 38	64 cm.	14	21·4
39 – 51	80 cm.	0	21·3
52·5 – 59	64 cm.	15	21.3
59·5 – 1.3	51 cm.	30	
3·5 – 7·5	30 cm.	71	21·2
8 – 13	15 cm.	166	21·2

Having shown that alcohol added to the water in the funnel increases the rate of bubbling by diminishing the surface-tension, we expected to find that alcohol, added to the beaker in which the leaves were, would produce either no effect or less than the normal effect. It turned out however that the addition of alcohol to the beaker produced nearly the usual effect. This proved that the accelerating effect produced by alcohol when the cut stalk and the leaves are in the same vessel of water is not simply due to diminished surface-tension, although this cause undoubtedly has some effect.

At this point Mr Blackman made the useful suggestion that water should be added instead of alcohol. It was found that this treatment greatly increased the rate of bubbling, and it at once became clear that this result and the chief part of the alcohol effect were due to the currents set up in the water by the act of pouring in the fluid, water or alcohol as the case may be. This was confirmed by gently stirring the surface-water with a rod, until, by the movement of small particles floating in the water, it could be seen that the lower strata were in rotation.

Under this treatment the rate of bubbling at once became nearly doubled. It should be noted that the plant was not visibly shaken or moved, so that the effect was not due to the bubbles being helped to escape by mechanical shock. This point is worth mention, because a blow on the table was found to cause a slight escape of gas from the glass capillary of the above described apparatus at a pressure which had ceased to produce a flow of bubbles.

Method.

The experiments were made in a dark room illuminated by one of the Incandescent Gas Company's burners, in which a white-hot mantle, surrounding an argand flame, gives a brilliant steady light. Between the gas and the vessel in which was the bubbling plant we placed a glass trough through which a current of water flowed. In this way the plants could be kept at a temperature sufficiently constant.

The plant to be experimented on was tied to a glass rod supported vertically in a beaker of water. In all the earlier experiments the yield of gas was estimated by taking with a stop-watch the time which elapses during the evolution of a given number of bubbles. The reciprocals of these times will be proportional to the rate of bubbling. The "rates" given in our experiments are simply derived from the reciprocals by moving the decimal place. The "rates" are comparable throughout any given experiment, but the rate in one experiment is not necessarily comparable with that in any other.

In our later experiments the gas yielded during half-an-hour was collected and measured in the modification of Timiriazeff's Eudiometer described and figured in Darwin and Acton's *Practical Physiology*[1].

As the times involved were short, and the temperature remained fairly steady during the experiments, we dispensed with barometric and thermometric corrections, and we believe that the readings of the eudiometer are fairly comparable *inter se* during any given experiment: although they certainly do not give absolute amounts with accuracy. It will be seen that the variations in the yield of gas as given by the eudiometer do not always agree with the rates obtained by timing the bubbles given off. When this is the case, we are inclined to trust the eudiometer readings rather than those of the stop-watch.

To stir the water uniformly, we hung a microscopic slide to an ordinary bottle-jack, such as is used for roasting meat. The glass slide makes five turns in about four seconds, and then reverses and rotates in the opposite direction.

Experiments with Elodea.

In the following experiments we have given the *average* rate of bubbling during periods in which the water was alternately left quiet and disturbed. Where the amount of gas yielded during the same period was measured the rate of yield per minute is given in terms of a graduation of the eudiometer: each graduation is equal to 0·01 c.c., and is of such a length that the quantity of gas can be roughly read to 0·002 c.c.

A single experiment is given in full, in order that it may be seen that the effect of stirring is fairly well kept up for a period of half-an-hour.

[1] Edition II. p. 45.

Experiment 1: 5th June, 1895. *Elodea.*

a.m.	Rate[1]		Temp.
10.37	100		15·4
	100		
47	76		
47·5	76		
56	62	Still	
11	66		
7	55		
8	76		
	74		
10	62		15·4
12		Started jack	
12·5	166		
	166		
	200		
13	200		15·7
	200		
14	200		
15	200		
16	166		
18	166		
	166	Stirred	
20	166		
21	142		
23	142		
25·5	166		
27	166		
29	153		
30	41	(Sudden change in rate of	
32	40	bubbling for 3 minutes.)	
33	125		
	43		
35·5	181		
	181	Stopped jack	16
37	153		16·1
38	111		
40	90		
43	83		
46	86	Still	
50	80		
53	76		
	76		
55	76		
12.3	71		16·4

[1] *rate* means rate of bubbling calculated from the readings of the stop watch.

p.m.	Rate		Temp.
12.4		Started jack	16·6
	125		
	142		
	142		
5	166		
	166		
	142		
6	142		
9	166		
	153		
11	153		
15	142		
15·5	153	Stirred	
16	142		
18	142		
	142		
19	133		
21	125		
23	125		
25	117		
26	100		
27	100		
29	100		
30	105		
	117		
30	105		16·8

Experiment 2: 1st June, 1895.

Elodea in Tap-water.

a.m.	Time		Rate	Temp.
11.10 „ 26	16 m.	Still	45	19·4 C.
11.28 „ 53	25 m.	Stirred	71	19·7 20
11.57 12.11	14 m.	Still	27	20 20·1
12.12 „ 27	15 m.	Stirred	31	20·2

Experiment 3: 4th June, 1895.

Elodea in Tap-water.

a.m.	Time		Rate	Temp.
10.5 „ 45	40 m.	Still	40	16·1 C. 16·4
10.47 11.13	26 m.	Stirred	50	16·5 16·7
11.13 „ 53	40 m.	Still	16	16·8 16·9
12.7 „ 17	10 m.	Stirred	30	17 17·1
12.17 „ 37	20 m.	Still	13	17·2
12.42 „ 54	12 m.	Stirred	29	17·3 17·4

Experiment 4: 5th June, 1895.

Elodea in Tap-water.

a.m.	Time		Rate	Temp.
10.37 11.10	33 m.	Still	76	15·4 C.
11.12 „ 29	17 m.	Stirred	166	15·4
11.30 „ 32	2 m.	„	40	15·8
11.33 „ 35·5	2·5 m.	„	100	16·1
11.37 12.3	26 m.	Still	90	16·2
12.4 „ 37	33 m.	Stirred	142	16·6 16·8
12.38 „ 55	17 m.	„	34	17
12.55 1.3	8 m.	Still	22	17·1

Experiment 5: 6th June, 1895.

Elodea in Tap-water.

	Time		Rate	Temp.
9.33 10.26	53 m.	Still	76	13·6 C. 14·2
10.29 11.13	44 m.	Stirred	111	14·5 14·8

Experiment 6: 10th October, 1895.

Elodea. Fresh Tap-water.

a.m.	Time		Rate[1]	Quantity[2]	Temp.
10.35 11.5	30 m.	Still	18	·13	18 C.
11.11 „ 41	30 m.	Stirred	40	·27	18
12 „ 30	30 m.	Still	15	·15	18

Experiment 7: 12th October, 1895.

Elodea. Fresh Tap-water.

a.m.	Time		Rate	Quantity	Temp.
11.20 „ 51	31 m.	Still	14	·17	18 C.
11.54 12.26	32 m.	Stirred	30	·35	18
12.26 „ 50	24 m.	Still	6	·25	18

[1] *rate* means rate of bubbling.

[2] *quantity* means amount of gas collected, expressed in eudiometer-graduations per minute.

Experiment 8: 21st November, 1895.

Elodea. Old Tap-water.

a.m.	Time		Rate	Quantity	Temp.
10.40 11.15	35 m.	Still	52	·10	15 C.
11.20 „ 50	30 m.	Stirred	45	·14	15·5
11.55 12.25	30 m.	Still	30	·08	16
12.29 1.1	30 m.	Stirred	28	·11	

Experiment 9: 9th October, 1895.

Elodea in Fresh Tap-water.

a.m.	Time		Rate	Quantity	Temp.
10.24 „ 40	16 m.	Still	19	·31	16·5 C.
10.48 11.3	15 m.	Stirred	25	·58	16·5
11.17 „ 33	16 m.	Still	12	·36	16·5
11.39 „ 54	15 m.	Stirred	77	·48	
12.4 „ 19	15 m.	Still	45	·31	16·5
12.30 „ 45	15 m.	Stirred	22	·50	16·5

Experiment 10: 20th November, 1895.

Elodea in Tap-water which had stood 24 hours.

a.m.	Time		Rate	Quantity	Temp.
10.23 11.13	50 m.	Still	16	·08	15 C.
11.17 12	43 m.	Stirred	28	·10	15·5
12 „ 39	39 m.	Still	17	·06	16
12.39 „ 50	16 m.	Stirred	22	·12	16·5

Altogether eleven experiments were made with *Elodea*.

The results are fairly uniform, that is to say, the rate of bubbling as taken by the stop-watch is nearly always markedly increased. Exceptions occur, however: for instance, in the "stirred" periods of Experiment 8 stirring did not increase the rate of bubbling, but in both cases the amount of gas actually yielded was greater. The same thing is true of the final "stirred" period of Experiment 9. Indeed there were no exceptions to the statement that the actual yield of gas per minute was greater when the water was stirred. We are inclined to believe that the results obtained by measuring the gas are trustworthy because the amount of effect due to change of condition does not vary greatly. The following figures give, in round numbers, the effect of change from the "stirred" to the "still" condition.

180 : 100
140 „
175 „
161 „
155 „
165 „

Experiments with Hottonia palustris.

Only two experiments were made with this species: the results agree with those obtained for *Elodea*.

Experiment 11: 10th June, 1895.

Hottonia.

a.m.	Time		Rate	Temp.
11 „ 35	35 m.	Still	43	16·5 C.
11.38 12.8	30 m.	Stirred	71	17 17·4
12.8 „ 12	4 m.	Still	37	

Experiment 12: 15th August, 1895.

Hottonia. Old Tap-water.

p.m.	Time		Rate	Quantity	Temp.
12 „ 29	29 m.	Still	21	·22	24 C.
12.30 „ 51	21 m.	Stirred	35	·32	

Experiments with Potamogeton bilupsii.

This plant, which is said to be a cross between *P. coriaceus* and *P. plantagineus*, happened to occur in convenient quantities in the Botanic Garden and was therefore used.

The results were of a conflicting character: in two experiments in which freshly-drawn tap-water was used, the stirring undoubtedly produced an increase in the yield of gas. But in the other ten experiments for which water was used which had stood in an open vessel for twenty-four hours, the stirring produced either increase or decrease or else was without effect on the yield of gas.

Cause of the phenomenon in Elodea.

There can be no doubt that the increased yield of gas when the water is stirred is simply due to the movement of the water bringing the leaves rapidly in contact with fresh layers of water and thus hastening the process of diffusion. It illustrates in fact the same phenomenon as the well-known experiment in which a strong solution of a coloured salt (*e.g.* potassium bichromate) is covered by a layer of water in order to demonstrate the extreme slowness with which the yellow colour spreads to the upper layers if the jar is undisturbed.

It is clear that the effect of stirring on water-plants is a physical phenomenon in the sense that it is not necessarily connected with the act of assimilation.

This is proved by taking advantage of the fact that aquatic plants placed in water highly charged with gas give off bubbles in the dark[1].

We observed this form of bubbling by using a feeble illumination just enough to enable us to count the bubbles, but quite insufficient for assimilation. When the water was stirred the rate of bubbling was clearly increased. In other cases where the dark-bubbling had ceased, it was started again by means of stirring.

Assimilation.

The question whether the increase in the yield of free gas produced by stirring necessarily implies increased assimilation is one to which we cannot give a complete answer.

The solution involves a more complete control of the physical conditions than obtained in our experiments, as well as analytical data which we cannot at present supply.

The gaseous exchange of an aquatic plant does not necessarily involve any yield of free gas, it depends primarily on diffusion of dissolved gases both into and out of the plant. The escape of bubbles is, as Devaux and others have pointed out, an accidental occurrence depending on the existence of fortuitous openings into the intercellular spaces. It is clear therefore that the yield of free gas cannot, without further inquiry, be assumed to represent assimilation.

There is, moreover, as already pointed out, an evolution of gas quite independent of the action of light, depending on the gas-pressure in the water. Devaux has explained how it is that, in spite of the above considerations, the rate at which gas is given off does fairly well represent the activity of assimilation.

[1] See Devaux, *Annales des Sc. Nat.*, 1889, p. 38, where the literature of the subject is given.

The oxygen which arises as a bye-product of assimilation is less diffusible than the CO_2 absorbed, and this necessarily leads to a rise of gas-pressure in the intercellular spaces, and to an escape of the internal atmosphere at any openings which may exist. It therefore follows that the more vigorous the assimilation becomes the more rapid is the escape.

We have shown that in some cases at least stirring the water produces an increased yield of gas, there is therefore *prima facie* evidence that assimilation is actually increased. But unless we can ascertain in what way the amount of oxygen yielded *as free gas and by diffusion* is affected by stirring, we cannot be sure of the point.

We have made a few analyses of the gas collected over water from *Elodea*, using water which had stood for twenty-four hours as well as freshly-drawn water, so as to avoid errors from bubbling due to gas-pressure.

Elodea. Feb. 13, 1896. Fresh Tap-water.

			O per cent.
9.58	to 11.34 a.m.	still	22
11.38 a.m.	to 12.43 p.m.	stirred	20

Feb. 17, 1896. Fresh Tap-water.

9.55 to 11.10 a.m.	stirred	19
11.13 a.m. to 1.10 p.m.	still	22

Feb. 12, 1896. Old Tap-water.

9.55	to 11.32 a.m.	still	22
11.38 a.m.	to 12.44 p.m.	stirred	19

Feb. 18, 1896.

10	to 11.23 a.m.	stirred	19
11.26 a.m.	to 12.45 p.m.	still	20

The percentage of oxygen is remarkably low in these cases. Devaux found the percentage of oxygen yielded by *Elodea* in the light to be about 25 p.c., and we have, in earlier experiments, found a similar proportion. On the other hand we have also found *Elodea* yielding no more than 20 p.c. when the evolution of gas was certainly nearly all due to assimilation since it was diminished to a quarter of its former amount by darkening the plant.

Moreover, Devaux has shown that in darkness *Elodea* may give off gas containing only 16 p.c. of oxygen, and we have found that *Hottonia* may only give off 14 p.c. of oxygen. It seems possible therefore that a yield of 19 p.c. of oxygen may be an index of assimilative activity.

With *Hottonia* the few experiments we made were more satisfactory.

Aug. 1. Old Tap-water.

still 23 p.c. oxygen
stirred 22 „ „

Aug. 2. Old Tap-water.

still 23
stirred 23

In the case of *Potamogeton* the results were conflicting, but on striking an average it was clear that no marked effect on the percentage of oxygen was produced by stirring.

It is worth noting that the percentage of oxygen yielded by *Potamogeton* is always much higher than that given by *Elodea* or *Hottonia*, being occasionally as high as 58 per cent. We are unable to suggest a reason for this difference. For Elodea and Hottonia the case stands thus: stirring does not seem markedly to affect the percentage of oxygen, while it increases the yield of gas. If so, the total amount of free oxygen yielded must be increased, and this probably implies increased assimilation.

Although the evidence from gas analysis is not conclusive, we have shown by a direct evidence that assimilation is increased by movement of the water.

This was proved to be the case by cultivating water-plants in still and running water, and observing, by means of Sachs' iodine-test, the relative amounts of starch produced. The plants were de-starched by darkness before being used for the experiment, and care was taken to ensure identity of temperature and illumination. Two trials were made with *Elodea* and one with *Potamogeton*, with identical results; namely, that all the specimens from the running water contained distinctly more starch than those from the still water.

(4) *On a collection of plants from New Britain (Neu Pommern).* By I. H. Burkill, M.A., Gonville and Caius College.

In 1875 Baron A. von Hügel visited several islands in the Pacific, and brought back collections of plants from Ovalau in the Fiji group, Upolu in the Samoa group, from the Duke of York's Island (New Lauenburg), and from New Britain. The last-named island he was unable to visit in person, and had, therefore, to entrust the collecting entirely to his collector. Unfortunately a number of accidents caused the loss of a large part of the collections, and the remaining material only represents a portion of what had been obtained. These collections were subsequently presented to the University Herbarium.

The specimens from Ovalau proved to be all known Fijian forms; so also those from Upolu were, with one exception—a new orchid[1],—known Samoan forms. Our knowledge of the botany of New Britain is very incomplete, and consequently the following enumeration will be found to contain the names of many plants hitherto unknown from this island. At the end of this paper is a bibliographical list to which reference will be made by means of roman numerals.

One early botanist, Commerçon, who accompanied Bougainville in his voyage round the world, came very near to landing on New Britain; but the island, called New Britain in the narrative of the voyage, is in reality New Ireland. The part of New Ireland visited by him is in the south, or New Britain end of the island—a district, Bougainville says, 'not very rich for the botanist'[2]. Commerçon's collections, including more than 3000 species from all parts of the world, which he never lived to bring to Europe, lie in the Paris Museum; but it must be remembered that it is erroneous to refer to any of them as from New Britain.

The north end of New Britain, constituting the Gazelle Peninsula, though not yet completely explored, is known to be a very rugged region with steep volcanic mountains, rising to considerable heights in Mts Fitzgerald and Beautemps Beaupré. On the north coast of this peninsula lies Blanche Bay, in the neighbourhood of which these plants were collected. The bay lies under a range of hills rising to 2000 feet and containing an active volcano. Our knowledge of the botany of the island is practically confined to its vicinity. The interior, as indeed the greater part of the coast, remains a *terra incognita*.

New Britain is by no means isolated. The channel between it and New Ireland (New Mecklenburg) is only 30 miles wide, and in it lies the Duke of York's Island (New Lauenburg) with its attendant islets: through this long island of New Ireland we have a land-link to the island of New Hannover, and so to the Admiralty Isles. To the South-east at no very great distance lie the Solomon Isles; but unfortunately the island of Bougainville, which is the nearest, is the least known. The western extremity of New Britain approaches very near the coast of New Guinea, and the gap between the two islands is bridged over by a number of islets. From its geographical position, we should then expect New Britain to possess a flora of a Papuan type, and such indeed the fauna seems to be[3].

[1] Rolfe, *New Orchids. Decade* 17. *Kew Bulletin of Miscellaneous Information*, 1896.

[2] *A Voyage round the World.* English edit., London, 1772, p. 330.

[3] Wallace, *Australasia.* London, 1888, p. 469.

ENUMERATION OF SPECIES

obtained in the vicinity of Blanche Bay.

Pteridophyta.

Asplenium Nidus L.
Asplenium falcatum L.
Nephrodium latifolium Baker.
Nephrolepis acuta Presl.
Polypodium adnascens Sw.
Polypodium sinuosum Wall.
Polypodium Linnæi Bory.
Lycopodium carinatum Desv.
Selaginella canaliculata Baker.

Angiospermæ.

Monocotyledones.

Panicum javanicum Poiret.
Centotheca lappacea Desv. (recorded xviii.).
Mariscus Sieberianus Nees.
Calamus sp.
Cordyline terminalis Kth. var *Ti* Baker.

Alpinia (Hellenia) oceanica mihi (*A. nutans* K. Schum. non Rosc.) (recorded xviii.). Professor Schumann has described this species, although not yet found in New Guinea, in the flora of Kaiser Wilhelms Land[1], and, supposing it to be a form described by Rumphius, on which Linnæus founded his species *Globba nutans*, named it *Alpinia nutans*. I cannot agree with him in this complicating of the synonymy. The matter may be sifted somewhat as follows. In the *Herbarium Amboinense*[2] Rumphius describes five species of *Globba*, of which the last *Globba sylvestris* was found in two forms; these he distinguishes as *Globba sylvestris major* and *Globba sylvestris minor sive florida*[3]. Linnæus did not see even varietal difference between them, and called *Globba sylvestris* of Rumphius *Globba nutans*[4]. Subsequently Roscoe[5] thought his *Alpinia nutans* to be *Globba nutans* L., the two being in size and habit somewhat similar, but probably erred. Miquel[6] saw in *Alpinia gigantea* Bl. one element of Linnæus' species,

[1] Schumann u. Hollrung, *Flora von Kaiser Wilhelms Land*. Berlin 1889, p. 28.
[2] Amsterdam 1750. Lib. xi. Cap. xxix.
[3] *loc. cit.* Cap. xxxiii.
[4] *Mantissa*, ii. Stockholm 1767, p. 170.
[5] Roscoe in Smith, *Exotic Botany*, ii. London 1805, p. 93, and *Monandrian plants of the order Scitamineae*. Liverpool, 1828, t. 73.
[6] *Flora Indiae batavae*, iii. Amsterdam 1855, p. 605.

namely Rumphius' form *major*; and there are many reasons which make one regard this as correct. Now Schumann sees in his *Alpinia nutans* the other element, namely *Globba sylvestris minor*, —a decision the correctness of which is questionable. The following statements give the reasons for and against Schumann's identification.

Alpinia nutans K. Schum. agrees with *Globba sylvestris minor* in its nodding inflorescence, persistent bracts, leaves less than two palms' breadths broad, and narrow petals. In other points, however, it differs, thus:—leaves oblanceolate instead of lanceolate and not undulate (cf. Rumphius' figure LXIII.), tapering very rapidly above to a shortly acuminate point, flower not conspicuously red, yellow and white. The reader, remembering the difficulties which beset the identification of species of Alpinia, will form his own opinion upon the necessity of caution in this case. The probabilities seem to be in favour of the following decisions being correct.

Alpinia nutans Roscoe (1805), not known till described by Wendland[1] as *Zerumbet speciosum* in 1798.

Alpinia gigantea Blume (1830), apparently the plant figured by Rumphius as *Globba sylvestris major*, and therefore is *Globba nutans* L.

Alpinia nutans K. Schumann (1889) may possibly be, but probably is not, *Globba sylvestris minor*, but if so is also *Globba nutans* L. For this last species I propose the name *A. oceanica*.

Hollrung collected *A. oceanica* on the two islets of Mioko and Kerawara belonging to the New Lauenburg group[2]; Warburg has obtained it from Kerawara and the north of New Britain (XVIII.). In the Kew Herbarium are specimens from the following sources: Quadalcanar, Solomon Isles, Milne, 1855,—Admiralty Isles, Moseley, Challenger' Expedition,—and New Ireland, Barclay[3]. All these localities lie fairly close together, the most westerly being more than 1200 miles from Amboyna.

A. oceanica lies very near *A. scabra* Benth. from British India, and is easily distinguished from other species of the section Hellenia with a nodding inflorescence such as *A. decurva* Ridley.

Baron von Hügel's specimens lead one to suppose that the curvature of the inflorescence becomes more marked as it grows older, being very slight when the buds first begin to open. In cases where the flowers are much crowded the axis of the inflorescence becomes twisted so as to obscure the $\frac{1}{2}$ divergence which is present. Such might account for the want of it in Rumphius' figure.

[1] Schrader et Wendland, *Sertum Hannoverianum*, IV. Göttingen, 1798, 3, t. 19.

[2] *Flora v. Kaiser Wilhelms Land. loc. cit.*

[3] Bentham recognised it as a new species. *Enumeration of plants collected in the Feejee Islands, Tanna, New Ireland and New Guinea.* Hooker's *Lond. Journ. of Bot.* II. (1843), p. 235.

Dicotyledones.

Piper caninum Dietz.
Pouzolzia indica Gaudich. (recorded XVIII.).
Ficus tinctoria Forst.
Amarantus melancholicus L. var. *tricolor* Lam. (recorded from Blanche Bay III.), and also an intermediate form approaching the type.
Celosia cristata L.
Muehlenbeckia platyclados Meissn. (recorded VIII.).
Clematis aristata R. Br. var *Pickeringii* O. Kze.
Indigofera Anil L.
Desmodium umbellatum D.C. (recorded XVIII.).
Uraria lagopoides D.C.
Phaseolus Mungo L. (recorded IX. and XV.).
Vigna retusa Walp. (recorded XVIII.).
Cassia glauca Lam.
Soulamea amara L. (recorded VIII.).
Mallotus ricinoides Müll. Arg.
Macaranga sp. near *M. tanarius* Müll. Arg. ♂ only. The pith has been bored, probably by an ant; the leaves, which are very similar to those of *M. tanarius*, bear on the upper surface near the base 5—7 nectaries. Doubtless it is myrmecophilous.
Cordiœum variegatum Bl. both the variegated form and that with green leaves.
Phyllanthus Finschii K. Schum. (recorded XVIII.). To the known distribution of this plant,—viz. New Guinea (XV.) up to 1800 metres[1], Gazelle peninsula and Mioko (XVIII.),—I can add New Ireland, from whence a specimen collected by Barclay is preserved in the Kew Herbarium. The proportion of ♂ flowers to ♀ must be very high if the trees are equally divided between the two sexes.
Schmidelia littoralis Bl.
Colubrina asiatica Brongn.
Calophyllum inophyllum L.
Triumfetta suffruticosa Bl. (?)
Thespesia populnea Soland.
Abutilon graveolens W. and A.
Urena lobata L. (recorded VIII.) var. *scabriuscula* Mast.
Hibiscus rosa-sinensis L. flore pleno.
Hibiscus tiliaceus L.
Sida rhombifolia L.
Melochia indica Wall.
Dischidia Collyris Wall. (recorded XVIII.).

[1] Warburg, *Bergpflanzen aus Kaiser Wilhelms Land.* Engler's *Bot. Jahrb.* XVI. (1893), p. 14.

Ipomœa Turpethum R. Br.
Physalis minima Don. (recorded III.).
Solanum tetrandum R. Br.
Hemigraphis reptans Engl.

Eranthemum Huegelii mihi n. sp. Caulis fulvide scaber. Folia lanceolata, in petiolos sensim contracta, supra minute scabrida, subtus nervis exceptis glabra, nervis lateralibus utrinque 10—12 arcuatim ascendentibus. Inflorescentia foliis longior, floribus in cymulis, infimis 2—3-floris, supernis 1-floris, dispositis, pedicellis quam bracteis subulatis longioribus, calyci æquilongis. Calyx, apicibus glabrescentibus exceptis, glaber, laciniis anguste linearilanceolatis quam tubo 3—4-plo longioribus. Corollæ tubus longus, lobis oblongis 3—4-plo longior. Antheræ ultra fauces exsertæ.

The leaves have the lamina about 15 cm. in length by 4 cm. in breadth, and resemble considerably those of *Eranthemum malaccense* Clarke; the petiole is less than 2 cm. long: the flower is most conspicuous, having a tube 3·5 to 4 cm. long and lobes upwards of 15 mm. long: the ovary is about 2 cm. long when ripe, and carries 4 seeds: the anthers are exserted to the distance of 3 mm.

Of two inflorescences, in one the lower part carries no developed flowers, and in the other the lowermost flowers appear to be cleistogamic, while the upper ones are chasmogamic. Cleistogamy is known in the Indian members of the genus from the researches of Kurz[1] and Scott[2].

E. affine Warbg. (XVIII.) connects *E. Huegelii* with *E. pacificum* Engl. (III.), but is quite distinct. *E. Huegelii* is a more robust plant than *E. malaccense*. *E. affine* and *E. pacificum* are known from the neighbourhood of New Britain, and *E. malaccense* only from the Western Malayan region. *E. Huegelii* belongs, like all these species here mentioned, to Lindau's genus *Pseuderanthemum*[3].

Eranthemum sp. near *E. laxiflorum* A. Gray.
Tournefortia argentea Forst.
Leucas flaccida R. Br.
Orthosiphon stamineus Benth.
Callicarpa pedunculata R. Br.
Callicarpa eriochlona Schauer. This plant, first described from the Philippine Isles and since found in New Guinea, is by no means identical with *C. cana* L. to which it has been united.

[1] *Dimorphism in Eranthemum. Journ. of Bot.* x. (1872), p. 46.
[2] *Dimorphism in Eranthemum. Journ. of Bot.* x. (1872), p. 161.
[3] *Xantheranthemum und Pseuderanthemum, zwei neue Gattungsnamen d. Acanthaceen. Gartenflora*, 1893, p. 612.

C. cana has a hoary appearance due to stellate hairs which branch once or rarely a second time; while *C. eriochlona* is very densely covered on the stems and below the leaves with a dense wool of tawny shaggy hairs branching 4—8 times or even more.

Morinda citrifolia L.
Benincasa cerifera Savi.
Scævola Kœnigii Vahl.
Wedelia strigulosa D.C.

In concluding it may be well to gather together the shreds of information which we possess on New Britain botany, and see what can be constructed out of them. In the various papers hitherto published we find mention of just over 160 species,—a fact which testifies to our ignorance of its flora. Of these species a considerable portion consists of wide-spread plants common on the Pacific and Malayan coasts, and some such as *Tournefortia argentea* and *Bœa Commersoni* R. Br.[1] which are confined to a coral formation. About two-thirds of the whole number are known from German New Guinea, one-seventh from the little-known Admiralty Isles, a fair percentage from the Solomon Isles, and barely one-half from Fiji. A small number of new species, described from specimens from New Britain, have not yet been discovered elsewhere; but it is very improbable that most of these are endemic forms.

An island which helps to connect three regions such as Australia, Malaya and Polynesia, may be expected to yield interesting results in botanical geography; but at present many years must pass before this is brought about.

A few of the specimens in Baron v. Hügel's collection could not be determined: among them were three species of Ficus which were too imperfect, a *Macaranga* (?) probably new, and some others. Finally, I wish to tender my sincere thanks to the members of the staff of the Kew Herbarium, to Mr C. B. Clarke and to Baron Sir Ferdinand von Mueller for the considerable assistance which they have given me, and most especially to Baron A. von Hügel for the many kindnesses which I have received from him.

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Monday, 24 *February,* 1896.

Professor J. J. Thomson, President, in the Chair.

J. Reynolds Green, Sc.D., Trinity College; Charles Cave, B.A., Trinity College; J. Parkin, B.A., Trinity College, were elected Fellows of the Society.

The following Communications were made to the Society:

(1) *Experiments on Liquid Air.* By Professor Dewar.

(2) *On the Generalization of certain Properties of the Tetrahedron.* By J. Brill, M.A., St John's College.

1. In a paper[1] published in the *Messenger of Mathematics* for last August, I obtained a generalization of some of the properties of the plane triangle with the aid of ordinary algebra. I now propose to utilize the theory of binary matrices to obtain a generalization of the corresponding properties of the Tetrahedron. The methods used will be found to bear a close analogy to those used in a paper printed in the sixth volume of the *Proceedings* of the Society[2].

2. In order to facilitate the working out of the subject, we will adapt the notation used by Hamilton in his theory of quaternions to the theory of binary matrices, which is really the theory of quaternions in a somewhat different form[3].

Let m be a binary matrix whose characteristic equation is

$$m^2 - 2\lambda m + \mu = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

We will define the operators U and W with the aid of the equations

$$Um = \lambda, \quad (Wm)^2 = \mu,$$

where, for the sake of definiteness, we suppose that Wm represents the positive value of $\sqrt{\mu}$.

The operator K will be defined by the equation

$$Km = 2\lambda - m,$$

so that we have

$$m + Km = 2Um \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

We also have

$$m \,.\, Km = Km \,.\, m = 2\lambda m - m^2 = \mu = (Wm)^2 \ldots\ldots\ldots (3).$$

In addition to the above operators, we will introduce an operator V defined by the equation

$$m = Um + Vm,$$

so that we have

$$Km = Um - Vm \ldots\ldots\ldots\ldots\ldots\ldots (4),$$

[1] "Note on the Application of Analysis to Geometry." *Messenger*, xxv. 49—59.

[2] "A New Geometrical Interpretation of the Quaternion Analysis." *Proc. Camb. Phil. Soc.*, vi. 156—169.

[3] It will be seen that K and V correspond to Hamilton's K and V, while U corresponds to his S, and W to his T. Hamilton has no symbol corresponding to L.

and therefore

$$(Wm)^2 = m \,.\, Km = (Um + Vm)(Um - Vm) = (Um)^2 - (Vm)^2.$$

Thus we obtain, for the characteristic equation of the matrix Vm, the equation

$$(Vm)^2 + (Wm)^2 - (Um)^2 = 0,$$

from which it follows that

$$UVm = 0, \quad WVm = \{(Wm)^2 - (Um)^2\}^{\frac{1}{2}} \ldots\ldots\ldots\ldots (5).$$

We also have

$$\begin{aligned}(Km)^2 &= (Um)^2 - 2UmVm + (Vm)^2 \\ &= 2(Um)^2 - 2UmVm - (Wm)^2 \\ &= 2UmKm - (Wm)^2,\end{aligned}$$

or

$$(Km)^2 - 2UmKm + (Wm)^2 = 0,$$

from which it follows that

$$UKm = Um, \quad WKm = Wm \ldots\ldots\ldots\ldots\ldots\ldots (6).$$

Now suppose that we have a second matrix m', whose characteristic equation is

$$m'^2 - 2\lambda' m' + \mu' = 0 \ldots\ldots\ldots\ldots\ldots\ldots (7).$$

Then, in addition to equations (1) and (7), we have an equation involving both m and m', of the form

$$mm' + m'm - 2\lambda' m - 2\lambda m' + 2\nu = 0 \ldots\ldots\ldots\ldots\ldots (8).$$

The equations (1), (7), (8) are said to constitute the catena of relations satisfied by m and m'.

We will introduce one more symbol L, defined by the equations

$$Lmm' = Lm'm = \nu \ldots\ldots\ldots\ldots\ldots\ldots (9).$$

We have

$$\begin{aligned}mKm' + m'Km &= m(2Um' - m') + m'(2Um - m) \\ &= 2mUm' + 2m'Um - (mm' + m'm) \\ &= 2Lmm' \ldots\ldots\ldots\ldots\ldots\ldots (10),\end{aligned}$$

by means of equation (8).

In the sequel we shall sometimes be concerned with the matrix unity. When it is necessary to bring this matrix in evidence we shall denote it by the symbol ω, otherwise we shall treat it in the customary manner. The said matrix has both its latent roots equal to unity, and thus we shall have

$$U\omega = 1, \quad W\omega = 1, \quad K\omega = \omega, \quad V\omega = 0,$$
$$Lm\omega = L\omega m = Um.$$

The catena of relations satisfied by m and m' may be written in the form

$$m^2 - 2mUm + (Wm)^2 = 0,$$
$$m'^2 - 2m'Um' + (Wm')^2 = 0,$$
$$mm' + m'm - 2mUm' - 2m'Um + 2Lmm' = 0.$$

From these we may readily deduce the characteristic equation of a matrix of the form $xm + ym'$, where x and y are scalars. We have

$$\begin{aligned}(xm + ym')^2 &= x^2m^2 + xy(mm' + m'm) + y^2m'^2 \\ &= 2(xm + ym')(xUm + yUm') \\ &\quad - \{x^2(Wm)^2 + 2xyLmm' + y^2(Wm')^2\}.\end{aligned}$$

Thus we obtain

$$U(xm + ym') = xUm + yUm' \ldots\ldots\ldots\ldots\ldots (11),$$

and, as a special case,

$$Uxm = xUm.$$

Thus the operator U is commutative with scalars, and is also distributive in its operation. It follows from equations (2) and (4) that the operators K and V possess the same properties. Further, the formula (11) is capable of generalization by successive application, so that we obtain

$$U\Sigma xm = \Sigma xUm, \quad K\Sigma xm = \Sigma xKm,$$
$$V\Sigma xm = \Sigma xVm \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12).$$

We also have

$$\{W(xm + ym')\}^2 = x^2(Wm)^2 + 2xyLmm' + y^2(Wm')^2.$$

This equation also may be generalized by means of a property of the symbol L, which we proceed to develope. We have

$$\begin{aligned}2L(\Sigma xm \,.\, m'') &= (\Sigma xm)Km'' + m''K\Sigma xm \\ &= \Sigma x(mKm'' + m''Km) \\ &= 2\Sigma xLmm'' \ldots\ldots\ldots\ldots\ldots\ldots (13).\end{aligned}$$

Also, if in (8) we write $m' = m$, it reduces to (1), so that we have

$$Lm^2 = (Wm)^2.$$

Thus we obtain

$$\begin{aligned}(W\Sigma xm)^2 = L(\Sigma xm)^2 &= \Sigma x^2Lm^2 + 2\Sigma xx'Lmm' \\ &= \Sigma x^2(Wm)^2 + 2\Sigma xx'Lmm' \ldots (14).\end{aligned}$$

The characteristic equations of mm' and $m'm$ may readily be deduced from the catena of relations satisfied by m and m'. They will be found to be

$$(mm')^2 - 2mm' \{2UmUm' - Lmm'\} + (Wm \,.\, Wm')^2 = 0,$$
$$(m'm)^2 - 2m'm \{2UmUm' - Lmm'\} + (Wm \,.\, Wm')^2 = 0.$$

Hence we have

$$Umm' = Um'm = 2UmUm' - Lmm' \quad \ldots\ldots\ldots\ldots\ldots\ldots (15),$$

and

$$Wmm' = Wm'm = Wm \,.\, Wm' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16).$$

We must here take notice of a peculiarity of the symbol L, which does not belong to the other operative symbols we have made use of, viz. that $L(mm' \,.\, m'')$ is not equivalent to

$$L(m \,.\, m'm''),$$

nor may we replace $L(mm' \,.\, \omega)$ by Lmm'. This is evident from the definition of L, and may also be brought out with the aid of equation (15), which may be made to furnish us with expressions for the two quantities to be compared.

Finally, we have

$$\begin{aligned} Km' \,.\, Km &= (2Um' - m')(2Um - m) \\ &= 4UmUm' - 2mUm' - 2m'Um + m'm \\ &= 4UmUm' - 2Lmm' - mm' \\ &= 2Umm' - mm' \\ &= Kmm'. \end{aligned}$$

3. The matrices that we shall make use of in this paper will, for the most part, be expressed in a special form. We will suppose p and q to be two constant matrices satisfying the catena of relations

$$ap^2 - 2hp + b = 0,$$
$$aq^2 - 2gq + c = 0,$$
$$a(pq + qp) - 2gp - 2hq + 2f = 0,$$

where a, b, c, f, g, h are scalar constants; and the matrices with which we shall be chiefly concerned will be of the form

$$m = x + py + qz,$$

where x, y, z are scalars.

Now we have

$$aUp = h, \quad aUq = g, \quad a(Wp)^2 = b, \quad a(Wq)^2 = c,$$
$$aL\omega p = aLp\omega = aUp = h,$$
$$aL\omega q = aLq\omega = aUq = g,$$
$$aLpq = aLqp = f.$$

Hence, by means of the first of equations (12), we have

$$aUm = ax + hy + gz.$$

From equation (14) we obtain

$$a(Wm)^2 = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy.$$

If we introduce a second matrix of the given form, viz.

$$m' = x' + py' + qz';$$

then, with the aid of equation (13), we readily deduce

$$aLmm' = axx' + byy' + czz' + f(yz' + y'z) + g(zx' + z'x) + h(xy' + x'y).$$

4. We now come to the establishment of a connection of our analysis with geometry. We will suppose in the first place that our axes of coordinates constitute a right-handed screw system. Next, we will suppose that we have a set of three points whose coordinates are (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), and also a second set whose coordinates are (x_1', y_1', z_1'), (x_2', y_2', z_2'), (x_3', y_3', z_3'). We will consider these two triads of points as determining two planes; and will suppose the origin to be so placed that the cyclical order of the subscripts given above corresponds, in each instance, to a right-handed rotation about that normal to the plane of the triad which is drawn away from the origin. Further, we will write

$$2X = \begin{vmatrix} y_1, & z_1, & 1 \\ y_2, & z_2, & 1 \\ y_3, & z_3, & 1 \end{vmatrix}, \quad 2Y = \begin{vmatrix} z_1, & x_1, & 1 \\ z_2, & x_2, & 1 \\ z_3, & x_3, & 1 \end{vmatrix},$$

$$2Z = \begin{vmatrix} x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \\ x_3, & y_3, & 1 \end{vmatrix},$$

and will suppose that X', Y', Z' denote quantities formed in a similar manner from the coordinates of the points constituting the second triad.

We will now write

$$m = X + pY + qZ,$$
$$m' = X' + pY' + qZ';$$

and, then, by means of the results obtained in the last article, we have

$$a(Wm)^2 = aX^2 + bY^2 + cZ^2 + 2fYZ + 2gZX + 2hXY,$$
$$a(Wm')^2 = aX'^2 + bY'^2 + cZ'^2 + 2fY'Z' + 2gZ'X' + 2hX'Y',$$
$$aLmm' = aXX' + bYY' + cZZ' + f(YZ' + Y'Z) + g(ZX' + Z'X) + h(XY' + X'Y).$$

We will suppose (x_0, y_0, z_0) to be the coordinates of some point lying on the line of intersection of the two planes defined by our two triads of points. Then the equation of any plane passing through the said line of intersection may be written in the form

$$(x-x_0)(X+kX')+(y-y_0)(Y+kY') + (z-z_0)(Z+kZ')=0 \ldots\ldots\ldots\ldots(17).$$

Now consider the cone

$$A(x-x_0)^2+B(y-y_0)^2+C(z-z_0)^2+2F(y-y_0)(z-z_0) + 2G(z-z_0)(x-x_0)+2H(x-x_0)(y-y_0)=0 \ldots..(18),$$

where

$$A=bc-f^2, \quad B=ca-g^2, \quad C=ab-h^2,$$
$$F=gh-af, \quad G=hf-bg, \quad H=fg-ch.$$

To find the values of k corresponding to the two tangent planes to (18) passing through the line of intersection of the planes defined by our two triads, we have to express the condition that the normal through (x_0, y_0, z_0) to (17) lies on the normal cone of (18). This gives us

$$a(X+kX')^2+b(Y+kY')^2+c(Z+kZ')^2 + 2f(Y+kY')(Z+kZ')+2g(Z+kZ')(X+kX') + 2h(X+kX')(Y+kY')=0.$$

If this be expanded, it will be seen that it can be written in the form

$$(Wm)^2+2kLmm'+k^2(Wm')^2=0\ldots\ldots\ldots\ldots(19).$$

If k_1 and k_2 be the roots of (19), we have

$$k_1+k_2=-\frac{2Lmm'}{(Wm')^2},$$

$$k_1k_2=\frac{(Wm)^2}{(Wm')^2};$$

and therefore

$$\frac{1}{2}\left\{\left(\frac{k_1}{k_2}\right)^{\frac{1}{2}}+\left(\frac{k_2}{k_1}\right)^{\frac{1}{2}}\right\}=-\frac{Lmm'}{Wm\,.\,Wm'}.$$

Now k_1/k_2 is one of the anharmonic ratios of the set of four planes constituted by the two planes defined by our two triads and the two tangent planes to (18) through the edge in which they meet. If we replace this ratio by the symbol $e^{2i\theta}$, we have

$$Lmm'=-WmWm'\cos\theta.$$

5. Suppose that

$$(x_1, y_1, z_1),\ (x_2, y_2, z_2),\ (x_3, y_3, z_3),\ (x_4, y_4, z_4)$$

denote the coordinates of four points, so situated that the origin of coordinates lies within the tetrahedron formed by them. We will write

$$2X_1 = \begin{vmatrix} y_2, & z_2, & 1 \\ y_3, & z_3, & 1 \\ y_4, & z_4, & 1 \end{vmatrix}, \quad 2X_2 = \begin{vmatrix} y_3, & z_3, & 1 \\ y_1, & z_1, & 1 \\ y_4, & z_4, & 1 \end{vmatrix},$$

$$2X_3 = \begin{vmatrix} y_4, & z_4, & 1 \\ y_1, & z_1, & 1 \\ y_2, & z_2, & 1 \end{vmatrix}, \quad 2X_4 = \begin{vmatrix} y_1, & z_1, & 1 \\ y_3, & z_3, & 1 \\ y_2, & z_2, & 1 \end{vmatrix}.$$

We will also write Y's with the same subscripts to denote what these expressions become when for the y's are substituted the corresponding z's, and for the z's the corresponding x's. Also we will use Z's with the same subscripts for the expressions obtained when for the y's are substituted the corresponding x's, and for the z's the corresponding y's.

In the above it is to be understood, as before, that our axes of coordinates constitute a right-handed screw system. Also the corners of the tetrahedron are understood to be so placed that the cyclical order of the subscripts in the expression for any X is supposed to be that given by a right-handed rotation about the outward drawn normal to the corresponding face of the tetrahedron.

We have

$$X_1 + X_2 + X_3 + X_4 = 0,$$
$$Y_1 + Y_2 + Y_3 + Y_4 = 0,$$
$$Z_1 + Z_2 + Z_3 + Z_4 = 0.$$

Therefore, if we write

$$m_1 = X_1 + pY_1 + qZ_1,$$
$$m_2 = X_2 + pY_2 + qZ_2,$$
$$m_3 = X_3 + pY_3 + qZ_3,$$
$$m_4 = X_4 + pY_4 + qZ_4,$$

we have

$$m_1 + m_2 + m_3 + m_4 = 0 \quad \text{.....(20).}$$

From this we immediately deduce the equation

$$Km_1 + Km_2 + Km_3 + Km_4 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (21).$$

From (20) we have

$$m_1Km_1 + m_2Km_1 + m_3Km_1 + m_4Km_1 = 0,$$

and from (21) we deduce

$$m_1Km_1 + m_1Km_2 + m_1Km_3 + m_1Km_4 = 0.$$

Adding these two equations, and making use of equations (3) and (10), we obtain

$$(Wm_1)^2 + Lm_1m_2 + Lm_1m_3 + Lm_1m_4 = 0 \quad \ldots\ldots\ldots\ldots (22).$$

For simplicity we will denote the edges of the tetrahedron by the symbols (12), (13), (14), (34), (42), (23), the numbers denoting the subscripts of the coordinates of the points joined by these edges. We shall also denote the angles of the anharmonic ratios corresponding to the edges (12), (13), (14), (34), (42), (23) by the respective symbols α, β, γ, θ, ϕ, ψ. Further, we shall write

$$aX_1^2 + bY_1^2 + cZ_1^2 + 2fY_1Z_1 + 2gZ_1X_1 + 2hX_1Y_1 = P^2,$$

and shall denote by the symbols Q, R, S the quantities corresponding to P when the subscripts 2, 3, 4 are substituted for the subscript 1.

This being premised, we immediately obtain for the interpretation of equation (22), the relation

$$P = Q\cos\theta + R\cos\phi + S\cos\psi.$$

Similarly we should obtain

$$\begin{aligned} Q &= R\cos\gamma + S\cos\beta + P\cos\theta, \\ R &= S\cos\alpha + P\cos\phi + Q\cos\gamma, \\ S &= P\cos\psi + Q\cos\beta + R\cos\alpha. \end{aligned}$$

Eliminating P, Q, R, S from these four equations, we obtain

$$\begin{vmatrix} -1, & \cos\theta, & \cos\phi, & \cos\psi \\ \cos\theta, & -1, & \cos\gamma, & \cos\beta \\ \cos\phi, & \cos\gamma, & -1, & \cos\alpha \\ \cos\psi, & \cos\beta, & \cos\alpha, & -1 \end{vmatrix} = 0.$$

Thus the six angles α, β, γ, θ, ϕ, ψ are connected by a relation exactly similar to Cayley's relation connecting the dihedral angles of a tetrahedron.

If m_5 be the matrix corresponding to any fifth plane, we readily obtain from equations (20) and (21) the result

$$Lm_1m_5 + Lm_2m_5 + Lm_3m_5 + Lm_4m_5 = 0.$$

Hence if χ_1, χ_2, χ_3, χ_4 denote the angles of the anharmonic ratios corresponding to the respective sets of four planes constituted by the given plane, one of the faces of the tetrahedron, and the two tangent planes to (18) through the line in which they meet, we have

$$P\cos\chi_1 + Q\cos\chi_2 + R\cos\chi_3 + S\cos\chi_4 = 0.$$

6. From equations (20) and (21) we have

$$m_1Km_1 = (m_2 + m_3 + m_4)(Km_2 + Km_3 + Km_4).$$

Expanding this we obtain

$$(Wm_1)^2 = (Wm_2)^2 + (Wm_3)^2 + (Wm_4)^2 + 2Lm_3m_4 + 2Lm_4m_2 + 2Lm_2m_3,$$

that is

$$P^2 = Q^2 + R^2 + S^2 - 2RS\cos\alpha - 2SQ\cos\beta - 2QR\cos\gamma.$$

Similarly we should obtain

$$Q^2 = R^2 + S^2 + P^2 - 2SP\cos\psi - 2PR\cos\phi - 2RS\cos\alpha,$$
$$R^2 = S^2 + P^2 + Q^2 - 2PQ\cos\theta - 2QS\cos\beta - 2SP\cos\psi,$$
$$S^2 = P^2 + Q^2 + R^2 - 2QR\cos\gamma - 2RP\cos\phi - 2PQ\cos\theta.$$

By adding these four relations we obtain a symmetrical relation, which may be obtained direct from equations (20) and (21) as follows. We have

$$(m_1 + m_2 + m_3 + m_4)(Km_1 + Km_2 + Km_3 + Km_4) = 0,$$

which on expansion becomes

$$(Wm_1)^2 + (Wm_2)^2 + (Wm_3)^2 + (Wm_4)^2 + 2Lm_3m_4 + 2Lm_4m_2 + 2Lm_2m_3 + 2Lm_1m_2 + 2Lm_1m_3 + 2Lm_1m_4 = 0.$$

Interpreting this we have

$$P^2 + Q^2 + R^2 + S^2 = 2RS\cos\alpha + 2SQ\cos\beta + 2QR\cos\gamma + 2PQ\cos\theta + 2PR\cos\phi + 2PS\cos\psi.$$

We also have a set of equations of the form

$$(m_1 + m_2)(Km_1 + Km_2) = (m_3 + m_4)(Km_3 + Km_4).$$

On expansion this gives

$$P^2 + Q^2 - 2PQ\cos\theta = R^2 + S^2 - 2RS\cos\alpha.$$

The other relations of this set are

$$P^2 + R^2 - 2PR\cos\phi = S^2 + Q^2 - 2SQ\cos\beta,$$
$$P^2 + S^2 - 2PS\cos\psi = Q^2 + R^2 - 2QR\cos\gamma.$$

7. If we write down the first three results of Art. 5 in their symbolical form, we have

$$(Wm_1)^2 + Lm_1m_2 + Lm_1m_3 + Lm_1m_4 = 0,$$
$$Lm_1m_2 + (Wm_2)^2 + Lm_2m_3 + Lm_2m_4 = 0,$$
$$Lm_1m_3 + Lm_2m_3 + (Wm_3)^2 + Lm_3m_4 = 0.$$

From these three equations we readily deduce

$$\begin{vmatrix} (Wm_1)^2, & Lm_1m_2, & Lm_3m_1 \\ Lm_1m_2, & (Wm_2)^2, & Lm_2m_3 \\ Lm_3m_1, & Lm_2m_3, & (Wm_3)^2 \end{vmatrix} + \begin{vmatrix} Lm_1m_4, & Lm_1m_2, & Lm_3m_1 \\ Lm_2m_4, & (Wm_2)^2, & Lm_2m_3 \\ Lm_3m_4, & Lm_2m_3, & (Wm_3)^2 \end{vmatrix} = 0.$$

Thus, taking account of equation (20), we obtain

$$\begin{vmatrix} (Wm_1)^2, & Lm_1m_2, & Lm_3m_1 \\ Lm_1m_2, & (Wm_2)^2, & Lm_2m_3 \\ Lm_3m_1, & Lm_2m_3, & (Wm_3)^2 \end{vmatrix}$$

$$\begin{vmatrix} L(m_2+m_3+m_4)\,m_4, & L(m_2+m_3+m_4)\,m_2, & L(m_2+m_3+m_4)\,m_3 \\ Lm_2m_4, & (Wm_2)^2, & Lm_2m_3 \\ Lm_3m_4, & Lm_2m_3, & (Wm_3)^2 \end{vmatrix}.$$

$$= \begin{vmatrix} (Wm_4)^2, & Lm_4m_2, & Lm_3m_4 \\ Lm_4m_2, & (Wm_2)^2, & Lm_2m_3 \\ Lm_3m_4, & Lm_2m_3, & (Wm_3)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (Wm_2)^2, & Lm_2m_3, & Lm_4m_2 \\ Lm_2m_3, & (Wm_3)^2, & Lm_3m_4 \\ Lm_4m_2, & Lm_3m_4, & (Wm_4)^2 \end{vmatrix}.$$

This result may be written in the form

$$P^2 \begin{vmatrix} -1, & \cos\theta, & \cos\phi \\ \cos\theta, & -1, & \cos\gamma \\ \cos\phi, & \cos\gamma, & -1 \end{vmatrix} = S^2 \begin{vmatrix} -1, & \cos\gamma, & \cos\beta \\ \cos\gamma, & -1, & \cos\alpha \\ \cos\beta, & \cos\alpha, & -1 \end{vmatrix}.$$

We may also obtain similar relations connecting Q and R with S. Thus we have

$$P^2 : Q^2 : R^2 : S^2 :: \begin{vmatrix} -1, & \cos\gamma, & \cos\beta \\ \cos\gamma, & -1, & \cos\alpha \\ \cos\beta, & \cos\alpha, & -1 \end{vmatrix} : \begin{vmatrix} -1, & \cos\alpha, & \cos\phi \\ \cos\alpha, & -1, & \cos\psi \\ \cos\phi, & \cos\psi, & -1 \end{vmatrix}$$

$$: \begin{vmatrix} -1, & \cos\psi, & \cos\beta \\ \cos\psi, & -1, & \cos\theta \\ \cos\beta, & \cos\theta, & -1 \end{vmatrix} : \begin{vmatrix} -1, & \cos\theta, & \cos\phi \\ \cos\theta, & -1, & \cos\gamma \\ \cos\phi, & \cos\gamma, & -1 \end{vmatrix}.$$

(3) *On the absolute Minimum of Optical deviation by a Prism.* By J. Larmor, M.A., St John's College.

When a ray of light crosses a prism in a principal plane, it is of course well known, and easy to verify graphically, that the deviation suffered by the ray is least when it crosses the prism symmetrically. It seems also to be recognized that no deviation smaller than this can be obtained when the ray does not pass in a principal plane; though I have not met with any valid demonstration[1] of this result. The following proof may therefore be worth recording.

The inclinations η and η' of an incident and refracted ray to any plane normal to the refracting surface, for example their inclinations to the principal plane of the prism, obey the law of sines

$$\sin\eta = \mu \sin\eta'.$$

Hence, after passing across a prism, the emergent ray is inclined to the refracting edge at the same angle $\frac{1}{2}\pi - \eta$ as was the incident ray.

When directions are projected on to a spherical surface, let E represent the direction of the edge of the prism, and P, Q those of the incident and emergent rays. Then EP and EQ are each $\frac{1}{2}\pi - \eta$; and if the arcs EPp and EQq are each a quadrant, p and q will represent the projections of these rays on the principal plane of the prism.

[1] The proof quoted by Czapski, *Treatise*, p. 156, from Heath, *Treatise*, p. 31, does not seem to be valid.

These projected rays are refracted across the prism in accordance with the law of sines, the index being not μ but

$$\mu \cos \eta'/\cos \eta, \text{ where } \sin \eta = \mu \sin \eta'.$$

If D denote the actual deviation PQ, and d the projected deviation pq, then, from the isosceles spherical triangle,

$$\sin \tfrac{1}{2}D = \sin \tfrac{1}{2}d \cos \eta.$$

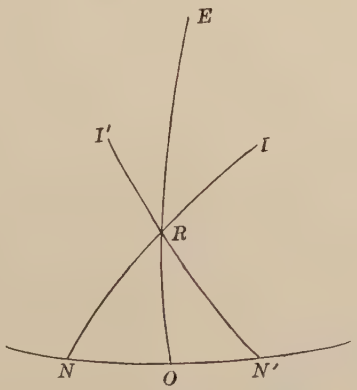
Thus, of all incident rays which have the same inclination $\frac{1}{2}\pi - \eta$ to the edge of the prism, that one has its projected deviation d, and therefore also its true deviation D, least, whose projection passes across the prism symmetrically. This least value is given by the equation

$$\sin \frac{1}{2}(d + A) = \frac{\mu \cos \eta'}{\cos \eta} \sin \frac{1}{2}A,$$

where A is the angle of the prism.

As $\mu \cos \eta'/\cos \eta$ is greater than μ, it follows that d is greater than D_0, the minimum deviation for an actual ray passing in the principal plane: but as it is also greater than D, no inference can be drawn in this way as to the relative magnitudes of D and D_0.

We may however find the absolute minimum of D by comparing with one another the rays, corresponding to different values of η, whose projections cross the prism symmetrically. In the annexed spherical diagram E represents the edge, N, N'



the normals to the faces, I, I' the incident and emergent rays, and R the ray inside the prism. The pole of NN' is E, and EI, EI' are equal; the symmetry of the projection of the rays on the plane NN' requires that ON, ON' shall be also equal,

so that *ERO* is the plane bisecting the external angle between the faces. Thus as $II' = D$, $NN' = A$, we have

$$\frac{\sin \frac{1}{2}D}{\sin \frac{1}{2}A} = \frac{\sin (i - r)}{\sin r},$$

where i, r are the angles of incidence and refraction *IN*, *RN*.

It is easy to see by geometrical construction that the right-hand side of this formula increases with r. It then follows that D is least when r is least, that is, when R is at O: thus the absolute minimum of deviation belongs to the ray passing in the principal plane of the prism.

Monday, March 9, 1896.

PROFESSOR J. J. THOMSON, PRESIDENT, IN THE CHAIR.

Mr R. Hargreaves, late Fellow of St John's College, was elected Fellow of the Society.

The following Communications were made to the Society:

(1) *Notes on the Geological History of Monocotyledons.* By A. C. SEWARD, M.A.

In discussing the relative antiquity of the two great divisions of Angiospermous plants, the statement is commonly made that Monocotyledons occur in older rocks than those containing the earliest traces of Dicotyledons.

The evolution of the Angiosperms and the lines of development of Monocotyledons and Dicotyledons, are among the most interesting botanical problems; and it is of the utmost importance to critically examine the evidence afforded by palæobotany as to the order of appearance of these two classes of plants.

Fragments of parallel-veined leaves and pieces of stem structures have frequently been described as species of Monocotyledonous genera; such determinations, however, as are based on mere external and superficial resemblance, cannot as a rule be accepted as trustworthy.

The cone-like fossils from Jurassic and Lower Cretaceous rocks described under the generic name *Kaidacarpon*, and referred to the *Pandanaceæ* on the grounds of a similarity of form and structure to the inflorescence of recent genera of that family, are most probably Araucarian cones, and should be placed in the genus *Araucarites*.

A case in point is the species *Kaidacarpon minus* Carr. from the Lower Greensand of Potton; this it is proposed to transfer to *Araucarites.*

In the Geological department of the British Museum there are several specimens of fossil stems discovered many years ago in the Iguanodon quarry at Maidstone, and named by König *Dracæna Benstedtii.*

The reasons for making use of the recent generic name are quite inadequate, and it is much more probable that the stems are Cycadean and not Monocotyledonous. In some species of *Zamia,* e.g. *Z. Skinneri,* Warsz, *Z. Loddigesii,* Miq. and others the stem is without the characteristic armour of leaf-bases, and presents an appearance very similar to that of the Cretaceous fossils from Maidstone. The pre-Cretaceous, and some of the Lower Cretaceous fossils described as Monocotyledons, cannot be quoted as trustworthy evidence bearing on the question of Angiospermous development. As yet we are without any definite proof of the existence of either Monocotyledons or Dicotyledons before Cretaceous times, and the statement that the former preceded the latter does not seem to be supported by the facts of palæobotany.

(2) *A description of the Crania found at Girton in* 1881. By R. J. Horton-Smith, B.A., St John's College.

The following Crania of which a short description is here given were excavated from the Anglo-Saxon cemetery discovered at Girton in 1881.

The work of excavation was undertaken by Mr F. J. H. Jenkinson, and a description of the results is to be found in the *Journal* of the Cambridge Antiquarian Society of that year. In the cemetery many crania were found, together with a large amount of pottery of early Anglo-Saxon date.

It would seem that before the Anglo-Saxons made use of this piece of ground as a cemetery it had been devoted by the Romans for a similar purpose. This one might have expected, inasmuch as it lies alongside the old Roman road, and this supposition is supported by the fact that some Roman cinerary urns were discovered at a lower level than those of the Anglo-Saxons; and with these urns were also found a carved lion's head and a military torso, both of Roman workmanship.

All the human remains, however, were found with the Anglo-Saxon urns and other relics, and as no Roman coins were to be seen with the skeletons, we may fairly presume that the skeletons

belong, not to the Romans, but to the Anglo-Saxon race. This fixes the earliest possible date of the cemetery at about 410 A.D., the year in which the Roman legion were recalled from Britain. The latest date possible is not so easy to determine, but inasmuch as no Christian emblems or relics of any description exist in these graves, we shall not in all probability be wrong in assuming that the Anglo-Saxons buried here had not been converted to Christianity. Their conversion took place about 650 A.D.; and hence the date of this cemetery may be said to lie between the years 410 and 650 A.D.

The importance of these skulls lies in the fact that they help us to draw the boundary line between the East Anglians and East Saxons, for we can see by a study of them whether Girton was in the East Anglian or East Saxon territory. As far as one can judge from such a limited number of crania as eight—for these are all that have reached us—I think Girton must be considered as an East Anglian possession.

In a paper to be published shortly in the *Journal* of the Anthropological Institute I have given what I consider to be the typical indices of both the East Anglian and Saxon skulls.

As the paper has not been published yet, I may perhaps be allowed to reproduce them here. By "East Anglian" skulls, I mean skulls which were found, some at Hauxton, some in Cambridge itself.

	Cephalic Index	Height Index	Cranial Capacity	Orbital Index	Gnathic Index	Nasal Index	Kollmann's Upper Facial Index	Kollmann's Total Facial Index	Virchow's Upper Facial Index	Virchow's Total Facial Index	Naso-Malar Index
East Anglians	74	71	1550	85	96	(1) 50 (2) 44	56*	—	78*	129*	108
Saxons	72	70	1480	81	94	(1) 49 (2) 45	54	88·5	72	119	(1) 113 (2) 107
Mean of Girton Skulls	74·4	72·7	1410 (?)	81·8	95·5	48·9	56·7*	90·3*	77·6*	124·7*	108·4

* Mean of Male Indices only.

By glancing at the comparative table given above, it will be seen that the East Anglian skull differs from the Saxon in being broader, rather more lofty, and of greater capacity. Its orbital index is higher and, above all, the face of the East Anglians is distinctly longer than that of the Saxons.

In most of these points the skulls found at Girton show a greater resemblance to the Anglian than to the Saxon type. The two main exceptions are the cranial capacity and the orbital index, where they approach the Saxon type: with regard to the capacity, I would say that the mean index given for these Girton skulls is very doubtful. I have been able to take the capacities of no more than four out of these eight skulls, and of these four one is a female skull. The index given is the mean of the three male crania, two of which differ widely from the third, while No. 272, considering it is a female skull, has in reality a very high capacity. It is therefore quite possible, that if there had been more of these skulls at my disposal I should have found the mean index to be, not 1410, but somewhere about 1550. And unsatisfactory also is the orbital index given. In all cases I examined both orbits where it was possible, and the total number examined was ten. Now considering how widely different the indices are, varying from 73·7 to 92·1, it is obvious that not much stress can be laid on the mean index, which comes to 81·8.

Taking all in all, then, I think it may be said that these crania belong to the East Anglian and not the Saxon race.

Before giving the measurements of the various skulls, I may perhaps make a few remarks on the peculiarities of each.

No. 267. On the right side the Temporal Bone meets the Frontal. The supraorbital notches are extremely large, measuring 3 mm. from above downwards and 6 mm. transversely.

No. 268. Most of the Sagittal Suture is synostosed, and the skull has been posthumously deformed: I have therefore omitted the indices of this skull, when taking the mean indices of the series.

No. 269. In this female skull the Metopic Suture persists.

No. 271. This skull belonged to an adult or rather old male: an epactal bone is present. On the top of the cranium in the middle line is to be found an instance of "prehistoric trephining." The hole is just by the bregma. It is of course hard to say with absolute certainty whether this hole owes its origin to primitive surgical skill, or not, but there is no doubt that the hole was made either on purpose or by accident during life, and judging from the edges it seems probable that the individual survived the operation some little time.

No. 272. Female skull, containing a very large number of Wormian bones. These are to be found in the squamous and lambdoidal sutures of both sides and in the coronal suture on the left side. Epipteric bones exist on either side and the Metopic Suture persists.

No. of Skull	Sex	Cephalic Index	Height Index	Cranial Capacity	Orbital Index		Gnathic Index	Nasal Index	Kollmann's Upper Facial Index	Kollmann's Total Facial Index	Virchow's Upper Facial Index	Virchow's Total Facial Index	Naso-Malar Index (Oldfield Thomas)
					Right	Left							
267	♂	75·7	73·8	1335	77·1	79	95·9	48·5	56·7	90·3	82·2	130·8	107·6
271	♂	74·3	70·4	1525	77·9	76·2	99	50	—	—	71	113·5	106·9
273	♂	77·6	74·9	—	—	73·7	90·9	48·9	—	—	75·6	—	109·8
274	♂	73·1	73·7	1375	—	85·2	95·4	49·1	—	—	81·9	129·8	110·1
269	♀	72·6	72·9	—	92·1	88·6	92·8	47·6	—	—	79·5	—	111·1
270	♀	73·9	—	—	—	—	—	—	—	—	—	—	—
272	♀	73·9	70·4	1480	82·4	85·3	99	49·1	54	—	73·2	—	105·2
268†	♂	67·7	—	—	77·1	—	—	42·2	—	—	87·2	—	—
Mean indices of all except No. 268		74·4	72·7	1410	81·8		95·5	48·9	56·7*	90·3*	77·6*	124·7*	108·4

† Posthumously deformed. * Mean of Male Indices only.

(3) *On the Recurrence of Ice Ages.* By Prof. T. McKenny Hughes, M.A., F.R.S.

Part IV. On some scratched stones from the Permo-Carboniferous rocks of South-eastern Australia and their mode of occurrence.

I have on a former occasion[1] laid before the Society a large number of sections and specimens bearing upon the question of the recurrence of ice ages and discussed the inferences which it appeared to me might be drawn from the evidence. I then laboured under two difficulties; one was that I had no specimens to exhibit in illustration of the glaciation of the basin of the Indian Ocean, and no trustworthy information as to the exact manner of occurrence of the glaciated rocks in it. The other was that a plausible and attractive theory referring the phenomena to extra terrestrial causes was in favour and the conclusions I urged were proportionately depreciated.

Now, however, through the kindness of Professor Edgeworth David of the University of Sydney I am able to exhibit specimens of the striated rocks which he obtained from the boulder beds of South-eastern Australia and photographs of sections in the deposits in which they were found.

[1] *Proceedings Camb. Phil. Soc.*, 1894, Vol. viii., pp. 98 and 219.

Moreover the chief arguments formerly urged in favour of the popular astronomical theory have been found to have been based upon error, and, though some of its supporters still cling to it for auld lang syne, they are waiting till some new facts may turn up in support of it. It seems therefore opportune to urge the terrestrial theory again.

There seems to be an attempt to retreat behind the well-known fact that ice makes ice, or the cumulative effect of refrigeration, especially beyond a certain point. But it seems to be forgotten that this applies equally to terrestrial causes, and that if a mountain range were upheaved, say, a thousand feet above what would be the estimated snow line for that locality, the existence of that snow and ice would so lower the temperature that practically the snow line would creep down far below the altitude estimated from considerations of latitude, longitude and level only.

Further, I must point out in passing that the demonstration offered by Professor David does not provide a bill of indemnity for those who have been claiming credit for the discovery of glacial beds and have offered in proof only slickensided pebbles and similar unsatisfactory evidence. At the Geological Society when Professor David exhibited, among others, these specimens which he has kindly presented to the Woodwardian Museum, a stone was exhibited from the Indian boulder beds. This I had no hesitation in pronouncing to be merely a slickensided pebble, a view in which Professor David entirely concurred. Dr Heim has written to me in answer to enquiries I had addressed to him in regard to the stone I had noticed in the Museum at Zurich and said to have been found *in situ* in the "crystalline boulder beds," or "palæozoic glacial bed" in the Salt range of Sandschal in India. "The striations," he says, "are different on different distinctly cornered sides. That is a phenomenon of quite another order than glacial striation."

As I have before explained, the character which is most common in pebbles which have been subjected to crushing action in a conglomerate, and most clearly distinguishes them from glacially striated rocks is the manner in which they often appear to be pinched out and scored in the same direction on both sides up to the edge; so that, whereas the same set of glacial striæ seldom run far over the margin on the surfaces of any glacial boulder, and flat sides are the rule, in crushed conglomerates, on the other hand, the grooves follow the curvature of the stone, and the edge is sometimes crushed up as if the pebble had been nipped out.

One of these Australian specimens shows furrows due to the weathering of the rock along weaker divisional planes, but these cannot be confounded with the true glacial striæ. We may, I

think, reject the possibility of the scratched stones collected by Professor David being due to earth-movements in a conglomerate.

In order however that we may avoid all possible sources of error we must consider the curious cases of polished floors and striated fragments due to landslips. Of these I exhibited examples[1] from Wales and the south of England on the former occasion. The condition of the surfaces would be in many of these identical with that produced by glacial action. In both cases in order to produce it the stones must be in contact. The striations on fragments from landslips have however a greater tendency to be parallel, whereas in all glacial boulder clays it is seen that the direction of the striæ has been frequently changed, owing to the shifting position of the stones as they have been carried forward in the frozen mass and scrunched against one another on a far longer journey and over a much more irregular surface than in the case of landslips.

Heim and Baltzer attach considerable importance to the difference in character and effect produced by slips of superficial *débris* (Schuttstürze) and falls of solid rock (Felsstürze), and Mr Wickham King recalls examples from the Dumberg landslip near Elm in which the fallen rock produced a bruise from the force of impact and a groove beyond it where the mass was forced forward. Heim points out that when the soil is of a clayey nature the slickensided surface is as smooth as a mirror, with fine parallel striations in the direction of the movement. A still more conspicuous polishing of the surface is always produced upon the solid rock under the slipped mass: and sometimes it is only the solid rock at the base that is polished. The polished surfaces in the slipped mass generally occur in the lowest layers, and sometimes, instead of there being one uniform surface so polished, there are several such planes succeeding one another at various angles. These polished surfaces often extend with remarkable constancy over very wide areas. In the channels cut to carry off the water under the landslip at Villnockenn on the Bozberg railway, which was of small vertical descent but of great breadth it was possible to trace a perfectly smooth surface polished like glass and extending for several hectares (a hectare is something under $2\frac{1}{2}$ acres).

Dr Baltzer mentions that, after the fall and slip of *débris* at Bilten, clearly defined striations, caused by the stones in the sliding mass, were observed on the upper slopes, some of them running in sinuous curves. He also states that ruts, like those made by the wheels of a cart, have been observed after a landslip in the Suabian Alps; and that, after the landslip of Bottstein, the

[1] Nos. 32, 33, 41, *op. cit.* pp. 120, 121.

inclined beds of Opalinthusthone, wherever laid bare, were found to have a perfectly smooth and polished surface[1].

If a mass of slipped earth with scratched stones were found resting on a rounded, smoothed, and striated surface and were consolidated into a compact rock it might be difficult in sections of limited extent to feel sure whether we had not before us an ancient consolidated boulder clay—and, if such a mass were by any peculiar combination of circumstances transported piecemeal and the stones scratched by landslips were dropped in clay, so as to appear as isolated boulders, it would be difficult to pronounce very positively as to its origin.

But the account given of the mode of occurrence of these ancient Australian boulder clays makes it extremely improbable that they can have been produced directly or indirectly by landslips. In no case do they appear to have been formed on or near the flanks of any steep slopes, but when they rest on a rounded and polished and grooved surface of Archæan or Ordovician rock, the direction of the striæ on the solid floor is always from the south, showing that the phenomena are not produced by any merely local agency.

Some of the boulder clays are interstratified in a vast series of sandstones and conglomerates of marine origin, which attain a thickness in places of 5000 feet. This also points to some remote origin and a mode of transport other than currents of water.

The marine deposits in which they occur are identified with the Glossopteris and Gangamopteris beds of Tasmania, South Central Africa, India, and Central America. This gives us no data for an exact correlation with any known horizon in Europe. They may have been laid down at any time in the vast interval between the deposit of our newest Carboniferous beds and the first conglomerates of the Poikilitic series—a period so vast that it may be looked upon as comparable with the time that has elapsed from the age of the New Red to the present day. Those who adopt the very unsatisfactory name Permo-Carboniferous seem to favour this view. There are those however who think that some of them may belong to the same age as part of the Poikilitic or even Jurassic series. It is clear therefore that in the present state of our knowledge the occurrence of glacial deposits in these beds lends us no aid in settling the question of alternate or contemporaneous glaciation of either hemisphere.

But they do afford very strong evidence in favour of the view that somehow or another the incoming and disappearance of glacial conditions is always connected with great earth movements.

Here we have a depth of 5000 feet at least getting silted up

[1] "Ueber Bergstürze in den Alpen." *Neuen Jahrbuch für Mineralogie &c.* 1875 and footnote.

by material carried from adjoining lands where glaciers crept down valleys to the sea. That cannot have been a country totally covered with ice and snow, for the glaciers must have passed through regions where rocks were exposed and broken up into fragments which fell upon the ice. They must have been long glaciers, for there are no scratched stones in short glaciers, in which there is not sufficient time for the fragments to get into the crevasses and work their way down to the stony mass where alone striæ could be produced.

Besides this there must have been land on which ferns and other such plants grew.

It is clearly therefore incorrect to speak of the glaciation of South-eastern Australia. All that has been proved is that somewhere to the south of Australia there was land with glaciers from the end of which icebergs carrying clay and scratched stones drifted north. When the water was too shallow to float them they touched the bottom and grooved the rocks on which they grounded. Afterwards when the subsidence of the area had gone on and the water was deeper they melted and dropped their loads over a sea bottom covered with the *débris* carried down by torrents from lands which had not so severe a climate. Far from being a district of ice and snow, South Australia was then deep down beneath the waters of the sea, with probably a compensating elevation of the land in the adjoining regions.

Sir John Lubbock[1] has pointed out that an examination of the form of continents suggests a great and comparatively recent submergence of the Southern hemisphere. The long pear-shaped promontories pointing south, whether continuous as in the case of Africa or interrupted as in the case of Australia, are according to this view only the backbones of continents plunged deeper and deeper under the water as we follow them to the south. This implies some great earth movement affecting nearly the whole southern hemisphere. But local conditions must have determined what exact effect should follow in each area. For instance if there were an upheaval of 25,000 feet along some transverse fold, would it make no difference whether that affected a portion of the earth's surface which was 20,000 feet deep under water, or already 20,000 feet above the sea level? These are far within observed heights and depths.

Again, supposing any pull upon earth's crust, of such a kind as men have had in their minds when speculating upon the dragging of the waters of the ocean from one hemisphere to another, were to tend to pull a portion of the surface out of place would it make no difference that the trough in which there had been a sea 20,000 feet

[1] *Nature*, 1887. *Geographical Journal*, Dec. 1896.

deep was now filled with solid minerals $2\frac{1}{2}$ times the specific gravity of water—all of which had been transferred from an adjoining continent 20,000 feet high?

In this connection how suggestive is the fact that "all the extreme depths in the ocean are near land or shallow water, and apparently follow the trend of such upheaved parts of the earth's crust[1]."

It is also worth noting that the deepest depressions yet found in the ocean, namely, between 30,132 feet and 30,930 feet, were those proved by Captain Balfour about 500 miles N.E. of New Zealand along approximately the same belt of latitude as that of the depression containing glacial deposits in South-eastern Australia.

If regional movements depend upon terrestrial causes such as denudation and deposition, and if shrinkage of the nucleus and astronomical causes determine only the time when the necessary readjustments shall take effect, it is clear that the general continuity and never the permanence of oceanic and continental areas must follow as a matter of course. Migration along the growing land leaves at the end of a long period a record of similar fauna and flora, continuous, but not contemporaneous. Hence the difficulty of assigning an exact date to the deposits containing the wide-spread Glossopteris and Gangamopteris.

Here we have at last exact observations on an ancient glacial deposit occurring in a region where the conditions are now such that no glaciation is possible, and we find that no glaciation took place in that area on the former occasion either, but only that glaciated stones were carried there.

We have evidence of great earth movements such as would account for all the phenomena over the whole of that region, even down to recent times.

It would be well if astronomers and physicists would cease to consider only the view that the different amount of heat received from the sun alternately in either hemisphere is sufficient to account for the variations of climate which have been experienced by different regions. They are not asked to account for any deformation of the whole mass of the earth, nor for any violent dragging of the topographical poles from the ends of the axis of rotation. What they are now asked to consider is, whether there are no forces which have a tendency to distort and dislocate portions of the surface of the crust; forces which shall be always tending towards a readjustment of equilibrium over those regions where transference of vast masses of material from one area to another has deranged it; which shall have some tendency to

[1] Wharton, W. I. L. *Nature*, Vol. 53, Feb. 27, 1896, p. 393.

produce results upon the great table-lands and protuberant masses analogous to what are called "marginal effects" in earthquakes; which might explain what are called overthrusts, that is, nearly horizontal slides of masses of rock; it may be where the friction has been reduced by the liquefication of deep-seated strata when the pressure is relieved by the pull, or it may be where there already existed molten lakes below, or where fractured and lubricated divisional planes occur in the solid rock.

It is a matter of observation that such movements have taken place. Can they be explained by any well-supported theory of earth movements which shall be consistent with all the other operations which we know have been going on?

If astronomers and physicists would account for the observed periodicity of earth movements by allowing the sun and moon to pull the trigger when denudation and deposition with secondary internal changes have loaded the gun they would be speculating along lines suggested by observation from many different points of view.

(4) *On some Chipped Flints from the Plateau Gravel of Salisbury and elsewhere.* By Prof. T. McKenny Hughes, M.A., F.R.S.

It has long been an accepted fact that flints showing traces of man's handiwork are found in river gravels which from their relation to the physical geography of the district and by the shells and bones found in them are referred to a very remote antiquity. These implements are connected by similarity of form and by the associated remains with others which have been found in caves, where a much larger and more varied assortment of remains of human art and appliances have led to the inference that the man of that period was not at all low down in the scale of civilization but might be compared rather to the Esquimaux of to-day.

Seeing therefore that the earliest men of whom we know anything for certain do not exhibit characters pointing to a low stage of development, we are always looking out for traces of some still earlier race. Many a time has the discovery been announced and primæval man been referred further and further back until some have even assigned a place in this hierarchy to "man's precursors."

I would invite the attention of the Society this evening to the evidence which has been recently brought forward in support of the view that among the flints and in the patches of gravel which occur scattered, both over the beds which rise from below the chalk, and over Tertiary beds which rest on the chalk, as well as over the chalk hills themselves, we have evidence of man's

work of a much earlier date than that of the river gravels in which flint implements have long been recognized. It has been proposed to group these "most ancient stone" implements under a new name, palæotatoliths, or for shortness, palæotoliths.

The subject falls into two parts, (1) the age and origin of the beds in which the flints in question are found, and (2) the character of the flints themselves.

The upper chalk is mostly covered with a thick layer of what is known as "Clay-with-Flints." This consists of the insoluble residue left when, by the action of the surface water charged with carbonic and other acids, the carbonate of lime has been removed. But it is generally the lowest part only of this deposit which is obviously derived directly from the chalk. In it the flints are not worn nor broken, or, if they have been broken, the several parts have not been separated, but all the upper part of the Clay-with-Flints consists of broken and apparently weathered flints in a matrix of red clay. This can only be explained on the supposition that there has been a kind of soil-creep going on for ages. Had it been due to the agency of floods the stones and clay would have been sorted and the deposit would have been more or less stratified. The subsidence necessarily accompanying such enormous subterranean denudation would initiate and keep up the movement of the superficial mass. A rough measure of the amount of such subterranean denudation may be obtained by an inspection of the nearest sections of upper chalk where it will be seen at a glance what an immense thickness of the chalk with flints would have to be removed to furnish, say, 10 feet of the insoluble residuum. Every here and there, especially on its margin, the clay has been all washed out of the "Clay-with-Flints" and carried away, and a universal covering of white-weathered fragments of flint is distributed all over the surface. These flints are also on the move. They creep down the slopes and accumulate on every ledge and terrace. They more obviously travel down the "dry chalk valleys," where also the motion is largely due to subterranean denudation, the water which runs through the porous gravel removing the soft and soluble chalk below it and only appearing at the surface in very wet seasons when there is more water than can pass even through this very open gravel. If there happen to have been any remnants of older superficial deposits left on the hill side, they too start as the underlying chalk gets destroyed, and travel down by all the processes of soil-creep, finally becoming worked into the general superficial mass.

When these various deposits come within the action of either occasional or perennial streams they are handed on more rapidly and are dropped as ordinary river gravels or silts.

Professor Prestwich[1] has given an instance of a sudden outwash due to heavy rain. In this way such superficial deposits have often been carried down and deposited in stratified masses hanging on the hill side.

The relative levels of these gravels may be complicated by contemporaneous or subsequent earth movements, but it will be long before they lose the character they acquired from their mode of origin, as they consist chiefly of surface-weathered flints.

These are all operations going on still, and the recent causes and effects being obvious, we reason back by analogy to the explanation of the more ancient results. Take such a district as that between Six Mile Bottom and Linton.

The higher ground, but not the highest only, is covered with boulder clay. The chalk is rapidly perishing by subterranean chemical and mechanical erosion. The upper chalk furnished the flints which creep down hill by various processes, a large proportion of them being at one time or another exposed on the surface. Finally on the Six Mile Bottom side they creep down the long "dry chalk valley" by Lark's Hall to the railway station, carrying with them from above fragments out of the Boulder Clay, and lower down incorporating the remnants of terraces of river gravel of the age of the Mammoth and Tichorine Rhinoceros.

Over the undulating plateau on top, the stony patches creep outwards and downwards as the inequalities admit of movement.

Down the steep slope by Linton the *débris* slips fast, overlying the patches of boulder clay and still older superficial deposits on the brow of the hill and creeping out over the lower beds of the chalk. If the outcrop of chalk is being eaten back, these stony masses will by and by lie on isolated hills and old terraces far out in front of the new chalk escarpment.

In the country round Salisbury we find very similar conditions prevailing and similar results following. There is an extensive plateau, or rather "plain of marine denudation" over which there are patches of Tertiary loams &c. resting, along the margin of the Hampshire basin, on the upper or flint-bearing Chalk.

On lower ground along the valleys there are terraces of Mammoth gravel. The "trail," consisting of the sweepings of ancient and modern surface soils, has in one place crept off the chalk on to the Tertiary beds, in another has travelled down and got mixed up with the gravel of the old river terraces, which were themselves largely made up of still more ancient surface *débris* with a few sarsen stones, black flint pebbles from the lower

[1] *Quart. Journ. Geol. Soc.*, Vol. xlv. 1889, pp. 278, 279.

Tertiary beds, and boulders of various far-travelled rock which had got distributed over the surface by earlier operations, not always easy to explain after so many changes have taken place.

It is from one of these patches of gravel, resting on a bed of sandy loam of Tertiary age, at Alderbury, three miles south of Salisbury, that most of the flints I now lay before the Society in illustration of my paper were procured. Some of them were collected and given to me by Dr Blackmore, who kindly drove us to the locality and gave us much information about the district. I exhibit also some of the Kentish palæotoliths of Prof. Prestwich and Mr Harrison, kindly lent by Mr Fisher, and some collected by myself, near Cambridge. The flints and fragments show evidence of having been long exposed on the surface. The sandstones are often fretted and eaten away rather than rolled. Some of the flints have a good coating of soft white silica, from which the more soluble colloidal portion has been removed, and they exhibit traces of the innumerable accidental chippings and crushings and breaking up under the influence of frost and sun to which surface-flints are liable; while some were deeply stained by dark red oxide of iron when buried in the gravel into which they had found their way after they had been long knocked about and chipped and weathered on the surface of the ground.

It was clear that the particular beds we were examining were not merely a surface soil formed in place, but a violently transported drift from a chalk surface covered with weathered flints, such a deposit in fact as would be carried down a slope of chalk in storms when the rainfall was greater than the soil could absorb, and ran off in torrents, tearing up the rubbly disintegrated chalk and hurrying along the surface soil and flints. In such a case a chalky gravel is formed with irregular beds of fine marl, the matrix being almost entirely made up of chalk. When this is afterwards exposed to the slow decomposing action of percolating waters and the carbonate of lime is carried away, only the insoluble flints and sand and iron oxides remain.

Of this we see distinct proofs in the Alderbury beds. The flints did not all lie as they would in an ordinary river-gravel with their longer axes horizontal or so inclined as to enable the mass to offer best resistance to the current, but they were frequently arranged vertically in loops and hollows into which they had sunk owing to the gradual but irregular removal of the included masses of chalky material.

In one place I observed that a part of the chalky matrix had been banded by infiltered iron oxides, so that the iron ran in wavy lines conforming to the outline of the included fragments. When the calcareous portion was dissolved away, those stood out in separate septarian bands of iron oxides.

Mutatis mutandis we may offer the same explanation of the origin of the superficial deposits along the North Downs of Kent and of the mode of occurrence of gravels largely derived from surface flints and from the destruction of the top of the chalk which occur in that area, not only over the surface of the chalk itself but over beds which crop out from below the chalk in front of the great escarpment that overlooks the weald and over the Tertiary beds which cling to the dip slope of the chalk on the north as described in the works quoted above.

The reason why it is of the greatest importance to clear up this question is that, if the views now put forward are correct, these deposits may be of any age and that they are even being formed still. Some of the gravels on ledges and terraces, and the outlying knolls which represent old ledges and terraces, may be patches of trail or be due to rain-wash and soil-creep which have used up remnants of older river gravels, while some of them may be ancient river gravels which have largely if not wholly derived their material from the surface.

Sometimes it may be possible to refer a given bed to one or other of these two gravels so different in mode of transport, if not in the origin of the material.

If many, perhaps most, of the patches in question have been formed by the local rainwash and soil-creep, the height of the deposit above the sea or above the existing rivers goes for nothing, but, if we must suppose that an escarpment has receded far since it fed the now outlying hills with its *débris*, we must admit that as an argument in favour of an enormous lapse of time since the objects found in the gravel were deposited where now found, or we must believe in a much more rapid destruction of the chalk than has been until recently advocated[1].

To consider now the character of the flints found on such surfaces and in such deposits as those just described.

A blow or crush takes off pieces of flint with a bulb of percussion and a conchoidal fracture. Changes of temperature make it crack and often remove large lenticular flakes which show no bulb. This is what so commonly spoils flint implements which have been lying long exposed on the surface of the ground.

None of these operations necessitate the presence of man.

Any movement which would force one flint against the other would be likely to produce fractures. The stampede of a herd of oxen or deer must make many a flake and chip the edge of many a flint.

The whole of the surface from Balsham to Six Mile Bottom is covered with flints showing every variety of fracture. Most

[1] See Howorth, *Geol. Mag.* Feb. 1896, pp. 58 *et seq.*

of these are broken-up tuberous flints, some few are pieces of tabular flint, that is, flint which has replaced the chalk along lines of joint oblique to the bedding and across the bands of tuberous flints.

Of course the parts of each flint which are most likely to get chipped are the more exposed points and thinner edges or the curved margins which would hold a round stone when pressed into the hollow. Thus we can easily make a selection of forms which look as if they had got bruised at the point or edge by being used. We can find many tabular flints which have got chipped along an exposed corner or edge, many tuberous flints which fit comfortably into the hand at one end, and have at the other or on one side a point or edge which has got chipped.

If over such a land surface we have here and there some unfinished implements and misfits, both palæolithic and neolithic as well as recent strike-a-lights, and now and then all get washed together, or creep down the hill into the same bed of gravel, it might have been and it has been suggested, that a large number of the natural pieces of flint owe their chips to their having been picked up and used by primæval man. The palæotoliths, says Mr Montgomerie Bell[1], "are not shaped into particular forms by the will and skill of the workman as palæolithic flints are. They are chipped flints rather than shaped flints, used tools, not made tools." There is now in Jermyn Street a collection of flints, which I made to illustrate the view that almost all the artificial forms of implement, certainly all the common ones, were suggested by the forms into which flint naturally breaks. If there be anything in this, we might expect to find rude transitional forms among surface-weathered flints.

The points therefore which I would accentuate are that, unless the mode of origin by rainwash and soil-creep above described be borne in mind, these high-level gravels and flints may be referred by some to a period the remoteness of which is measured by the time it has taken to cut the valley down or the escarpment back to the level of the existing rivers or the base of the present slopes; that all the specimens I exhibit are natural forms chipped by the accidents common to all surface-soils; that they are identical with and represent every type of the palæotoliths upon the character and mode of occurrence of which the theory of the existence of man in Britain before the palæolithic age has been founded.

We are all aware that the genuineness of the first found palæolithic implements was questioned by many who now believe in them, especially when *à priori* reasons made them doubtful as to the possibility of such a discovery.

[1] *Brit. Assoc. Rep.*, Edinburgh, 1892, Trans. Sects., p. 900.

But the difficulty of convincing the public or even the scientific world of the authenticity and genuineness of any of the specimens was greatly increased by the enormous mass of false or inconclusive evidence that was then adduced in favour of the discovery. Those therefore who believe most firmly that traces of man earlier than those now known as palæolithic will be found, should be the most careful in their examination of the evidence adduced while the enquiry is in progress.

(5) *On the Leakage of Electricity through Dielectrics traversed by Röntgen Rays.* By Prof. J. J. Thomson, M.A., F.R.S., and J. A. McClelland, *Fellow of the Royal University of Ireland, Trinity College, Cambridge.*

Experiments proving that dielectrics when traversed by Röntgen rays conduct electricity were described by one of us in papers read before the Philosophical Society on Jan. 28, 1896, and before the Royal Society on Feb. 13, 1896. The following paper contains an account of further experiments made to investigate the laws governing the passage of electricity through dielectrics exposed to Röntgen's rays.

Method of making the Experiments. The induction coil and the bulb for producing the rays were inclosed in a large iron tank, fitted with a metallic lid. A small hole was bored in the lid and the bulb placed underneath this hole. The radiation from the bulb passed through the hole and fell upon the dielectric whose conductivity was to be investigated. In the experiments with gases two different methods were employed; in the first of these, which we shall call method (A), the lower end of a piece of glass tubing about 4 cm. in diameter was closed by a piece of thin sheet aluminium, a paraffin stopper was placed in the upper end of the tube, a stout metallic rod passed through the paraffin and supported a metallic disc with its plane parallel to the plate of aluminium. The rod and disc were connected with one pair of quadrants of a quadrant electrometer. The plate of aluminium rested on the iron tank, which was connected with the earth. To determine the rate of leak through the dielectric the two pairs of quadrants of the electrometer were connected together and with one terminal of a battery of small storage cells, the other terminal of which was connected with the earth. The quadrants of the electrometer were then disconnected from each other and from the battery. Before the Röntgen rays were turned on the insulation was so good that there was no movement of the spot of light

reflected from the mirror of the quadrant electrometer, as soon however as the rays passed through the dielectric there was leakage through the dielectric, the potential of the disc approached that of the aluminium plate and the spot of light moved across the scale. The rate of leak was determined by measuring with a stop-watch the time taken by the spot of light to pass across 50 scale divisions. The wire attached to the middle of the electrometer carried a vane which make the deflections of the electrometer dead beat.

In the other method, which we shall call method (B), a mica window was inserted in the side of a glass vessel shaped like a lamp-glass, the window was placed opposite to the hole in the tank containing the coil and bulb. The open ends of the glass vessel were fitted with paraffin plugs, through each of these plugs a metal rod passed carrying a metal disc, the two discs were parallel and opposite the window. As long as the gas between the discs insulated no electricity could pass from one disc to the other. One of the discs was kept permanently connected with one terminal of a battery of small storage cells, the other terminal of which was connected with the earth, while the other disc was connected to one pair of quadrants of the electrometer. The other pair of quadrants were connected to earth. If the two pairs of quadrants were connected together and then disconnected the spot of light reflected from the mirror would remain at rest if the gas between the discs (which are at different potentials) remained an insulator, when however electricity can leak through the gas the potentials of the two discs will tend towards equality and the spot of light will move across the scale. The spot of light remained at rest when the gas between the plate was not traversed by the Röntgen rays; as soon however as these passed through it the spot of light began to move, and the conductivity of the gas was determined by measuring the time taken for the spot of light to pass over 50 scale divisions.

Conductivity of different gases when traversed by the Röntgen rays. The following results were determined by the method A. The vessel was filled with air at atmospheric pressure and temperature and the rate of leak when the rays were passing through the air determined. The air was then pumped out and the vessel filled with the gas to be examined and the rate of leak again determined; after this the gas was again pumped out and the vessel refilled with air and another determination of the rate of leak made; this process was repeated several times.

For these determinations it is necessary to use a bulb whose rate of emission of the Röntgen rays remains steady. There is sometimes a little difficulty in getting the bulb into this state, many bulbs which will produce satisfactory photographs are so

variable that they are useless for these measurements. We generally found that a bulb got steadier after being used, indeed we never found a bulb that was steady enough for our purpose when first taken off the pump.

The following table contains the rate of leak of different gases at atmospheric pressure, and unless when otherwise stated at a temperature of about 15° C., the rate of leak through air at this temperature and at atmospheric pressure being taken as unity.

Name of Gas.	Molecular Weight.	Relative rate of Leak.
Hydrogen	2	·5
Coal gas		·8
Ammonia gas	17	·9
Sulphuretted Hydrogen	34	3·5
Carbonic acid gas	44	1·2
Sulphur dioxide	64	3
Nitrogen peroxide (N_2O_4)	92	3
Chlorine	71	12
Bromine (at the temperature of boiling bromine)	160	9
Iodine (at the temperature of boiling iodine)	230	7
Hydriodic acid gas	126	4
Mercury vapour (at the temperature of boiling mercury)	200	10

It will be seen from the preceding table that the electrical conductivity of the heavier gases when exposed to the Röntgen rays is greater than that of the lighter gases, the order for electrical conductivity does not however exactly correspond with the order of the molecular weight. A very marked feature of the table is the high conductivity of the halogens, in fact the leakage of electricity from a charged disc through a tube filled with chlorine is a very sensitive and convenient measure of the intensity of the Röntgen rays, and was used by us in many of the experiments described in the later part of this paper.

The high conductivity of mercury vapour is very remarkable, for this gas only allows the ordinary electric discharge to pass through with great difficulty, whereas under the influence of the Röntgen rays it is one of the best conductors among the gases we investigated.

In the case of most of the preceding gases the rate of leak was determined when the charged disc was electrified positively as well as when it was electrified negatively, but no difference was ever detected between the rate of leak in the two cases.

Influence of pressure on the rate of leak. A series of experiments, using the method A, were made to determine the rates of leak through air at different pressures. As the following numbers show, the rate of leak diminishes as the pressure diminishes though the rate of diminution in the rate of leak is not so rapid as that of the pressure.

Rate of leak through air at different pressures.

Pressure.	Time in seconds taken for the spot of light to pass over 50 scale divisions.	$\{\text{Pressure}\}^{\frac{1}{2}} \times$ time.
755	26	715
565	32·5	758
375	39	752
130	57	655
55	81	600
15	109	422

Second set of experiments.

750	34·5	931
470	41	902
320	51	918
170	74	962
45	97	650

Rate of leak through sulphuretted hydrogen at different pressures.

771	15	417
551	18·5	454
341	23·5	454
101	48	480
31	97	533
11	140	462

Rate of leak through sulphur dioxide at different pressures.

771	22	611
501	28	616
291	42	714
150	60	730
11	160	528

Rate of leak through coal-gas at different pressures.

769	58	1606
579	70	1680
374	81	1563

Rate of leak through hydriodic acid gas at different pressures.

769	7	194
489	8·5	189
289	10·5	178
89	20·5	193

These numbers show that over a considerable range of pressures the rate of leakage through the gas, *i.e.* the electrical conductivity of the gas, is approximately proportional to the square root of the pressure. Now we know from the theory of the dissociation of gases that in cases where the dissociation is small, *i.e.* where the number of ions is small compared with the number of the undissociated molecules, the number of ions is proportional to the square root of the pressure. Hence the results of our experiments indicate that the electrical conductivity of gases traversed by the Röntgen rays is proportional to the number of ions, and as long as the gas and the number of ions is the same is independent of the number of undissociated molecules, or in other words, the electrical conductivity is a function of the number of ions and not of their mean free path.

Effect of temperature upon the conductivity of a gas at constant pressure. A few experiments were made to determine the effect of temperature upon the conductivity of air at constant pressure. It was found that the conductivity at high temperatures was slightly less than that at the temperature of the room.

Further experiments showed that as the gas was heated up the conductivity attained a maximum value, after which it began to diminish.

As an instrument for measuring high temperatures was not available at the time of making these experiments, further consideration of these effects must be reserved for a future paper.

Effect of Variation in the Potential Difference on the rate of leak under the influence of the Röntgen rays. A very remarkable and characteristic property of the conductivity produced by these rays in a gas, is that the rate of leak as the potential difference increases soon reaches a maximum value and becomes almost independent of the potential difference. Many experiments were made to investigate this point, using both methods A, B. The

following may serve as specimens of such experiments; the method used was B, and the gas in the tube was air.

Potential difference in volts between the discs in the tube.	Time required to leak through 50 scale divisions.
2	58
4	44
6	30
8	25
10	25
12	28
16	26
35	26
73	25
108	25
138	24
278	23

Another set of measurements gave, when the gas in the tube was chlorine,

35	8
75	8
132	7

A third set with a chlorine tube with two sheets of tinfoil between the exhausted bulb and the discs,

35	18
75	17
132	15

These numbers show that the rate of leak and the current through the gas is almost independent of the electromotive force. If we regard the conduction as electrolytic the gas being ionised by the Röntgen rays, the fact that the current is independent of the electromotive force indicates that the velocity of the ions is independent of this force. This result is consistent with the law connecting the conductivity with pressure for we saw that this implied that the conductivity was proportional to the number of ions but independent of their mean free path. Now if the conductivity depended upon the velocity acquired by the ions under the action of the electromotive force it would depend upon the coefficient of viscosity between the ions and the molecules, and hence upon their mean free path. The current increases with the potential difference when this is small, but soon attains a maximum value. This effect would result if the passage of the Röntgen rays through the gas produced chains

of molecules or aggregations of some kind in which the component atoms with their electrical charges could rearrange themselves with facility; the time T required for this rearrangement being independent of the intensity of the electric field. The effect of such a rearrangement would be that in a time T a definite quantity of electricity passed from one end to the other of the chain of molecules. If these chains of molecules were distributed in an unpolarized way through the gas their breaking down would not lead to a current, as as much electricity would pass in one direction as in the opposite. As soon however as the chains are polarized there will be a current. Suppose now that these chains get polarized under the influence of an external electromotive force and that the polarization is analogous to that of the molecular magnets in a piece of soft iron under an external magnetic field. Then until all the chains get pulled so as to set in one direction the current will increase with the potential difference, but as soon as all the chains get pulled into one direction the current will reach a maximum value and be independent of the electromotive force.

Rate of leak through solid dielectrics traversed by the Röntgen rays. Solid dielectrics when traversed by these rays become conductors of electricity, but the conduction through these substances, inasmuch as it approximately obeys Ohm's law, differs in one very important respect from that through gases. The conduction through solid dielectrics is also remarkable for the extent to which polarization is developed. If the two discs in method AB are imbedded in paraffin, and the paraffin when exposed to the Röntgen rays has a current sent through it, then if the discs are removed from the battery and connected together, and the paraffin kept exposed to the rays a current in the opposite direction flows through the paraffin. If the Röntgen rays are shut off from the paraffin, the polarization can be kept locked up in this substance almost indefinitely, and can after any lapse of time be liberated by again exposing the paraffin to the influence of these rays. Solid sulphur showed exactly similar polarization effects to those shown by paraffin.

The amount of this polarization may be realized from the fact that the times taken to leak through 50 divisions under a constant electromotive force, were in four consecutive experiments taken quickly one after another 20, 47, 68 and 80 seconds respectively. In comparing the rates of leak under different electromotive forces, it is therefore essential to get rid of polarization between each experiment. In the following experiments this condition was carefully attended to.

Leakage of electricity through paraffin under different electromotive forces.

Voltage.	Time required to leak through 50 divisions.	Product of voltage and time.
278	14	3892
208	21	4368
138	29·5	4071
73	56·5	4124
35	140	4900

Thus the conduction approximately obeys Ohm's law.

Leakage through sulphur.

278	10	2780
208	13·5	2808
138	21	2898
73	37	2701
35	67	2345

And again the conductivity approximately obeys Ohm's laws.

On the law connecting the rate of leak through a dielectric with the distance of the dielectric from the vacuum tube. In the experiments made to investigate this law, the vacuum tube was placed immediately behind a small hole in the metal tank enclosing the coil and tube. Method B was used to measure the rate of leak, the vessel in which the discs were placed being filled with paraffin, as the leak through this gas is very rapid.

The bulb used in the experiments (1), (2), (3) below was of the shape shown in Fig. 1. The distance of the tip of the negative

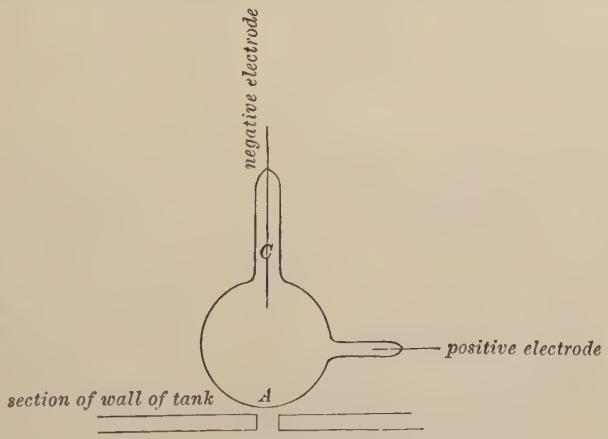


Fig. 1.

electrode from A was about 5 centimetres, and the length of the electrode about 4·5 centimetres. The bulb was placed so that the negative electrode was perpendicular to the plane of the hole in the side of the tank, and its prolongation passed through the centre of the hole. The discs in the chlorine tube were placed opposite the hole with their planes parallel to the normal to the plane of the hole.

EXPERIMENT 1.

Distance between the middle of the discs and the point A of the bulb.	Time in seconds required to leak 50 scale divisions.
6·7	3
13·4	6·5
20·1	13
26·8	18
40·2	36
53·6	48·5
67	80

The results of these experiments are plotted in the curve in Fig. 2. The abscissæ are proportional to the distance of the middle

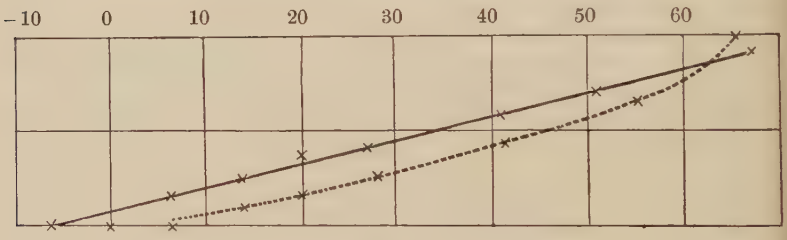


FIG. 2.

point of the discs from the point A of the bulb. The ordinates on the dotted curve are proportional to the time required to leak 50 divisions, those on the continuous curve are proportional to the square roots of these times. The continuous curve is a straight line, passing through a point on the axis which corresponds to a point C, 7 cm. behind A. Hence this experiment shows that the times required for a given leak are proportional to the squares of the distance of the discs from C, or that the rate of leak is inversely proportional to the square of the distance from C. This suggests that the Röntgen rays originate chiefly in the neighbourhood of C. There is no evidence of any coming from the glass in the neighbourhood of A, though this is brightly phosphorescent.

EXPERIMENT 2. (Made on a different day with the same bulb arranged in the same way as in Experiment 1.)

Distance from centre of the discs to *A*.	Time required to leak 50 divisions.
5·5	2·5
11	5
16·5	7
22	12·25
27·5	15·5
33	25
38·5	35

The curves plotted in Fig. 3 show the results of these measurements, the ordinates are proportional to the square roots of the

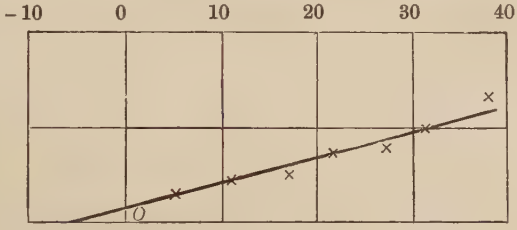


FIG. 3.

times of leak and the abscissæ to the distances from *O*. We see again that a straight line passing through approximately the same point as before represents fairly well the relation between the distance and the square root of the time required to leak 50 divisions. Thus the results obtained by experiments (1), are confirmed by (2).

EXPERIMENT 3. Same arrangement as before.

Distance of discs from *A*.	Time required to leak 50 divisions.
6·5	less than 2
19·5	13
32·5	32
52	61

The results are plotted in Fig. 4, and confirm the conclusion already drawn. In the next (3) experiments, a tube of another pattern was used. This is represented in Fig. 5, *A* is a flat aluminium plate, *B* and *C* aluminium wires fused in the tube. The dimensions of the tube are marked on the figure. The bulb was placed so that *D* was close to the hole. In Experiment 4,

when the plate A was the positive electrode and the wire B the negative, the following results were obtained.

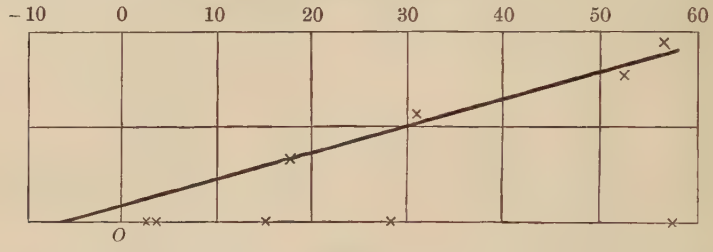


Fig. 4.

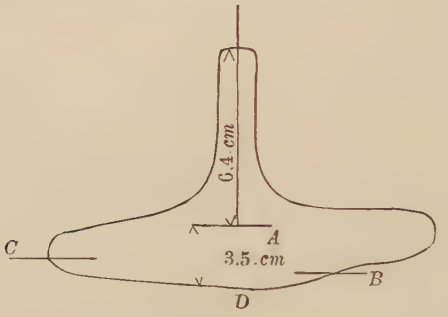


Fig. 5.

Experiment 4.

Distance of the centre of the discs from D.	Time required to leak 50 divisions.
4·5	6
9	10
13·5	15·5
18	25
22·5	30
27	40
31·5	60

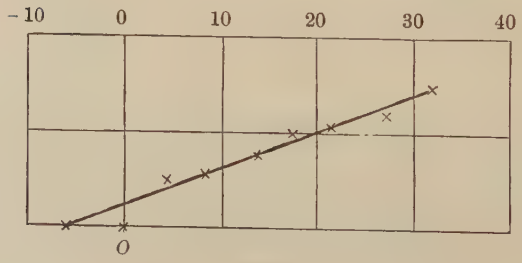


Fig. 6.

The results are plotted in the curve shown in Fig. 6, where the ordinates are the square roots of the times and the abscissæ the distance of the centre of the discs from *D*. We see that the measurements are represented by a straight line cutting the horizontal axis at a point corresponding to a point in the tube about 6 cm. behind *D*. Thus the rate of leak varies inversely as the square of the distance from a point distinctly behind the flat aluminium plate, which in this experiment is used as the positive electrode.

Experiment 5. In this experiment the same tube was used as in Experiment 4, but in this case *C* was the positive electrode, while *B* was again the negative electrode. The following results were obtained.

Distance of centre of discs from *D*.	Time required to leak 50 scale divisions.
4·5	6·75
9	15
13·5	29·5
18	54·5
22·5	85

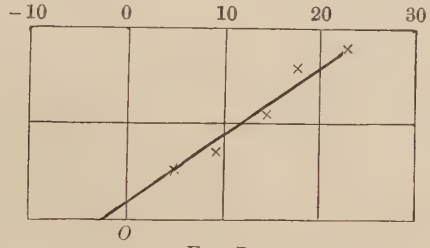


Fig. 7.

These results are plotted in the curve shown in Fig. 7, where the ordinates are proportional to the square roots of the time of leak and the abscissæ proportional to the distances of the centre of the discs from the point *D*. The observations are well represented by a straight line cutting the horizontal axis at a point corresponding to the point in the tube about 3·5 centimetres behind *D*. This point is very nearly on the flat plate *A*, so that in this case the rate of leak varies inversely as the square of the distance from the plate.

Experiment 6. In this experiment *C* was the negative and *B* the positive electrode. The following results were obtained.

Distance of centre of discs from *D*.	Time required to leak 50 divisions.
6	4·5
12	11
18	22
24	33
30	60

These results are plotted in the curve shown in Fig. 8, where the ordinates are proportional to the square roots, times of leak

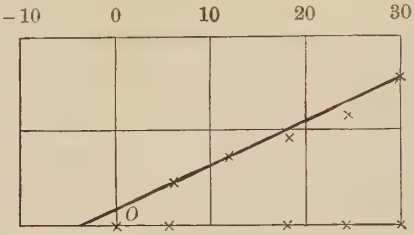


Fig. 8.

and the abscissæ proportional to the distances of the centre of the discs from *D*. The observations are represented by a straight line cutting the horizontal axis at a point corresponding to a point in the tube about 4 centimetres behind *D*, and this point is approximately on the flat plate *A* and thus in this case as in the preceding, the rate of leak varies inversely as the square of the distance from the flat plate. Thus reversing the electrodes has produced no change of the place of origin of the Röntgen rays.

The origin of these rays in these tubes seems to be the flat plate, though when this is made the positive electrode the origin seems slightly displaced. To test this point a set of experiments were made with a tube whose shape and dimensions are shown in Fig. 9. In the experiments made with this tube the plate was the

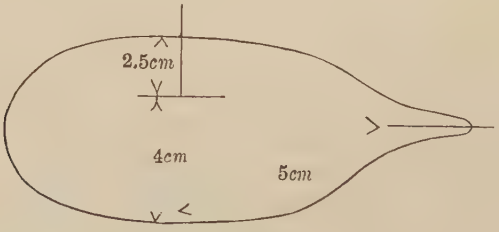


Fig. 9.

positive and the wire the negative electrode. The following results were obtained.

Distance of centre of discs from *A*.	Time required to leak 50 scale divisions.
5	5
10	14·5
15	23
20	34
25	44·5
30	65

These results are plotted in the curve shown in Fig. 10. The ordinates are again proportional to the square roots of the time and the abscissæ to the distances of the centre of the disc from *A*. The results are represented by a straight line which cuts the horizontal axis at a point corresponding to a point in the tube about 7 centimetres behind *A*, *i.e.* the rate of leak varies inversely as the square of the distance from a point behind the plate.

Absorption of the Röntgen rays by sheets of tinfoil. A number of experiments were made on the effect produced by inserting between the vacuum tube and the gas exposed to the Röntgen rays sheets of tinfoil in gradually increasing numbers. The result of these experiments would seem to indicate that the rays are not all of one kind, for it was found that the effect produced by the insertion of a given number of leaves of tinfoil was greater when the number of leaves already placed between the vacuum tube and the gas was small than when it was large. This effect would occur if some of the rays were readily absorbed by the tinfoil, while others passed through with greater facility. Thus when the rays had already passed through a number of layers of tinfoil most of the more absorbable rays would be already absorbed, and any more leaves of tinfoil would only have the less absorbable ones to deal with. The effect is very marked with some bulbs, though it is hardly noticeable in others.

We shall first give a specimen of the results obtained with a bulb showing this effect.

Number of leaves of tinfoil between the bulb and the gas.	Time required to leak through 50 divisions.
0	8
1	10
2	11
3	11
4	12
6	16
8	19
10	22
12	25
16	29
20	36
24	44
28	48
36	60
44	61

Thus to take a few examples, the effect of increasing the number of leaves from 2 to 10 is to diminish the rate of leak

in the proportion of 2 to 1, increasing the number from 12 to 20 decreases it in the proportion of 1·44 to 1, from 20 to 28 in the

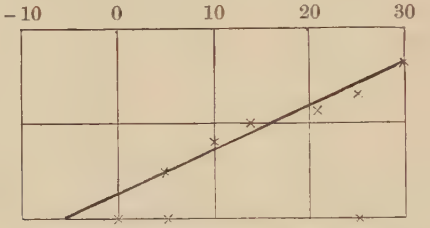


Fig. 10.

proportion of 1·33 to 1, from 28 to 36 in the proportion of 1·25 to 1, and from 36 to 48 only in the proportion of 1·01 to 1.

With this bulb the effect is clearly marked. But in the following experiments made with another bulb it is much less noticeable.

Number of leaves of tinfoil between the bulb and the gas.	Time required to leak through 50 divisions.
0	3
2	5
4	8
6	10
8	15
10	20
14	32
18	51

In this case increasing the leaves from 2 to 6 increases the time of leak in the proportion of 2 to 1, from 6 to 10 in the same proportion, from 10 to 14 in the proportion of 1·6 to 1, and from 18 to 14 again in the same proportion. Thus, though there are traces of the effect, it is not nearly so marked as in the preceding bulbs; we therefore conclude that the different rays are emitted by different bulbs in different proportions. We have always found this effect most marked in the most efficient bulbs, *i.e.* the bulbs which produced the greatest leakage.

We have in conclusion much pleasure in thanking Mr E. Everett for the assistance he has given us in the preceding experiments.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Monday, 27 *April,* 1896.

PROFESSOR J. J. THOMSON, PRESIDENT, IN THE CHAIR.

At a meeting held in the University Chemical Laboratory, the following were elected Fellows of the Society:

Professor J. B. Bradbury, M.D., Downing College.
A. J. Wallis, M.A., Corpus Christi College.
C. T. R. Wilson, M.A., Sidney Sussex College.

The following Communications were made:

(1) *On photographing the whole length of a Spectrum at once.* By PROF. LIVEING.

Professor Liveing exhibited photographs of a variety of spectra in which the whole length of the spectrum between the wave-lengths 550 and 214 was depicted on a celluloid film at one operation. A concave grating of $10\frac{1}{2}$ feet radius was used, with the slit in the centre of curvature, and the slide which held the sensitive film formed part of a cylinder with a radius of $5\frac{1}{4}$ feet, so that, when the axis of this cylinder was midway between the slit and grating, every part of the spectrum was perfectly focussed on the film. The length of the photograph between the limits of wave-length above-mentioned was 65 centimeters. To obviate the confusion caused by the overlapping of the spectra of different orders, he projected on to the slit the image of the source of light by means of a combination of two quartz lenses with a quartz prism of 30° between them. The slit being vertical the edge of the prism was made horizontal, with the result that the more

refrangible rays were somewhat diverted downwards, and the image of the slit produced by them fell on the film at a different level from that produced by less refrangible rays. Consequently the spectrum of the second order in the photograph was about half the length of the lines lower than the part of the spectrum of the first order at the same place, and the two orders were at once distinguished.

It is not at all difficult with a table of sines to set out a scale of wave-lengths for a spectrum formed in this way, and such a scale once made would of course apply to all photographs taken with the same instrument, provided there were no inequalities in the shrinking of the films when drying after development. Unfortunately Professor Liveing had found that the films, inelastic as they seem even when wet, did not shrink uniformly, and he was therefore turning his attention to glass plates. Glass plates will not bend to cylinders of so short a radius as $5\frac{1}{4}$ feet without danger of fracture, but thin sheet glass can easily be bent to a cylinder of double that radius. He was therefore fitting up a suitable camera for use with a large grating of 21 feet radius, and hoped to obtain photographs with it which should not only be useful for reference, but from which wave-lengths could be read off to a close approximation. At present he could only read off wave-lengths from the films by photographing a known spectrum, such as that of iron, at the same time and on the same film as the other spectrum, and measuring the distance of the unknown line from the nearest lines of known wave-length, a much more troublesome process than the simple application of a scale.

(2) *On dioxymaleic acid and its derivatives.* By H. J. H. Fenton, M.A., Christ's College.

By oxidation of tartaric acid, in presence of a ferrous salt, a substance results which has powerful reducing properties, and which gives a beautiful violet colour with ferric chloride. This substance has been isolated and proves to be an acid having the formula $C_4H_4O_6$. It crystallizes with $2H_2O$ in flat diamond-shaped plates, having a pearly lustre. The isolation of the acid was a matter of extreme difficulty owing to the unstable nature of its aqueous solution, which decomposes slowly at ordinary temperatures, and very rapidly when heated, giving glycollic aldehyde and carbon dioxide. Glycollic aldehyde has, in this way, been isolated as a syrupy liquid; when heated to 100° in a vacuum it undergoes polymerization, a sweet-tasting solid gum being produced which has the formula $C_6H_{12}O_6$.

By oxidation with bromine the original acid is converted quantitatively into dioxytartaric acid. It is easy to prepare considerable quantities of free dioxytartaric acid in this way, and the author suggests the use of this acid as a delicate reagent for sodium.

The constitution of the original acid has been established by determination of its molecular weight (in various ways), its basicity, the behaviour of acetyl chloride, benzoyl chloride, aniline and other reagents, and by its relation to succinic tartaric and dioxytartaric acids. It is shewn to be a tetrahydroxylic, dibasic, unsaturated acid belonging to the maleic series. Its formula is therefore

$$\begin{array}{l} OH \cdot C \cdot COOH \\ \| \\ OH \cdot C \cdot COOH. \end{array}$$

By action of hydrogen bromide, an isomeric modification has been obtained having a distinct crystalline form and yielding a more stable aniline salt. It is suggested that this may be the corresponding dioxyfumaric acid.

(3) *On the atomic weight of Oxygen.* By A. Scott, M.A., Trinity College.

Mr Scott gave a short account of the present state of our knowledge as to the atomic weight of oxygen, and said that it might be regarded as conclusively proved that if H = 1, O = 15·87 to 15·88. Morley determined the densities of hydrogen, and oxygen, the ratios by volume in which the gases combine (by a somewhat indirect method) and finally combined known weights of hydrogen and weighed the water produced. Thomsen made similar determinations but with far less pretension to the highest accuracy attainable. The results were

	Morley.	Thomsen.
Weight of a litre of oxygen at 0° C. and 760 mm. at sea level, lat. 45°.	1·42900	1·42906
ditto for hydrogen.....	·089873	·089947
Ratio of densities	15·9002	15·8878
Ratio of combining volumes	1 : 2·00269	1 : 2·00237
Atomic weight of oxygen...	15·879	15·869

The ratios by volume in which the gases combine agree well with that published by the author directly three years ago, viz.

1 : 2·00245 at about 15°,

and 1 : 2·00285 at 0° C.

(4) *On the combining volumes of Carbon Monoxide and Oxygen.* By A. Scott, M.A., Trinity College.

Mr Scott described some preliminary experiments made to determine the ratio by volume in which carbon monoxide and oxygen unite to form carbon dioxide and to determine at the same time the volume of the latter gas in terms of the others. Experiments so far shewed that the ratio was very nearly 2 : 1 for the combining gases, but that satisfactory determinations of the volume of carbon dioxide produced had not been obtained as yet.

(5) *The constituents of Indian hemp resin.* By T. H. Easterfield, M.A., Clare College, and T. B. Wood, M.A., Caius College.

Cannabis Sativa (syn. indica) is made use of by nearly all the nations of Southern Asia and Northern Africa on account of the toxic constituents which it contains. It is stated that the plant when grown in Europe has no narcotic action, but this statement requires confirmation. The most common use to which the plant is put is as a substitute for opium or tobacco amongst smokers, but sweetmeats, drinks, and pills are also prepared, and all possess narcotic effects to a greater or less degree. The word Haschish is generic and applies to any preparation from the hemp plant ready for immediate use.

Many investigators have already endeavoured to isolate from the hemp plant the substance to which the characteristic physiological action is due. Amongst the earliest workers in this direction T. and H. Smith of Edinburgh (1847) should be specially mentioned. They prepared a resin from the alcoholic extract of the plant which produced the effects obtainable from Haschish. At a later date Personne attributed the effects of Cannabis to volatile oils; other observers again have stated that alkaloids are present in the plant but they have not succeeded in shewing that the alkaloidal substances can produce the same effects as are usually produced by Haschish. The experiments of the authors have shewn that the resin of T. and H. Smith contains a large percentage of the narcotic constituent, but that the resin itself is not a pure chemical compound.

That the narcotic constituent is most certainly to be found in the plant resin is emphatically shewn by the fact that the preparation which is universally regarded by the natives of India as the most powerful and dangerous drug which the plant can yield is nothing more nor less than the natural resin exuded in large quantity by the ripe plant but more particularly by the full grown, unfertilised female plants. To this resin the name "Charas" is

given; it is collected by the natives of India, who are said to walk through the ripe hemp fields dressed in leather jackets to which the resin adheres. This substance is sold in the bazaars at about 10 rupees per seer, but only small quantities may be bought at one time without a special license. The substance is mixed with tobacco and smoked; it produces almost instant intoxication followed by no unpleasant symptoms. One of the most remarkable points about Cannabis intoxication is the sensation of complete lack of responsibility which accompanies it, and there is little doubt that the great popularity of Haschish is due to this fact.

No chemical investigation of Charas has been recorded, and since the narcotic constituent is so highly concentrated in this substance it appeared desirable to make a complete examination of the substance; the authors would here express their indebtedness to Mr C. A. Silberrad of Peterhouse, who has procured for them a considerable quantity of Charas, and has supplied them with much information regarding the use of the substance in India where he is now resident.

Chemical examination of Charas.

The crude Charas is a substance with a greenish-brown earthy appearance; held between the fingers it becomes intensely sticky; its smell is highly characteristic. In order to isolate the constituents the Charas was extracted with alcohol, ether, or other organic solvent, whereby a large quantity of insoluble matter was separated. The alcoholic solution was concentrated and eventually submitted to fractional distillation, at first under atmospheric pressure, and finally in a partial vacuum. By this treatment the following substances were obtained:—

(*a*) *Insoluble residue.* This substance has only been superficially examined; it consists to a very large extent of the hairs of the hemp plant; it contains upwards of fifty per cent. of ash and about three per cent. of nitrogen. Systematic examination for alkaloids gave negative results.

(*b*) The distillate from the alcoholic extract coming over below 300° C. at atmospheric pressure.

(*c*) The portion distilling between 270° and 290° at 15 to 60 mm. pressure.

(*d*) A small quantity of pitch which was not submitted to further examination.

Constituents of fraction (*b*).

Terpene, $C_{10}H_{16}$. By re-fractionation (*b*) was separated into two-constituents: one of these boiled between 160° and 180° and came over for the most part at 165° to 175°; the substance is a terpene.

Analysis.

Calculated for $C_{10}H_{16}$	Found
C = 88·2	87·8
H = 11·8	12·0

Vapour density.

Calculated	Found
68	65

The specific gravity at 17° = 0·819.

The substance is laevo-rotary, it forms a mono-hydro-chloride.

Sesqui-terpene, $C_{15}H_{24}$.

The second substance isolated from fraction (*b*) boiled at 258° —259°

Analysis.

Calculated for $C_{10}H_{16}$	Found
C = 88·2	87·9
H = 11·8	11·6

Vapour density.

Calculated for $C_{15}H_{24}$	Found
102	99·8 and 101·8

The specific gravity = ·898 at 18°.

This compound has been isolated by other observers as a constituent of Personne's "Cannabene."

Constituents of fraction (*c*).

When fraction (*c*) was allowed to stand crystals separated; these were found to be sparingly soluble in alcohol, and were thus readily separated. Analysis shewed the substance to be a higher paraffin, and since the melting and boiling points of the substance agree with those which can with considerable certainty be predicted for normal-nona-kosane there is some probability that the compound is this hydrocarbon.

Analysis.

Calculated for $C_{29}H_{60}$	Found
C = 85·3	84·9
H = 14·7	14·7

Physical constants.

	Calculated for $C_{29}H_{60}$	Found
Melting point	= 63—64	63·5—64
Boiling point at 15 mm.	= 285—286	285—290

Narcotic Red Oil, "Cannabinol," $C_{18}H_{24}O_2$.

This forms more than 96 per cent. of fraction (*c*); it distilled constantly at 265° C. (20 mm.). Analysis agreed with the formula $C_{18}H_{24}O_2$.

Analysis.

Calculated for $C_{18}H_{24}O_2$	Found
C = 79·4	79·1
H = 8·8	9·0

The figures are the mean of four concordant analyses made upon different preparations.

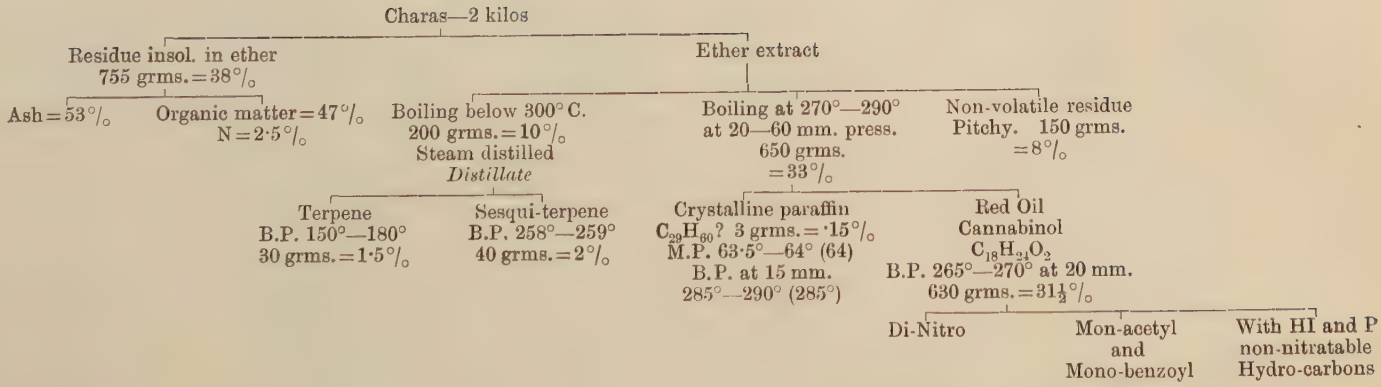
The molecular weight by Raoult's freezing-point method was found to be 273, the formula $C_{18}H_{24}O_2 = 272$.

The substance is liquid above 60° C., below that temperature it is a cherry-red, semi-solid resin. It is insoluble in water, but dissolves easily in most organic solvents.

The compound yields mon-acetyl and mono-benzoyl derivatives, both of which can be distilled under diminished pressure; it is hence to be regarded as a hydroxyl compound; it yields nitro and apparently sulphonic derivatives and must therefore be a closed chain derivative. Further experiments upon the constitution of this remarkable substance are in progress. The authors propose the name Cannabinol for the compound.

Cannabinol has been isolated by the authors from all the usual pharmaceutical preparations of Cannabis Indica. The experiments of Mr C. R. Marshall, Assistant to the Downing Professor of Medicine, shew conclusively that the narcotic action of the hemp plant is to be ascribed to the presence of this compound.

The main results arrived at in the present investigation are summarised in the subjoined table:


Charas—2 kilos
Residue insol. in ether
755 grms. = 38%
Ether extract
Ash = 53%
Organic matter = 47%
N = 2·5%
Boiling below 300° C.
200 grms. = 10%
Steam distilled
Distillate
Boiling at 270°—290°
at 20—60 mm. press.
650 grms.
= 33%
Non-volatile residue
Pitchy. 150 grms.
= 8%
Terpene
B.P. 150°—180°
30 grms. = 1·5%
Sesqui-terpene
B.P. 258°—259°
40 grms. = 2%
Crystalline paraffin
C29H60? 3 grms. = ·15%
M.P. 63·5°—64° (64)
B.P. at 15 mm.
285°—290° (285°)
Red Oil
Cannabinol
C18H24O2
B.P. 265°—270° at 20 mm.
630 grms. = 31½%
Di-Nitro
Mon-acetyl
and
Mono-benzoyl
With HI and P
non-nitratable
Hydro-carbons

(6) *Note on the pharmacological action of cannabis resin.* By C. R. Marshall, M.B. (Vict.), Assistant to the Downing Professor of Medicine.

The pure products isolated by Messrs Easterfield and Wood were passed on to me for pharmacological investigation. They included the resin *cannabinol*, the terpenes, and a small quantity of the residue. As it was probable that the active principle lay in the resin, my attention was first turned to this body: a few experiments were made on animals and afterwards I took some myself. The effects produced were similar to those described by other observers as resulting from the crude drug. These did not occur after the use of the other substances, and it therefore seems probable that the effects of the hemp plant are due to the cannabinol which it contains. This is further supported by the fact that Messrs Easterfield and Wood have shewn that cannabinol exists in charas (the most potent of cannabis compounds) to the extent of 33 per cent., and that it can be obtained, in amounts varying with the activity, from all the cannabis preparations they have examined. In this communication I merely wish to establish the activity of cannabinol, and I shall therefore confine myself to a description of its effects without commenting upon it further.

I was under the impression that I had taken a small dose of impure cannabinol sometime previously without any result, so, about 2·30 on the afternoon of Feb. 19 last, whilst engaged in putting up an apparatus for the distillation of zinc ethyl, I took ·1 g.—·15 g. of the pure substance from the end of a glass rod. The substance very gradually dissolved in my mouth, it possessed a peculiar pungent aromatic and slightly bitter taste, and seemed after some time to produce slight anæsthesia of the mucous membrane covering the tongue and fauces. I forgot all about it and went on with my work; but soon after the zinc ethyl had commenced to distil—about 3·15—I suddenly felt a peculiar dryness in the mouth, apparently due to an increased viscidity of the saliva. This was quickly followed by paræsthesia and weakness in the legs. Gradually my mental power diminished—I was no longer able to control the steps of the operation and commenced to wander aimlessly about the room. I had the most irresistible tendency to laugh; everything seemed ridiculously funny. At times I felt more rational; but these lucid intervals gradually grew shorter and I fell under the full influence of the drug.

I was now in a condition of acute intoxication: my speech was slurring; my gait ataxic. I was free from all sense of care and worry and consequently felt extremely happy. Fits of laughter

occurred, especially at first, and sometimes the muscles of my face were drawn to an almost painful degree. The most peculiar effect was a complete loss of time relation: time seemed to have no existence. I was constantly taking out my watch thinking that hours must have passed, whereas only a few minutes had elapsed. This I believe was due to a complete loss of memory for recent events. The occurrence of lucid intervals was also peculiar in many ways. They seemed to come on suddenly, sometimes, but not always, as if the result of an effort of the will; they lasted a variable time, being shortest when the symptoms of intoxication were most marked; and in them I could converse in a rational manner and even direct the work of the laboratory to a certain extent. Except in a few instances I do not remember falling under the full influence of the drug again. About 6 o'clock I had two cups of coffee and afterwards feeling somewhat better walked home. The fresh air and exertion revived me. I ate a good dinner, afterwards read a little, and retired to bed at 11 o'clock without having experienced any symptoms of sleepiness.

During the action of the cannabinol my pulse rose from 60 to 96 per minute; sensibility as determined by pinching was blunted; and my appearance was described as "ashy pale." The pupils were dilated somewhat, but throughout reacted well to light and accommodation. At no time do I remember having had any hallucinations; no unpleasant after-effects were experienced; and the substance seems to possess no constipating action.

Smaller doses produced similar though less marked effects; with doses of less than ·01 g. no distinct action was obtained.

Monday, 11 *May*, 1896.

Professor J. J. Thomson, President, in the Chair.

The following communications were made to the Society:

(1) *Note on the formation of the germinal layers in* Amphioxus. By E. W. Mac Bride, M.A., Fellow of St John's College.

The account of the early development of Amphioxus incorporated in text-books is due to Hatschek ("Studien über die Entwicklung des Amphioxus," *Arb. Zool. Inst. Wien*, 1881). Briefly summarized it is as follows: the segmentation of the egg results in the formation of a regular blastula, which becomes converted into a gastrula by regular embolic invagination. The mesoblast

arises as successive pairs of hollow diverticula of the gut, each of which gives rise to an adult myomere. When about half-a-dozen pairs of somites have been formed the most anterior part of the gut buds off two forwardly directed diverticula, which are at first symmetrically placed and equal to one another, but of which later the left becomes a small thick-walled pouch, which acquires an opening to the exterior. The right on the other hand is converted into a thin-walled vesicle, which extends forward and constitutes the cavity of the snout.

When rather more than a dozen somites have been formed the remainder no longer arise as outgrowths of the gut, since a dorso-lateral fold of the alimentary canal (by the division of which the first somites were formed) becomes now completely separated from the rest of the alimentary tract and constitutes a kind of primitive streak, out of which the remaining somites are formed.

Quite recently Basilius Lwoff ("Die Bildung der primären Keimblätter und die Entstehung der Chorda und des Mesoderms bei den Wirbelthieren," *Bull. de Moscou*, 1894) has given a divergent account of the early development of Amphioxus. According to him the mesoblast is a dorso-lateral solid outgrowth of the gut, in which the cœlomic spaces (cavities of the myomeres) appear later as secondary splits: it is true that the pressure of the notochord on the mid-dorsal wall of the gut depresses it and raises up the sides as two folds, but these have nothing to do with the formation of the cœlom.

The development of Amphioxus is exceedingly important from a theoretical point of view, since in it we meet with the only instance of a Vertebrate egg (outside the secondarily modified eggs of the Mammalia), where the yolk is small in quantity and evenly distributed, and where consequently the early developmental processes are not impeded by its presence. Hence it is necessary to interpret the development of other Vertebrates in terms of the development of Amphioxus, and not *vice versâ:* and in view of this consideration, and also of the doubt in which Lwoff's paper had enveloped our ideas on the subject of the formation of the layers, I undertook a reinvestigation of the whole subject, and hope to be able to publish my completed results this autumn. I am greatly indebted to Mr Sedgwick for placing at my disposal a large amount of valuable material collected in Sicily by himself in 1889 and by Mr Willey in 1890.

If we examine a section of a just completed gastrula we observe that it is roughly triangular in shape with the apex directed ventrally, and that each of the lateral angles is produced into a hollow ridge. In a slightly later stage we find that the cavity of this ridge is shut off from the enteric space in front, whereas behind it still opens into it. In a still later stage we find the

same appearance as before posteriorly and also in the most anterior sections: but between there is a place where the lumen has become obliterated, so that the front part of the cavity has now no connection with the hinder part, and thus from a forwardly-directed hollow diverticulum of the gut a mesoblastic somite has been cut off. As the larva grows in length this hollow diverticulum grows also, and from its anterior end a succession of somites are segmented off. In the meantime however, owing to the consumption of the yolk, the larva has diminished very much in diameter, and the cavity of the groove which we may term the cœlomic groove has disappeared owing to the apposition of its opposite walls; consequently it appears as a solid outgrowth of the gut-wall, with which however it does not, so far as I have seen, lose its connection.

The mesoblast of Amphioxus may thus be said to originate as one pair of pouches, opening into the gut behind, which gradually become divided into somites; and it is of great interest to notice that according to recent investigations this is true also of the lower Reptilia. (Will, "Zusammenfassende Uebersicht," *Zool. Centralblatt*, April 15, 1894.) The vesicles, however, which later become the first pair of myomeres have a different origin. They arise as an independent pair of gut pouches situated anterior to the pouches we have just been considering, and placed also nearer the middle line; that on the left side retains for some time its connection with the gut: and although later the continuity of the two lumina ceases to be observable, owing no doubt to the same cause as that which obliterates the lumen of the cœlomic groove, the continuity of the gut and somite walls is traceable in all the stages I have as yet examined. As Hatschek's "nephridium" is found in precisely the same position in the later larva as this neck of communication between the left anterior somite and the gut, I have no doubt that the two structures are identical. The tube known as Hatschek's nephridium is placed in front of the mouth in a horizontal position, and Van Wijhe ("Ueber Amphioxus," *Anat. Anz.*, 1893) has found that it communicates with the gut; this observation I can confirm, and can add that at its anterior end it opens into the left anterior somite. Van Wijhe imagined that it was derived from the connection between the gut and the left of those anterior outgrowths from the gut, described by Hatschek, which we may term head-cavities; but these latter soon lose all trace of their primitive communication with the gut.

On reviewing the facts now related we see first that the cœlom of Amphioxus is an indubitable enterocœle, and secondly that the formation of the mesoblast in this animal is easily referable to the type which Bateson has described in the case of

Balanoglossus (*Q. J. M. S.*, 1884). The head cavities represent the proboscis cavity, the first myomeres the collar cavities, and the great posterior mesoblastic pouch which becomes segmented into somites the trunk cavity.

Zoological Laboratory,
May 15, 1896.

(2) *Note on the continuity of Mesenchyme cells in* Echinid larvæ. By E. W. Mac Bride, M.A., Fellow of St John's College.

About a year and a half ago Mr Sedgwick read a paper before this Society ("On the Inadequacy of the Cell-theory and on the Development of Nerves," A. Sedgwick, *Proc. Camb. Phil. Soc.*, Vol. VIII., Part IV.) in which he strongly insisted on the view that free stellate mesoblast cells such as are commonly figured are not to be found in the Elasmobranch embryo, but that the appearances thus interpreted were really only the thickened nodes of a protoplasmic network.

Last vacation I had the opportunity of examining numerous larvæ of Echinus esculentus, and my observations support those of Mr Sedgwick in a very striking manner.

The larvæ specially referred to were in the gastrula stage and possessed a comparatively narrow alimentary tract, outside which was a wide blastocœle or primary body cavity; and in this latter were numerous mesenchyme cells.

These mesenchyme cells were of two main kinds: (1) aggregated masses of rounded cells, which formed the matrix of the future larval skeleton, (2) wandering cells or amœbocytes. Of these latter again there were two varieties, of which the first were obviously stellate in form and connected with each other and the walls of the blastocœle by long processes. This kind of cell has been seen and figured by Théel ("The Development of Echinocyamus pusillus," Hjalmar Théel, *Royal Soc. Upsala*, 1892), who however regarded the union of the processes of these stellate cells as a secondary phenomenon.

The second kind of wandering cell is rounded in form, and is in fact precisely similar to the Amœbocyte familiar to all who have studied Echinoderm anatomy. It has always been described and figured as if it were perfectly free: yet if one examines a slightly compressed living gastrula of Echinus esculentus one observes that these rounded cells, which at first sight look as if they were completely free, are in every case connected either with neighbouring cells or with the walls of the blastocœle by excessively fine threads along which they apparently travel.

Whilst not wishing to go so far as Mr Sedgwick in denying reality to the conception of the cell, I am inclined to hold that the cell structure of the Metazoa is largely due to secondary differentiation, and that a multinucleate Protozoon like Actinosphærium is to be compared to the Metazoon body, and not to a single unit of the same.

(3) *Crania from Teneriffe.* By F. C. Shrubsall, B.A., Clare College.

When engaged in a course of study at the Vesalianum at Basel, Professor Kollmann pointed out the importance of the Guanches and advised a detailed examination of that race if ever an occasion should offer. This spring, having to spend a short time in Teneriffe, I gladly availed myself of the opportunity to measure and describe all the Guanche skulls of whose existence I could learn in the island. I first examined those in the possession of the British Vice-consul, and afterwards worked at various private collections, both in Santa Cruz and Orotava, besides which I have measured the crania in the Vesalianum at Basel, those in the Anatomical museum at Cambridge, and those in the museum of the Royal College of Surgeons, London. To the owners and curators of these collections I must return my sincere thanks for the great kindness and courtesy with which they gave me every facility in their power for my study.

Altogether I have examined 54 male and 39 female skulls, and some 200 long bones, while in the seriations at the end of the paper I have included the various indices of the skulls in the collection from the Canary archipelago at Las Palmas, a printed copy of the measurements of which was lent me by Don Ramon Gomez of Puerto-Orotava.

The following are the details of the examination:

Cranium.

On taking the cranial capacity of skulls from Teneriffe it is at once apparent that while they can roughly be divided into two series, the large and the small, the classification thus established does not extend to other features, all the variations of face and cranium hereinafter to be described being found in either series. The average capacity of the large male skulls is 1550 cc. (maximum 1600 cc., minimum 1425 cc.), while that of the series of small male skulls is 1180 cc. (maximum 1250 cc., minimum 1155 cc.). The total mean of the male skulls is 1451 cc., and the average capacity of the females is 1375 cc., a mean sexual difference of 75 cc. or $\frac{1}{18}$ of capacity. This at first sight anomalous result being doubtless

due to the fact that all the female crania available for the purpose of this measurement belonged to the type of large skulls. On seriating the table of capacities maxima appear on the curve at 1525 and 1160, shewing the existence of the two series. The mean capacity of male skulls from Teneriffe is given by Verneau as 1672 cc. and by Flower as 1426. Comparing the capacity of these crania with that of other representatives of the Mediterranean race we have

Cromagnon, No. 1, 1590,
Solutré, No. 5, 1500,
Caverne de L'Homme Mort, 1606,
Grotte Baye, 1534,
Basques, 1564,
Teneriffan (excluding small skulls), 1550.

Crania from Teneriffe are on the average long and fairly broad, the greatest transverse diameter being usually bitemporal and the length being due to development of the occipital region. The average length of male skulls is 181 mm., and the average breadth 139 mm., giving a mean cephalic breadth index of 76·8, while in females the average length 178 mm. and the average breadth 139 mm., giving an index of 78·1; the mean index for males and females combined being 77·3.

Comparing this with the mean indices of crania from the different islands of the Canary archipelago as represented in the museum at Las Palmas and with other races, we have

Teneriffe, 78
Grand Canary, 76·7
Gomera, 77·2
Hierro, 73·2
} mean for the whole archipelago, 76·3,

Caverne de L'Homme Mort, 73·2,
Grotte de Baye, 78,
Spanish Basques, 77·6.

These figures shew a general relationship between the Cromagnon race and the former inhabitants of Teneriffe, which is further brought out by a comparative seriation of the cephalic indices of adult skulls.

Index	Teneriffe	Grand Canary	Gomera	Total Archipelago	Caverne de L'Homme Mort	Spanish Basques	Les Palmas Collection. Canary Archipelago. Males & Females	Teneriffe Females	Grand Canary Females	Gomera Females
68		1		6	1		6			
69										
70			1	3	2	1	4			1
71	1	1		2	3		3		2	
72	1	2		8	3		12	1	1	1
73	1	5	3	9	3	6	11		2	1
74	4	2	4	10	3	5	14	3	3	1
75	11	9	1	21		7	20	1	2	2
76	17	3	3	22	1	9	33	6	3	5
77	11	6	3	23		10	24	6	1	
78	17	5	4	25	2	8	34	9	1	3
79	3	1	3	8		9	13	6		1
80	4	2	2	9		2	14	7	1	3
81	2		2	4		1	12	6	2	2
82			1	1		2	3		2	
83							1	2		
84			2	2			3			1

Comparisons are also furthered by classifying the indices according to Brocas divisions.

Table of Percentages of Cephalic Index.

	Males		Males		Females	
	Modern skulls from Ossuary at Orotava, Lajard	Guanches, Verneau	Teneriffe	Total Archipelago	Teneriffe	Total Archipelago
True Dolichocephalic	52	37	18	31·4	10·3	23·7
Sub-Dolichocephalic	28	40·7	52·9	39·9	27·6	27·6
Mesaticephalic	16	18·9	23·6	20·9	37·9	26·3
Sub-Brachycephalic	4	3·4	5·5	6·5	20·7	18·4
Brachycephalic				1·3	3·4	3·9

The wide range covered by the indices, viz. 13 units both in male and female skulls is sufficient to indicate a mixture of races, while the foregoing table shews that comparative uniformity must have existed at the time of the Spanish conquest of the Canaries.

With regard to height, the skulls in the main are low rather than high, the mean altitudinal index for males from Teneriffe being 70·5, and for females 72·2, as against male 72·5, and female 71·9 for the whole archipelago. Contrasting with other series, we have for *males*,

Guanches,	70·5,
Cromagnon, No. 1,	65·3,
Solutré, No. 5,	70·7,
Caverne de L'Homme Mort,	68·8,
Spanish Basques,	70·7,
Modern Teneriffan,	73·5;

and for *females*,

Guanches,	72·2,
Grenelle, No. 2,	72,
Solutré, No. 2,	74·6,

again shewing the close resemblance to the Cromagnon race of the former inhabitants of the archipelago. The division into classes by the altitudinal index is as follows:

	Males		Females	
	Teneriffe	Total Archipelago	Teneriffe	Total Archipelago
Chamæcephalic	24·1	18·9	30·8	27·4
Orthocephalic	56·9	59·8	50	58·9
Hypsicephalic	19	21·3	19·2	13·7

Contrasting breadth and height, the mean index for male Teneriffan skulls is 92·9, and for females 92·1. Compare

Cromagnon,	88·6,
Solutré, No. 5,	93·7,
Caverne de L'Homme Mort,	96·4,
Spanish Basques,	92·4.

Classifying by this index according to Brocas divisions, we obtain the following table:

	Males		Females	
	Teneriffe	Archi-pelago	Teneriffe	Archi-pelago
Akrocephalic	40	28·1	44·4	37·5
Metriocephalic	45·5	45·5	48·1	45·8
Tapeinocephalic	14·5	26·4	7·4	16·7

In the great majority of crania from Teneriffe the frontal region is well developed, the forehead being full, rounded and decidedly broad, although in some which constitute a distinct type by themselves it is low and receding.

The glabella is only moderately prominent, but the superciliary ridges are well developed even in the female, the subcerebral curve contributing on the average in the male 23·9 per cent. and in the female 17·2 per cent. of the total frontal curve.

The frontal eminences are usually prominent but do not reach to the level of the mid-sagittal line.

The sagittal curve in most typical cases runs uniformly back to the lambda, but in some crania the annular post coronal depression of the middle third of the parietal bone described by M. Vernau is to be seen. This depression, which somewhat resembles that produced by a bandage tied tightly round the head during early youth, is, according to that author, not an artificial deformity but a natural variation in the sagittal curve. After passing the lambda this curve often shews a distinct flattening over the upper part of the occipital bone, and when this is present there is a decided fulness of the whole occipital region, the projection backwards taking place immediately at the parieto-occipital suture. From the inion the curve passes sharply downwards and forwards to the opisthion, the bone in this region being flattened and strongly marked by muscular impressions. In many cases the conceptaculæ cerebelli are so far developed that the skull rests posteriorly upon them rather than upon the mastoid processes or the occipital condyles. The inferior portion of the occipital bone is horizontal. The relationships in point of size of the various portions of the curve are shewn in the following table, in which they are compared with the length of the total sagittal curve (= 100):

	Frontal	Parietal	Occipital
Males	33·5	34·4	31·7
Females	33·7	32·6	33·6

Making similar comparisons for other curves.

The relation of the supra-auricular to the total transverse curve (= 100) is in males 65·6 and in females 66·8, while that of the preauricular to the total horizontal (= 100) is for males 47·7 and for females 45·4 as compared with

45·3 in skulls from Caverne de L'Homme Mort,
and 46·1 in Basques.

However on seriating the results of this last relation, we find that very few crania present this actual mean relationship but group themselves round the numbers 41 and 52, indicating a greater frontal and occipital development respectively.

Viewed in norma occipitalis the skulls are somewhat pentagonal in form, but the parietal eminences are not very prominent, and the greatest transverse diameter in about 80 per cent. of crania lies between the temporal bones. In the better class of mummies, presumably those of chieftains of some importance, there is in several instances, more frequently so than in the less cared for bodies, a prominent median sagittal ridge along a groove in which the interparietal suture runs for the posterior half of its course.

The temporal crest is always well marked both in male and female crania, the double nature of the ridge being very distinct. The zygomatic processes are stout, strongly arched outwards and with a supramastoid ridge usually extending up on to the posterior inferior angle of the parietal bone.

The pterion is of the ordinary H shape and anomalies are rare in this situation. In many cases the sphenoid and frontal bones close behind the external orbital process are markedly bulged outwards, and in the average among males the transverse diameter from one pterion to the other is greater than the bistephanic breadth.

Viewed in norma verticalis the crania are plainly phænozygous, and appear either pentagonal or, as in a smaller number of cases, oval ; a slight projection of the face is also noticeable.

The mean dimensions appear in the following table:

	Males		Females	
	Absolute measurement in millimetres	Relative to Max. B = 100	Absolute measurement in millimetres	Relative to Max. B (= 100)
Minimum Frontal Breadth	95	68·3	94	67·6
Interpterion Breadth	112	80·6	107	77
Bi-stephanic Breadth	111	79·9	112	80·6
Bi-asteric Breadth	114	82	107	77
Maximum Transverse Breadth	139	100	139	100

The fronto-zygomatic index of skulls from Teneriffe is for males 83·3 and for females 89.

Cf. Caverne de L'Homme Mort, 89·9,
Basques, 91·8,
Grotte de Baye, 83·5.

This series of crania may readily be classified into two groups according to the complication of their sutures. In the first and smaller division the sutures are simple and wormian bones the exception; while in the second series the sutures, more especially the lambdoid and coronal, are excessively complicated, while it is rare to find even the sagittal suture simple and wormian bones are very common. A persistent metopic suture is to be found in about 8 per cent. of crania of the second series; it may be complete, or persistent near either the bregma or the glabella, the latter arrangement being the more frequent.

Synostosis usually commences in the sagittal and lower part of the coronal sutures; contrary to the usual custom the lambdoid suture is quite the last to close. M. Verneau indeed describes a case in which the squamous suture had closed up while the lambdoid was still open.

Wormian bones are the rule in the lambdoid suture, the commonest arrangement being a series with their long axes nearly vertical and their short axes arranged along the line of the suture, the two sides being frequently asymmetrical. There may be a single 'os-epactal' or an 'os-epactal' with a small median wormian bone in front of it.

Passing on from the study of the cranium to that of the face we find a considerably greater range of variation. Faces from

Teneriffe are for the more part orthognathous, broad above, narrowing down in the maxillary and mandibular regions, with low, broad very rectangular orbits, superciliary ridges well marked and frontal sinuses capacious and extensive; but one also meets with skulls presenting the features of long, uniformly narrow, slightly prognathous faces with high and rounded orbits. The nose is as a rule straight, prominent and leptorhine, but in other cases it is platyrhine, and the lower border instead of being sharply cut, as is typical, may be rounded off or even present simian grooves. The nasal spine is not prominent. The palate is of almost all forms and sizes, the most typical being the ellipse, although the parabolic form is also well represented, the alveolar border projects forwards and accounts for the slight alveolar subnasal prognathism frequently found in skulls from the Canary archipelago. The lower jaw is strong, broad at the condyles and angle, but narrow at the chin. The angle is frequently very markedly everted, shewing the strength of the muscles, thus correlating well with the strength of the zygomatic arch and the great wearing down of the teeth, all being doubtless due to the grinding efforts necessary to masticate the but partially cooked cereals which constituted then, as now, the staple diet of Teneriffe, and which also accounts for the frequent traces of alveolar abscess in Guanche skulls.

The various facial indices are tabulated in order below.

Superior facial index of Broca.

Mean male, 63; female, 65.

In the seriation table of this index the range is from 58—87 (29 units) exhibiting maxima at 62 and 66 for males and at 68 for females.

Cf. males,	Cromagnon, No. 1,	63·4,
	Grenelle,	66·7,
	Grotte de Baye,	66·4,
	Homme Mort,	67·6,
	Basques,	67·8.
females,	Grenelle, No. 2,	66·5,
	Solutré, No. 2,	66·4.

Upper facial index of Kollmann.

Mean for males, 50·2; mean for females, 47·7.

The skulls therefore lie on the border line of lepto and chamæprosopy, the percentages in either division being as follows:

	Males		Females	
	Teneriffe	Archipelago	Teneriffe	Archipelago
Chamæprosope	42·9	47·1	82·3	85·7
Leptoprosope	57·1	52·9	16·7	14·3

This shews that while the males tend towards leptoprosopy the females are almost entirely chamæprosopic, and if these indices are viewed together with the orbital and nasal indices the very disharmonic nature of the face, more especially in the female skulls, is at once brought to notice.

The distribution of skulls from Teneriffe among the types of Prof. Kollmann is shewn by the following table:

	Males	Females
Chamæprosope Dolichocephals	3·0	15·4
Chamæprosope Mesocephals	36·4	38·5
Chamæprosope Brachycephals	6·1	23·1
Leptoprosope Dolichocephals	12·1	
Leptoprosope Mesocephals	42·4	15·4
Leptoprosope Brachycephals		7·7

The mean *orbital index* is for males 83·6 and for females 87·2. Seriation of this index shews a range from 68—97 (29 units) with maxima at 80, 83, and 89.

Cf.
Solutré, No. 1, 74·3,
Homme Mort, 80,
Grotte de Baye, 81·4,
Spanish Basques, 82·6.

The orbits are frequently asymmetrical, the difference in index between the right and left sometimes amounting to 3 or 4 units.

The *nasal index* shews a range from 41—60 with maxima at 45—48 and 52. Mean indices being male 47·6; female 41·5.

Cf.
Cromagnon, No. 1, 45·1,
Grenelle, No. 1, 49·2,
Solutré, No. 5, 52·4,
Homme Mort, 45·4,
Spanish Basques, 43·8.

The nasal index of the ancient can be compared with that of the modern inhabitants of Teneriffe, in whom there is a considerable admixture of Guanche blood, by means of this table of percentages,

	Modern (from Ossuary at Orotava) *Lajard*	*Ancient* Inhabitants
Leptorhine	27	44
Mesorhine	57	35·4
Platyrhine	8	20·6

shewing the convergence to a mean type in the modern skull.

The general features of the face can easily be traced in a combined table of facial, orbital and nasal indices, in which I compare the results of my measurements in Teneriffe and elsewhere with those of the collection of skulls from the whole archipelago at Las Palmas and with those of M. Verneau in Teneriffe itself.

Males.

	Teneriffe. M. Verneau			Teneriffe			Archipelago		
	Facial	Orbital	Nasal	Facial	Orbital	Nasal	Facial	Orbital	Nasal
Microseme	25·9	55·6	74·1	38·5	48·5	50	44·2	46·3	50·4
Mesoseme	37	33·3	18·5	34·6	31·8	32·8	22·1	33·6	38·2
Megaseme	37	11·1	7·4	26·9	19·7	17·2	33·7	20·1	11·4

Females.

	Teneriffe. M. Verneau			Teneriffe			Archipelago		
	Facial	Orbital	Nasal	Facial	Orbital	Nasal	Facial	Orbital	Nasal
Microseme	30	42·9	61·5	25	29	39·4	36·2	29·5	38
Mesoseme	40	28·6	30·8	33·3	25·8	30·3	27·6	30·8	38
Megaseme	30	28·6	7·7	41·7	45·2	30·3	36·2	39·7	24

The *palatal index* has a range of 61—94 with maxima at 78 and 89, mean index for the series male, 79·3; female, 78·9.

Cf.
L'Homme Mort, 72·7,
Spanish Basques, 75·2,
Grotte de Baye, 75·5,
French Basques, 79·6.

Classifying these indices according to Virchow's divisions we find among males well represented lepto- and brachystaphylinic groups with somewhat smaller numbers intermediately.

	Males.		Females.	
	Teneriffe	Archipelago	Teneriffe	Archipelago
Leptostaphylin	56·5	59·5	58·6	66·2
Mesostaphylin	16·1	19·1	27·6	23·9
Brachystaphylin	27·4	21·4	13·8	9·9

The *mandible* is characterised by the narrowness of the chin, the strength of the muscular impressions and alveolar border; the eversion of the angle and the prominence of the genial tubercles.

The mean goniozygomatic index is in the male 77·6, and in the female, 72·5, cf.

Mediterraneans, 77,
Mean European, 77·6,
Arabs, 74·2.

Summing up the results of the foregoing measurements it appears that four types of skull existed in Teneriffe at a period anterior to the Spanish invasion.

I. The first and most important type in respect of numbers presents the features of a capacious skull with moderately prominent glabella and superciliary ridges, a well-developed frontal region and full rounded forehead. It has a sagittal curve presenting a post coronal depression, a lambdoid flattening and a great filling out and projection of the occipital region of the cranium, while the greatest breadth is bitemporal. The sutures are very complicated, metopism is not infrequent and wormian bones are very prevalent in the lambdoid suture. The face is short, broad above, narrowed below, orthognathous, with low broad very rectangular orbits, a leptorhine sharply cut nose, slightly sunken at the root, an elliptical palate and greatly worn teeth.

This type, which from its superior numbers and greater prevalence in Teneriffe than in the other islands of the group is to be regarded as that of the Guanches, may be shortly described as mesaticephalic, orthocephalic, orthognathous, microseme, leptorhine and leptostaphylin.

All the succeeding types of skull differ from the above in the absence of the lambdoid flattening of the sagittal suture and in the absence of the fulness and projection of the occipital region.

II. The second type, distinguished by a smaller capacity, a retreating forehead, greater height of skull and lower cephalic index combined with simpler sutures, comparative absence of wormian bones, much stronger muscular impressions, larger mastoids, and more prominent inion, is somewhat wedge-shaped when viewed in norma verticalis. It differs but little from the preceding in facial features, but has a narrower mandible and a parabolic palate. This type, which is well represented in Teneriffe and Hierro, may be described as dolichocephalic, orthocephalic, mesoseme, leptorhine and brachystaphylin.

III. The third type presents the features of an ovoid skull of small capacity with glabella, superciliary ridges and parietal eminences not very prominent, although the greatest transverse diameter is interparietal. It is dolichocephalic and hypsistenocephalic with a sagittal curve rising directly and uniformly from the nasion, a face prominent and slightly prognathous, a nose straight, sharply cut at lower border but not depressed at the root, orbits high and rounded and a palate decidedly long with the ends of its dental arcade seldom or never incurved.

This type of skull, found more frequently in Grand Canary than in Teneriffe, is apparently that described by M. Verneau as Semitic.

IV. The fourth type is a short broad skull of fair capacity, with a nose not sharply cut but rounded off at its lower border and often platyrhine or high mesorhine, with high and rounded orbits and a palate in some cases long and parabolic, but in a few, elliptical. This type appears to occur in the N. and E. of Teneriffe and in Gomera more freely than in the rest of the archipelago.

Long Bones.

As these were only studied in the actual burial caves and their vicinity, situations somewhat difficult of access, but a few slight observations were possible. Taking the bones in order:

The *humerus* as a rule was strong and well-marked, the deltoid impression and musculo-spiral groove being especially prominent,

the olecranon fossa was frequently perforated (in 23 per cent. of the bones examined). The average maximum length was in the male 323 mm. and in the female 304 mm.

The *radius* and *ulna* were in most cases strongly marked, but presented no special features of interest.

Average length, *radius* male 247 mm., female 232 mm., but, however, very few examples were measured of the forearm bones of the latter sex.

Ulna. Male 265 mm., female 243 mm.

The *femora* were large and strong, deeply marked by muscular impressions and often pilastered. The average length in the oblique position with both condyles touching the lower end of the measuring apparatus was for males 424 mm. and for females 414 mm.

The *tibiae* were strong and in some instances platycnemic, the average length being for males 365 mm. and for females 335 mm. The *fibulae*, which were deeply channelled, were of the mean length of 352 mm. in the case of the male and 331 mm. in that of the female bones. Calculating the height of the former living inhabitants from these measurements by means of M. Manouvrier's sliding scale it is found to have been 1642 mm. for the males and 1552 mm. for the females. The results from each series of bones is shewn in the subjoined table.

Calculated height of living individual in millimetres:

Bone	Male	Female
Humerus	1637	1560
Radius	1666	—
Ulna	1668	1578
Femur	1610	1538
Tibia	1644	1542
Fibula	1626	1542
Mean height	1642	1552

giving a mean sexual difference of 90 mm.

Comparing the mean obtained for Teneriffe with the heights arrived at by Manouvrier's scale for the races with which the crania were compared, we have

Teneriffe,	1642,
Cromagnon,	1750,
L'Homme Mort,	1620,
Grotte de Laugerie,	1650.

A seriation of the heights computed from bones in Teneriffe shews a range in the case of males of 1550—1710 (160 units) and in the case of females of 1460—1660 (200 units) with maxima at 1640 and 1550 respectively: this range is sufficient to indicate a probable original mixture of tall and short races, although it is clear from historical evidence that the height was fairly even throughout the island at the time of the Spanish invasion and that the Guanches were tall.

In an attempt to reconstruct the features of the former inhabitants we draw our evidence from three sources, the present population, the chronicles of the Spanish invasion and such ancient records of the history of the island as we possess.

Turning to the first, we find congregated in and near the towns on the north and east people with olive skins, dark eyes and black or dark brown hair, obviously of Spanish descent; lying further out and distributed in the country districts throughout the island is a fairer complexioned, brown eyed, chestnut haired race mixed among whom are isolated families of the former type, while in the rugged and less easily accessible southern side where the Guanches last held out and where their descendants are still supposed to remain in the greatest purity, there are many of fair complexion, blue eyes and fair or flaxen hair and golden haired children are quite common. From the second source we learn that many of the inhabitants of the north of the island at the time of the invasion were fairer than the Spaniards, although some of those in the South were dark, while the native records tell the adventures of a golden haired blue-eyed princess. When added to this we find in many mummies a profusion of short yellow hair, although this latter evidence is not strictly admissible owing to post mortem changes, there is some reason for concluding that at any rate one race in the island presented the features of a fair complexion, blue eyes and light coloured hair, and belonged essentially to the blonde type of Eurafrican peoples. It is also very probable that there were also in the island from early times people of darker complexion and hair. In the eastern islands of the Canary archipelago early voyagers describe a race with dark skins, dark hair and thick lips; some of these people doubtless of African origin might have spread into Teneriffe and account for some of the peculiar features of that country.

In conclusion there appear to have been in Teneriffe,

(1) A dominant race, the Guanches, tall, blonde, blue-eyed, muscular, with a skull of the first type.

(2) A race with a skull of type two, also tall and, as far as evidence from their mummies go, probably blonde.

(3) A Semitic race, with a skull of the third type, probably dark and of medium height.

(4) A brachycephalic race (type IV) probably short and dark.

The records in European history anterior to the XVth century relating to the Canary islands are very scanty, but from them we learn that the only races known to have been aware of the existence of that archipelago and thus to have come in contact with and possibly had some influence on its population are the Greeks and Phoenicians possibly, the Carthaginians certainly; the Mauritanians who invaded the islands but were repulsed except at Lanzarote and Furteventura; the Irish, whose missionaries under St Brondan appear to have reached the 'Fortunate Islands,' and finally the modern Europeans of whom one nationality, the Spaniards, conquered the Archipelago. At the time of the conquest the islands were certainly inhabited by mutually hostile tribes differing however only slightly in physical features and speaking similar languages, although on these last points there is a conflict of evidence between the various chroniclers of the invasion, and as the earliest accounts of these people agree in most respects with those of the 'conquistadores' and also with later descriptions, there can be no doubt that the islanders must have been left undisturbed for a great number of years, perhaps of centuries, uninfluenced by the great migrations which have shaken Europe and rendered racial differentiation therein so difficult. Possibly then the Guanches, or typical race of Teneriffe, may represent an early race of mankind existing down to the present day in comparative purity under conditions similar to those which have preserved the characteristic fauna of Australasia. Their physical features lend support to such an hypothesis; for, on the one hand, the Guanches resemble the Cromagnon race, more especially those relics of it discovered at Solutré, and also the Basques of Northwestern Spain, while, on the other hand, they have many features in common with the Kabyles, Berbers, and Hamitic tribes generally of North Africa. The other and less dominant types in the island have very probably been introduced at the time of the various discoveries of the archipelago by the Mediterranean races.

Table of Seriations.

Length, Height, Index.

Index	Grand Canary		Teneriffe		Gomera		Total Archipelago.	
	Males	Females	Males	Females	Males	Females	Males	Females
63			1				1	
64			1				1	
65		2		1				3
66		1	2			1	2	2
67			4				4	1
68	2	1	2	3		1	4	5
69	2	3	5	2	2		10	5
70	2	1	8	10	2	3	12	14
71	4	1	12	3	3	3	19	8
72	5	3	11	6	3	3	20	12
73	3	2	7	6	6	4	17	14
74	5	4	4	3	4	3	13	12
75	6		7	4			13	4
76	3	2	4	1	2	1	9	4
77	3	1	1	2	1		5	3
78	1	1		1	3		4	2
79	2		1				3	
80				1				1

Table of Seriations.

Breadth, Height, Index.

Index	Grand Canary		Teneriffe		Gomera.		Total Archipelago	
	Males	Females	Males	Females	Males	Females	Males	Females
78	1						1	
79								
80				1				1
81			2				2	
82						1		1
83								
84				1				1
85								
86			1	2	1	1	2	3
87	1		1	4			2	4
88		1	4	4	2	1	6	6
89		1	6	2	1	1	8	4
90	1	1	3	3	2	5	7	9
91			5	6		1	5	8
92	1	1	6	1	2		9	2
93	4	3	7	3	2	2	14	8
94	2	2	9	8	2	2	13	12
95	1	5	3	4	3		7	11
96	9	1	4	3	2	2	15	6
97		1	3		1		4	2
98	2	2	5	1	1	1	8	4
99	3		4	2	1	1	8	3
100	3	1			1		4	2
101	6	1	2				8	2
102	2					1	2	1
103	1		1				2	
104			1		2		3	
105					1		1	
106					1		1	
107					1		1	
108	1						1	

Table of Seriations.

Upper Facial Index of Broca.

Index	Grand Canary		Teneriffe		Gomera		Archipelago	
	Males	Females	Males	Females	Males	Females	Males	Females
56				1				1
57								
58			1	2			1	2
59					2	1	2	1
60			1		1		2	
61			1	1	1	1	2	2
62	1	1	5		3	1	9	2
63	3		2	5	3	1	8	7
64	1		1	2	2	2	6	6
65	3		1	1	2	2	6	3
66	2		5	2	2	1	10	3
67	2	3	4	2	1	2	8	7
68		2	2	2		1	2	5
69	3	2	1	4	1		5	6
70	2		1		2		5	
71	1		3	2			4	3
72	1	1	2	2			3	3
73	3						3	
74	2	2	1				3	2
75	1	2	2				3	2
76		1						1
77		1						1
78	1	1					1	1
79	2			1			2	1
80								
81								
82		1						1
83								
84								
85								
86								
87			1				1	
88				1				1

Table of Seriations.

Orbital Index.

Index	Grand Canary		Teneriffe		Gomera		Archipelago	
	Males	Females	Males	Females	Males	Females	Males	Females
68			1				1	
69								
70						1		1
71					1		1	
72								
73					1		2	
74			3				3	
75	2		2	1			4	1
76	1		3		1		5	
77	3	1	7	1	1		11	2
78	1	1	3				4	1
79	1	1	3	1	1		5	2
80	4	2	7	1	2		14	3
81		3	4	3	2		6	7
82	1	1	4	3	4	2	9	6
83	6		6	5		2	13	7
84	6		4	5	1	1	11	6
85	2	1	2	2	2	1	6	4
86		2	11	3	5	7	15	13
87	2		6				8	
88	1			1		2	1	3
89	3	5	8	12	4	3	16	22
90						1		1
91	1	1	1	2		1	2	5
92		1	3		1		4	1
93								
94	1	2	1	4			2	6
95	2				1		3	
96				1				1
97			1	3			1	3
100				1				1

Table of Seriations.

Nasal Index.

Index	Grand Canary		Teneriffe		Gomera		Total Archipelago	
	Males	Females	Males	Females	Males	Females	Males	Females
37			2				2	
38	1			2			1	3
39			1	1		2	1	3
40				2				2
41	2		3	1			6	1
42			5	1	2	2	7	3
43	1	2	2	2	1	1	5	5
44	2	2	6	4	3	1	11	6
45	4	1	9	4	1		14	6
46	3	1	4	3	4	1	11	5
47	5	1	6	3	3	1	13	4
48	4	2	7	6	1	4	13	12
49	2		6	4	2	1	10	6
50	4	5	1	2	4		9	8
51	2		4	4	3	2	9	6
52	3	2	7	1	2	1	12	5
53		1	1	3		2	1	6
54	2	1	2	1	1		5	2
55		1	3			1	3	2
56		1	4	3			5	4
57		1		1				2
58	1		1			1	2	1
59				2				2
60			1				1	
67			1	1			1	1

Table of Seriations.

Facial Angle.

Index	Grand Canary		Teneriffe		Gomera		Total Archipelago	
	Males	Females	Males	Females	Males	Females	Males	Females
68			1		1		2	
69								
70	1			2	1		2	2
71	1	1				2	1	3
72	4	2	1	1	1	1	7	5
73	3	1		2	2		5	4
74	2	1	3	3	3	2	10	8
75	7	2	5	7	3	3	16	12
76	3	1	6	2	5	1	14	4
77	3	3	4	2	7	6	14	12
78	3	2	4		2	3	9	5
79	1	2				2	1	4
80	2		3		2		7	
81	2		1				3	
82				1				1
83		1	2			1	2	2
84		1						1
85		1						1

Table of Seriations.

Height (by Manouvrier's method).

Index	Teneriffe.		Index	Teneriffe.	
	Males	Females		Males	Females
1460		1	1620	13	
1470			1630	18	
1480		1	1640	23	1
1490		1	1650	16	
1500		2	1660	19	1
1510		3	1670	9	
1520		3	1680	4	
1530		7	1690	3	
1540		11	1700		
1550	2	10	1710	2	
1560		8	1720		
1570		9	1730		
1580	8	3	1740		
1590	5		1750		
1600	7	2	1760		
1610	17	1			

Length of Long Bones and Height calculated therefrom.

Femora

Series I.

Sex	Length	Height
M.	413·5	1584
M.	413	1583
M.	439	1636
M.	417·5	1614
M.	412	1580
M.	422	1608
F.	397	1496
M.	410·5	1576
F.	408	1527
F.	406·5	1524
M.	419·5	1604
M.	425	1612
M.	439	1636
F.	447	1607
M.	454	1661

Series II.

F.	413	1536
M.	421	1606
M.	435	1629
M.	419	1602
F.	385	1462
M.	412·5	1581
M.	427	1615
M.	440	1638
M.	471·5	1693
F.	410	1530
M.	416·5	1593
M.	418	1599
M.	422	1608
F.	413	1536
M.	426	1614
F.	409	1529
F.	407	1525
M.	423	1609
M.	420	1606
M.	436	1627
M.	417	1595
F.	422	1552

Series III.

M.	440	1638
F.	418	1544
M.	425	1613
M.	429	1619
M.	445	1648
M.	453	1660
M.	437	1633

Series IV.

F.	413	1536
M.	440	1638
F.	419	1546
M.	414	1585
M.	425	1613
F.	400	1505
F.	411	1532
M.	437	1633
M.	441	1640
F.	429	1566
M.	455	1662
M.	462	1672
F.	434	1575
M.	441	1640
M.	453	1660
M.	459	1668
M.	427	1616
F.	412	1530
M.	440	1638
M.	455	1663
M.	445	1648
M.	437	1633
F.	431	1569
F.	418	1546
M.	443	1644
F.	429	1566
M.	481	1713
F.	412	1534
F.	417	1582
M.	432	1624
M.	428	1617
M.	416	1593
M.	426	1614

Tibia

M.	359	1632
M.	368	1651
F.	342	1557
M.	375	1661
M.	360	1634
M.	342	1599
F.	315	1491
F.	329	1529
F.	342	1557
M.	360	1634
M.	373	1661
F.	339	1550
M.	375	1664
M.	381	1677
M.	361	1636
M.	363	1640
M.	358	1630
M.	372	1659
M.	355	1624
F.	343	1560
M.	365	1644
M.	363	1640
M.	376	1666
M.	369	1653
M.	382	1679
M.	377	1668
F.	333	1538
M.	355	1624
M.	358	1630
M.	367	1648
M.	373	1661
M.	366	1646
M.	374	1662
M.	369	1653
M.	363	1640
M.	341	1595
M.	357·5	1629

Fibula

M.	343	1607
M.	356	1634
F.	339	1562
M.	352	1626
M.	361	1646
F.	341	1566
M.	344·5	1609
M.	345	1610
M.	357	1636
M.	359	1642
M.	361	1646
F.	323	1523
F.	327	1535
M.	345	1606

LENGTH OF LONG BONES AND HEIGHT OF LIVING CALCULATED THEREFROM.

Humerus						Radius			Ulna		
Sex	Length	Height	Sex	Length	Height	Sex	Length	Height	Sex	Length	Height
M.	322	1634	M.	328	1652	F.	307	1566	M.	252	1690
M.	331	1659	M.	316	1619	F.	297	1542	M.	248	1667
M.	327	1648	M.	323	1637	F.	302	1555	M.	245	1660
M.	310	1600	M.	325	1644	M.	311	1605	F.	242	1661
F.	299	1546	M.	329	1655	F.	297·5	1546			
F.	288	1512	M.	331	1659	F.	282	1513			
F.	298	1548	M.	340	1675	F.	298	1547			
M.	325	1643	M.	332	1662	M.	329	1666			
M.	316	1619	M.	328	1652						
F.	319	1601	F.	305	1564						
M.	322	1634	F.	319	1600						
M.	324	1640	M.	325	1644						
F.	306	1564	M.	333	1664						
F.	309	1570	M.	327	1648						
F.	281	1480	F.	299	1546						
M.	315	1616	M.	319	1626						
F.	298	1511	F.	323	1612						
F.	303	1556	M.	329	1655						
M.	315	1616	M.	331	1659						
M.	306	1584	M.	338	1677						
M.	322	1634	M.	326	1646						
F.	307	1567	M.	317	1621				M.	274	1710
M.	331	1659	F.	314	1583				M.	268	1677
F.	296	1538	M.	330	1657				M.	270·5	1694
F.	302	1554	F.	306	1564				M.	265	1669
M.	307	1585	F.	309	1570				M.	267	1674
M.	314	1614	M.	318	1624				M.	256	1638
			M.	324	1640				F.	231	1542
			M.	315	1616				F.	243	1583
			F.	298	1541				F.	255	1645

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MEASUREMENTS OF MANDIBLE IN MILLIMETRES.

MALES. Number...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Museum and Catalogue Number...	Cambridge		Santa Cruz			Orotava					Barnard Davis Collection			Royal College of Surgeons				
											505	508	510	566	567	568	569	569A
Bi-Condyloid Breadth	116	114·5	116	125	—	123	114	113	126	110	114	120	114	119	—	117	116	124
Bi-Gonial Breadth	95	115	102	96	98	101	102	84	106	83	97	94	90	92	100	106	95	99·5
Bi-Mental Breadth	34	37	45	—	—	—	—	46	51	47	44	44	34	29	33	43	21	42
Symphysial Height	29	29	33	33	30	31	24	25	38	29	31	30	30	33	31	28	28	36
Molar Height	27	27	31	30·5	29	29·5	25	26	40	30	29	24	23	26	26·5	34	24	30
Ramus Length	49	51	51	53	—	—	—	56	62	55	53	53	43	49	51	61	51	60
Ramus Breadth	42	37	37	42	33	37	38	28	40	35	34	36	35	34	35	34	35	34
Bi-Gonial Curve	198	—	—	—	—	—	—	184	203	198	185	195	177	174	198	241	196	194
Total Facial Height	111	111	113	117	115	111·5	115	—	—	—	110	112	104	111	121	113	102	134
Bi-Zygomatic Breadth	127	134	—	139	126	131	126	—	—	—	—	—	122	131	141	137	131	132
Gonio-Zygomatic Index	74·8	85·8	—	69·1	77·8	77·1	81	—	—	—	—	—	73·8	70·3	70·9	77·4	72·5	75·4
Total Facial Index (Kollmann)	87·4	82·8	—	84·2	91·3	85·1	91·3	—	—	—	—	—	85·2	84·7	85·8	82·5	77·9	101·5

FEMALES. Number...	1	2	3	4	5	6	7	8	9	10	11
Museum and Catalogue Number...	Santa Cruz					Orotava			Royal College of Surgeons		
									506	513	514
Bi-Condyloid Breadth	120	116	112	115	111	—	113	108	102	102	108
Bi-Gonial Breadth	91	86	83	94	94	85	92	85	93	81	89
Bi-Mental Breadth	45	46	44	—	30	44	45	43	34	33	23
Symphysial Height	28	26	31	32	25	28	32	28	22	24	27
Ramus Length	53	60	46	52	40	53	51	48	36	47	45
Ramus Breadth	32	30	28	38	27	36	37	29	29	31	34
Molar Height	26	26	28	30	20	30	31	27	22	27	32
Bi-Gonial Curve	180	188	160	—	—	210	171	180	170	163	186
Total Facial Height	—	—	—	101·5	90	115	—	—	103	102	108
Bi-Zygomatic Breadth	127	126	120	116	116	131	—	119	—	128	127
Total Facial Index (Kollmann)	—	—	—	87·5	77·6	87·8	—	—	—	79·7	85
Gonio-Zygomatic Index	71·7	68·3	69·2	81	81	64·9	—	71·4	—	63·4	70·1

Number...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Museum. Catalogue Number	*s8*	Cambridge University Museum							Private Collections, Santa Cruz, Teneriffe																Vesalianum, Basel															Private Collection at Orotava						Barnard Davis Collection					Royal College of Surgeons							
		s6	*s7*	*s1*	1717	1715	1721	1718																	*m15*	*m18*	*m20*	*m21*	*m22*	*m23*	*m25*	*m26*	*m27*	*m29*	*m30*							494	503	505	508	510	566	567	568	569	569a	570	571	573
Maximum Length	—	174	189	188	177·5	191	181	173·5	179	182	176	188	178	180	174	189·5	183	180	179	180	189	184	187	182	195	—	176	189	181	190	186	175	182	—	169·5	179	183	182	177	179	185	183	195	191	181	183	182	193	194	187	192	187	181	182
Maximum Breadth	—	135	137	139	138	—	140	148	135	142	138	144	134	139	135	144	142	137	132	140·5	143	140	145	143	146	—	136	143	137·5	144	145	129	144	—	129	140	138	134	138	132	136	139	146	140	133	138	143	146	139	145	146	140	137
Basi-Bregmatic Height	—	120	—	139·5	128	—	137	130	126	128	128	135	124	127	121	132	134	133	127	136	134	138	132	128	—	—	131	141	—	135	135	128	136	128	130	121·5	130	134	130	127	138	127	130	129	137	137	140	140	140	133	128	134	131	130
Minimum Frontal Breadth	94	89	94	95	92·5	96	100	96	95	99	95	89	96	94	97	98	96	92	92	95·5	96	93	99	98	97·5	—	96	101	95	87·5	98	88	81	90	86	94	96	88	94	92	104	98	103	92	98	93	102	105	104	98	99	99	91	94
Pterion Breadth	—	104	111	107	112·5	115	116	120	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	121	114	100	115	108	114	123	124	108	127	106	111	108
Asterion Breadth	—	101	—	116	111	118	116	120	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	113	120	—	117	106	108	107	112	107	110	108
Curves — Sub-Cerebral	21	29	27	31	18	16	21	20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	108	106	114·5	97	107·5	108	116	119	110	116	110	104	106
Curves — Total Frontal	—	116	125	124	117	131	130	129	130	112	112	115	—	—	—	122	121	118	121	130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24	21	26	26	—	24	25	28	27	19	28	23	22
Curves — Parietal	—	112	130?	134	107	139	135	—	112	130	115	129	—	—	—	131	129	112	131	147	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	113	123	123	132	121	142	120	123	128	130	—	125	132	137	132	126	126	122	120
Curves — Supra-Occipital	—	66	—	73	78	—	67	72	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	118	111	123	135	131	140	114	138	111	128	—	121	130	120	132	139	120	122	126
Curves — Total Occipital	—	110	113?	118	—	—	110	109	115	118	111	122	—	—	—	111	118	125	110	103	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	84	71	81	71	—	76	67	89	70	77	82	58	79
Curves — Total Sagittal	—	338	371	376	—	—	375	—	357	360	338	366	—	—	—	374	368	355	362	380	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	119	128	126	113	110	108	121	122	135	109	—	117	114	138	116	125	122	107	115
Curves — Supra-Auricular	—	281	295	295	290	—	306	297	280	288	290	291	—	—	—	300	303	299	292	310	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	350	372	372	347	362	390	355	383	374	367	365	363	376	395	380	390	368	351	361
Curves — Total Transverse	—	409	—	472	430	—	451	444	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	285	294	298	312	292	300	309	—	305	311	302	302	312	320	302	323	305	303	302
Curves — Pre-Auricular	—	255	245	225	230	—	234	233	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	435	—	419	440	424	428	455	461	427	451	449	430	428
Curves — Total Horizontal	—	493	515	512	501	—	509	498	516	524	510	530	—	—	—	515	516	507	468	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	253	—	—	245	230	246	254	240	228	241	221	226	220
Foramen Magnum Length	—	35	—	38	—	—	40	36	36	35	36	34	35	36	37	40	33	33	37	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	507	516	503	515	468	530	508	—	—	510	499	516	545	540	522	545	525	505	503
Foramen Magnum Breadth	—	31	—	30	—	—	34	32	32	30	28	27	28	33	31	33·5	31	34	27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	37·5	40	33	32	37	32	35	40	35	40	36	34	41·5	37	36	36·5	37	37	37
Basi-Nasal Length	—	95	—	105	101	—	96	96	100	95	102	105	102	95	93	101·5	100	104	92	95·5	97	95	91	105	—	—	—	—	—	—	—	—	—	—	—	30	32	31	29	27	30	30	34	32	31	29	29	31	31	30	30	31	30	31
Basi-Alveolar Length	—	90	—	100	99	—	92	89	92	89	95	100	98	91	89	94	92	98·5	83	88	90	86	87·5	95	—	—	94	104	—	105	106	97	97	91	91	95	98	98	96	92	103	101·5	101	101	100	104	99	104	100	99	102	99	102	104
External Bi-Orbital Breadth	101	98	104	115	105	102	109	108	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	93	89	—	100	94	88	88	90	87	89·5	101	95	—	83	95	99	91	90	91	94	93	98	97	97	94	98	101	94
Internal Bi-Orbital Breadth	92	93	97	105	97·5	96	100	101	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	108	105	103	107	102	111	111	112	108	108	112	103	104
Bi-Jugal Breadth	108	106	116	123·5	117	105	115	121	125	—	120	122	123	121	126	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	101	97	98	99	97	104	103	102	101	99	105	95	95
Bi-Zygomatic Breadth	—	125	127	134	134	—	—	143	135	—	130	130	136	132	129	139	126	131	126	134	129	130	130	127	136	—	122	137	133	130	127	111	140	—	120	117·5	130	—	—	126	134	—	—	—	—	113·5	—	—	113	108	119	118	121	114	116	126	111	112
Bi-Maxillary Breadth	99	91	93	110	101	—	102	98	—	—	—	—	—	—	—	110	91	97	91	98	96	96	97	92	95	95	—	95	92	103	96	—	84	94	95	95	99	85	—	91	102	103	—	—	—	94	91	99	90	105	101	91	95	96	101
Bi-Malar Breadth	112	107	116	124	119	—	112	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	116	—	—	110	107	119	112	118	113	118	117	110	104
Ophryo-Alveolar Height	80	90	95	101	82	80	84	80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	90	86	84	92	88	85	97	96	88	98	94	88	85
Naso-Alveolar Height	59	65	68	72	66	62	64	59	62	71	67	68	64	67	66	65	63	71	61	66	67	70	69	74	65	67	64	57	64	68	67	55	58	58	60	58	68	65	—	61	70	69	65	62	68	58	65	74	71	64	79	69	65	63
Spino-Alveolar Height	15	22	20	21	15	14?	17	15	17	24	16	21	15	13	20	14·5	15	20	13	21	19	18	25	22	7·5	20·5	19	—	16	21	21	13	17	12	11	11	17	14·5	—	13	16	18	16	14	21	20	20	21	18·5	20	23	17	17	20
Orbital Breadth	35	37	37	41	40	39	39	40	35	35	36	41	37	36	35	41	40	37	40	40	38	35	35	41	43	43	38	40	39	41	42	38	41	39	40	40	40	36	—	40	38	39	37	40	37	37	37	40	42	39	40	43	38	38
Orbital Height	30	31	29	32	30	33	29	33	30	27	30	33	30	29	28	31	32	32	32	35	34	28	30	34	35	34	32	31	31	33	35	26	32	29	31	30	30	30	—	32	32	32	32	34	32	30·5	32	35	34	32	29	31
Nasal Height	45	44	49	52	50	49	47	45	46	48	52	48	50	55	47	51	50	51	52	46	49	53	45	53	55·5	51·5	47	51	48	48	49	43	46	49	47	47	51	51	—	52	55	52	50	48	49	41	47	55	52	45	56	51	47	43
Nasal Breadth	23	23	24	22	23	26	23	24	23	25	25	26·5	26	28	22	23·5	21	22·5	23	25	25·5	23	25	22	27	23	22	28	22	26·5	22	26	21	25	21	23	23·5	22	—	23	26	23	21	25	—	24	22	25	25	25	23	24	21
Bi-Dacryc Breadth	26	22	24	23·5	23	20	23	23	24	26	25	25	22	25	23	25	21	24	23	24	23	22	24	25	24	24	24·5	27	26	25	22	23	23	22	23	21	23·5	20	—	23	30	24	20	20	—	25	27·5	25	21	25	20	22	20	20
External Palatine Length	48	52	58	55	54	—	53	48	—	—	—	—	—	—	—	56	52	55	46	50	51	49	53	56	—	—	—	—	—	—	—	—	—	—	—	47	57	52	—	46	52	55	49	52	49	51	51	50	45	56	57	57	50	53
Internal Palatine Length	38	46	55	47	—	—	48	44	51	52	47	50	52	45	50	52	46	54	40	42	44	45	52	54	45	51	52	—	51	50	49	48	50	52	46	44·5	51	51	—	40	50	51	42	49	39	42	46	46	—	50	51	52	44	48
External Palatine Breadth	54	57	66	66·5	62	—	64	56	—	—	—	—	—	—	—	66	63	61	60	67	62	62	69	67	—	—	—	—	—	—	—	—	—	—	—	57·5	64	58	—	60	59	62	59	56·5	61	54	61	60	68	60	68	69	—	64
Internal Palatine Breadth	32	31	37	32	—	—	38	36	31	42	34	42	44	33	38	37	35	40	32	38	33	40	46	42	42	35	36	—	40	42	41	35	35	41	37	40	40	32	—	32	46	35	32	31	32	30	34	34	—	33	33	38	30	39

INDICES.

Length-Breadth	—	77·6	72·4	74	77·7	—	77·3	85·3	75·9	78	76·4	76·6	75·3	77·2	77·6	76	77·3	76·1	73·7	78	75·6	76·5	77·5	78·5	74·6	—	77·1	75·4	75·6	75·6	78·4	73·7	79·1	—	75·8	78	75·4	73·6	78	73·7	73·5	76	74·9	71·2	77·3	72·7	75·8	74·1	75·3	74·3	75·5	78·1	77·3	75·3
Length-Height	—	69	—	74·4	72·1	—	78·7	74·9	70·8	70·3	72·7	71·9	69·7	70·6	69·5	69·8	73·2	73·9	70·9	75·6	70·9	75	70·6	70·3	—	—	75·3	75·1	—	71·9	73	74	75·6	—	77·4	67·6	71	73·6	73·4	70·9	74·6	69·4	66·7	67·5	75·7	74·9	72	72·5	72·2	71·1	66·7	71·7	72·4	71·4
Breadth-Height	—	88·9	—	103·6	92·8	—	97·9	87·8	93·3	90·1	92·8	93·7	92·5	91·4	89·6	91·7	94·4	97·1	96·2	96·8	93·7	98·6	91	89·5	—	—	96·3	98·6	—	93·8	93·1	99·2	94·4	—	100·7	86·8	94·2	100	94·2	96·2	101·5	91·4	89	94·9	97·9	103	101·4	97·9	95·9	95·7	88·3	91·8	93·6	94·9
Upper Facial (Kollmann)	—	52	53·8	53·7	49·3	—	—	41·3	45·9	—	51·5	52·3	47·1	50·8	51·2	46·7	50	51·4	53	49·2	51·9	50·7	53·1	58·2	47·8	—	52·4	41·6	48·1	52·3	52·8	49·8	41·4	—	50	49·4	52·3	—	—	48·4	52·2	—	—	—	—	47·5	49·6	52·8	51·8	48·9	59·8	50·4	50·4	48·8
Upper Facial (Broca)	—	72	74·8	75·4	61·2	—	—	56	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	72·1	64·9	61·7	70·1	67·2	74·2	68·6	68·1	65·9
Orbital	85·7	83·8	78·4	78	75	84·6	74·4	82·5	85·7	77·1	83·3	80·5	81·1	80·6	80	75·6	77·5	86·5	80	87·5	89·5	80	85·7	82·9	83·7	79	84·2	77·5	79·5	80·5	83·3	68·4	78	74·4	77·5	75	75	83·3	—	80	84·2	82	91·9	80	86·5	82·5	86·5	87·5	81	76·9	87·5	74·4	76·3	81·6
Nasal	51·1	52·3	49	42·3	46	53·1	48·9	53·3	50	52·1	48·1	55·2	52	50·9	46·8	46·7	42	44·1	44·2	54·3	52·1	43·4	55·6	41·5	48·6	44·6	46·8	54·9	45·8	55·2	44·9	60·4	45·7	51	44·7	48·9	46·1	43·1	—	44·2	47·3	44·2	42	52·1	—	58·5	46·8	45·5	48·1	55·6	44·6	45·1	48·8
Palatal (Virchow)	84·2	67·4	67·3	68·1	—	—	79·2	81·8	60·8	80·8	72·3	84	84·6	73·3	76	71·2	76·1	74·1	80	90·5	75	89	88·5	77·7	93·3	68·6	69·2	—	78·4	84	83·7	72·9	70	78·8	80·4	90·9	78·4	62·7	—	80	92	68·6	76·2	63·3	82	71·4	73·9	73·9	—	66	64·7	73·1	68·2	81·3
Alveolar (Flower)	—	94·7	—	95·2	98	—	95·8	92·7	92	93·7	93·1	95·2	96·1	95·8	95·7	92·6	92	94·7	90·2	92·1	92·8	90·6	96·1	90·5	—	—	98·9	—	—	95·2	88·7	90·7	90·7	98·9	91·6	94·2	103·1	96·9	—	90·2	92·2	98	90·1	89·1	91	90·4	93·9	94·2	97	98	92·2	99	99	94·2
Relation Sub-Cerebral—Total Frontal Curve	—	25	21·6	25	15·4	12·2	16·2	15·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20	17·1	20·3	20	—	19·2	18·9	20·3	20·5	15·1	22·2	18·9	18·3
Supra-Auricular—Total Transverse	—	68·7	—	62·5	67·4	—	67·8	66·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Pre-Auricular—Total Horizontal	—	51·7	47·6	43·9	45·9	—	46	46·8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	71	—	72·8	70·7	71·2	70·6	68·6	69·4	70·7	71·6	67·9	70·5	70·6
Frontal—Total Sagittal	—	34·3	33·7	33	—	-	34·7	—	33·6	31·1	33·1	31·4	—	—	—	32·6	32·9	33·2	33·4	34·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	49·8	—	—	48	46·1	47·7	46·6	44·4	43·7	44·2	42·1	44·8	43·7
Parietal—Total Sagittal	—	33·1	35	35·6	—	—	36	—	31·4	36·1	34	35·2	—	—	—	35	33·3	31·5	36·2	38·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	32·3	33·1	33·1	38	33·4	36·4	33·8	32·1	34·2	35·4	—	34·4	35·1	34·7	34·7	32·1	34·2	34·8	33·2
Occipital—Total Sagittal	—	32·5	30·5	31·4	—	—	29·3	—	32·2	32·8	32·8	33·9	—	—	—	29·7	32·1	35·2	30·4	27·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30·9	29·8	33·1	38·9	36·2	35·9	32·1	36	29·7	34·9	—	33·3	34·6	30·4	34·7	35·6	32·6	34·8	34·9
																																			34	34·5	30·6	32·6	30·4	27·7	34·1	31·9	36·1	29·7	—	32·2	30·3	34·9	30·5	32·1	33·2	30·5	31·9

MEASUREMENTS IN MILLIMETRES OF FEMALE SKULLS.

Museum. Catalogue Number	Cambridge University Museum							Private Collections at Cambridge				Basel,			Santa Cruz and Orotava						Barnard Davis Collection																Royal Coll. of Surg.		
Number...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
	*s*2	*s*3	*s*4	*s*5	—	1716						*m*19	*m*24	*m*28							493	495	496	497	498	499	500	501	502	504	506	509	511	512	513	514	572	573A	573B
Maximum Length	179	178	171	183	181	164	188	174	179	171	181	182	174	169	189	173·5	175	183·5	185	188	177	178	182	172	183	172	163	180	170	177	171	175	170	169	181	189	170	180	179
Maximum Breadth	138	140	138	137	143	132	143	136	138	139	140	137	134	135	136	142	—	—	140	140	132	140	142	136	146	140	141	143	136	137	132	140	131	140	134	143	132	144	144
Basi-Bregmatic Height	130	118	124	130	124	131	123	122	130	133	127	135	133	125	133	125	125	126	134	132	124	125	129	124	117	128	125	133	—	129	125	122	123	127	126	129	122	127	124
Minimum Frontal Breadth	95·5	86·5	86	92	96	92	98	96	95	96	95	100	93	89	107	92	96	101·5	98	98	86	91	99	90	94	94	90	96	94	95	88	90	91	92	89	97	93	92	101
Bi-Stephanic Breadth	118	104	111	111	119	108	113	—	—	—	—	—	—	—	—	—	—	—	114	106	106	115	120	112	90	110	115	119	108	108	109	114	103·5	114	98	114	107	118	119
Pterion Breadth	110·5	104	110	103	100	103	110	—	—	—	—	—	—	—	—	—	—	—	112	108	100	99	111	103	111	104	102	112	108	104	108	105	103	109	104	112	104	109	110
Asterion Breadth	106	102	100	108	110	100	120	—	—	—	—	—	—	—	—	—	—	—	108	108	105	109	104	99	111	106	106	100	102	101	102	100	103·5	110	108	113	103	110	105
Curves: Sub-Cerebral	20	21	21	21	21	19	23	—	—	—	—	—	—	—	—	—	—	—	19	22	21	20	23	15	23	22	17	18	16	18	18	17	22	17	22	19	19	25	25
Curves: Total Frontal	127	120	125	127	121	114	120	114	127	128	115	—	—	—	125	115	121	129	132	122	121	117	132	119	121	122	128	120	115	121	111	115	114	122	111	128	121	123	120
Curves: Parietal	126	115	111	110	122	129	112	100	126	120	116	—	—	—	121	122	123	122	132	124	123	134	122	125	115	114	109	124	131	114	108	130	110	115	125	122	101	121	137
Curves: Supra Occipital	71	78	55	92	78	74	90	—	—	—	—	—	—	—	—	—	—	—	61	65	71	—	75	61	—	73	68	91	—	76	89	70	66	70	66	80	87	65	69
Curves: Total Occipital	116	126	122	131	120	107	121	131	116	110	140	—	—	—	132	110	111	121	124	130	116	—	111	114	126	109	108	127	—	120	122	110	114	107	117	120	122	108	111
Curves: Total Sagittal	367	361	366	366	363	350	353	345	369	358	365	—	—	—	378	347	—	372	388	376	360	—	365	358	362	345	345	371	—	355	341	355	338	344	353	373	344	352	368
Curves: Supra-Auricular	282	292	296	302	248	293	288	275	282	292	298	—	—	—	298	285	—	—	292	288	281	296	315	303	299	294	290	305	260	296	291	291	288	308	287	301	286	310	302
Curves: Total Transverse	438	410	—	421	410	412	428	—	—	—	—	—	—	—	—	—	—	—	440	438	401	420	431	413	413	411	419	434	398	404	401	406	410	422	405	428	396	430	425
Curves: Pre-Auricular	209	193	221	224	262	210	268	—	—	—	—	—	—	—	—	—	—	—	264	260	227	210	231	216	222	219	209	211	210	206	210	226	228	231	206	240	215	231	240
Curves: Total Horizontal	509	509	495	511	518	483	516	499	509	500	505	—	—	—	537	505	—	—	512	525	490	510	518	489	521	495	484	508	501	517	487	490	490	492	500	526	482	512	512
Foramen Magnum Length	36	35	30	35	35	33	38	35	36	37	41	—	—	—	37	36	34	34	35	35	33	—	33	33	37	36	36	34	—	36	36	34	33	34	36	36	33	36	31
Foramen Magnum Breadth	30	30	27	35	30	29·5	35	28	30	30	32·5	—	—	—	32	30	31	30	30	30	30	—	29	28	—	32	33	29	—	29	30	30	31	32	30	31	25	31	27
Basi-Nasal Length	91	90	89	96	98	95	98	96	91	91	91·5	91	93	94	102	96	100	99	98	100	91	98?	101	83	93	95	93	100	—	98	96	90	96	94	98	101	92	98	91
Basi-Alveolar Length	88	91	86	88	—	83·5	—	88	88	86	87·5	87	89	84	93	88	—	95	—	—	85	82?	96	76	85	87	85	92	—	93	88	76	—	89	91	97	88	93	90
External Bi-Orbital Breadth	103	105	96	100	106	97	106	—	—	—	—	—	—	—	—	—	—	—	108	108	98	94	106	92	101	102	—	104	98	101	98	96	98	98	104	103	97	104	106
Internal Bi-Orbital Breadth	98	99	90	94	95	93·5	98	—	—	—	—	—	—	—	—	—	—	—	98	95	93	86·5	99	88	93	95	—	94	95	94	92	90	—	92	98	95	88	94	99
Bi-Jugal Breadth	112	113	107	110	115	105	115	123	112	114	—	—	—	—	—	—	—	—	111	121	107	—	117	—	105	112	—	110·5	101	109	104	—	—	107	114	110	—	110	114
Bi-Zygomatic Breadth	127·5	131	—	122	126	116	121	128	128	122	116	125	112	119	132	127·5	—	—	129	127	123	114	128	—	126	121	—	124	—	119	—	124	—	125	128	127	120	124	125
Bi-Maxillary Breadth	96	100	93	89	—	87	—	—	—	—	83	90	88	—	96	99	—	88	—	—	91	81	96	80	87	89	—	99	87	102	89	—	—	92	93	94	—	96	99
Bi-Malar Breadth	106	113	107	104	103	103	107	—	—	—	—	—	—	—	—	—	—	—	101	108	105	98	113	100	105	107	—	110	101	108	102	—	—	111	116	109	—	109	107
Ophryo-Alveolar Height	82·5	87	82	80	80	67	83	—	—	—	—	—	—	—	—	—	—	—	81	100	77	70	91	69	80	84	79	80	65	86	85	72	—	80	88	84	80	89	79
Naso-Alveolar Height	62	66	61	58	—	52	—	67	62	65	57	61	60	52	62	63	—	65	—	—	56	55	69	54	57	60	60	63	50	69	66	53	—	63	68	66	61	67	58
Spino-Alveolar Height	16·5	20	20	13	16	12	15	20	16·5	16	14	21	18	14·5	16·5	13	—	20	20	18	14·5	13	19	17	12	16	15	15	10	19	17	14	—	15	19	21	15	19	16
Orbital Breadth	36	40	37	37	37	37	38	38	36	35	38	37	35	36	41	40	—	39	37	36	36	33	39	34	34	36	36	36	35	38	37	35	—	35	38	37	34	37	36
Orbital Height	30	34	31	30	32	30	34	31	30	31	30	33	32	29	32	34	—	31	31	32	30	31	31	28	32·5	34	32	32	27	32	32	31	—	34	32	32	34	33	30
Bi-Dacryc Breadth	22	21	20	22	25	21	30	26	22	23	20	25	22	20	23	21	—	26·5	28	24	21·5	23	23	23	21	22	18	23	23	22	23	23	—	20	21·5	21	20	21	27
Nasal Height	47	47	42	45	47	41	45	47	47	48	44·5	42	43	39	50	50	—	49	48	50	43	43	52	39	44	46	45	48	41	50	50	43	—	47	50	46	47	50	43
Nasal Breadth	23	25	24	22	22	22	23	24	23	22	20	25	25·5	22	25	26	—	23·5	24	24	22	19	20	19·5	23	19	21	23	20	21	20	22	—	23	23	21	20	22	24
External Palatine Length	51	56	46	48	—	42	—	—	—	—	50	—	—	—	47	44	—	54	—	—	47·5	46·5	55	42	—	56	48	48	44	52	51	—	—	48	51	50	45	52	53
Internal Palatine Length	46	50	42	44	51	36	45	50	46	51	46	46	49	46	46	43	—	50	52	53	42	43	48	39	43	47	41	44	39	47	46	35	—	43	45	45	39	44	47
External Palatine Breadth	63	64	57	57	—	45	—	—	—	—	60	—	—	—	53	60	—	58	—	—	60	56	58	56	—	55	60	60	53	59	56	—	—	45	56	52	55	60	55
Internal Palatine Breadth	35	36	34	27	38	30	40	37	35	41	35	40	36	38	32	32	—	32	38	41	35	30	31	32	34	31	36	32	32	39	31	30	—	31	36	30	30	36	32

INDICES.

Length-Breadth	78	78·6	80·7	74·8	79	80·5	76·1	78·2	77	81·3	77·3	75·1	76·9	80·1	71·9	82·1	—	—	75·7	74·4	74·6	78·7	78	79·1	79·8	81·4	86·5	79·4	80	77·4	77·2	80	77·1	82·8	74	75·7	77·6	80	80·4
Length-Height	72·6	66·3	72·5	71	68·5	79·9	65·4	70·1	72·6	77·8	70·2	75	77·3	74·9	70·4	72·3	71·4	68·9	72·4	70·2	70·1	70·2	70·9	72·1	63·9	74·4	76·7	73·9	—	72·9	73·1	69·7	72·4	75·1	69·6	68·3	71·8	70·6	69·3
Breadth-Height	94·2	84·2	89·8	94·9	86·7	99·2	86	89·7	94·2	95·7	90·7	98·5	99·2	92·6	97·8	88	—	—	95·7	94·3	93·9	89·3	90·8	91·2	80·1	91·4	88·7	93	—	94·2	94·7	87·1	93·9	90·7	94	90·2	92·4	88·2	86·1
Upper Facial (Kollmann)	48·6	50·4	—	47·5	—	44·8	—	52·3	48·4	53·3	49·1	49·6	45·1	43·7	47	49·4	—	—	—	—	45·5	48·2	53·9	—	45·2	49·6	—	50·9	—	58	—	42·7	—	50	53·1	52	50·8	54	46·4
Upper Facial (Broca)	64·8	66·4	—	65·6	63·5	57·8	68·6	—	—	—	—	—	—	—	—	—	—	—	62·8	78·7	62·6	61·4	71·1	—	63·5	69·4	—	64·5	—	72·3	—	58	—	64	68·7	66·1	66·7	71·8	63·2
Orbital	83·3	85	83·8	81·1	86·5	81·1	89·5	81·6	83·3	88·6	81	89·2	91·4	80·5	78	85	—	79·5	83·8	88·9	83·3	93·9	79·5	82·4	95·6	94·4	88·9	88·9	77·1	84·2	86·5	88·6	—	97·1	84·2	86·5	100	89·2	83·3
Nasal	48·9	53·2	57·1	48·9	46·8	53·7	51·1	51·1	48·1	45·8	44·9	59·5	59·2	56·4	50	52	—	48	50	48	51·2	44·2	38·5	50	52·3	41·3	46·7	47·9	48·8	42	40	51·2	—	48·9	46	45·7	42·6	44	55·8
Palatal (Virchow)	76·1	72	81	61·4	74·5	83·3	88·9	74	76·1	80·4	76·1	86·9	73·4	82·6	69·6	74·4	—	64	73·1	77·3	83·3	69·8	64·6	82	79·1	66	87·8	72·7	82	83	67·4	85·7	—	72·1	80	66·7	76·9	81·8	68·1
Alveolar (Flower)	96·7	101·1	96·6	91·7	—	—	—	91·7	96·7	94·5	95·6	95·6	95·7	89·4	91·2	91·7	—	96	—	—	93·4	83·7	95	91·6	91·4	91·6	91·4	92	—	94·9	91·7	—	—	94·7	92·9	96	95·7	94·9	98·9
Relation Sub-Cerebral—Total Frontal Curve	15·7	17·5	16·8	16·5	17·4	16·7	19·2	—	—	—	—	—	—	—	—	—	—	—	14·4	18	17·4	17·1	17·4	12·6	19	18	13·3	15	13·9	14·9	16·2	14·8	19·3	13·9	19·8	14·8	15·7	20·3	20·8
Supra-Auricular—Total Transverse	64·4	71·2	—	71·7	60·5	71·1	67·3	—	—	—	—	—	—	—	—	—	—	—	66·4	65·8	70·1	70·5	73·1	73·4	72·4	71·5	69·2	70·3	65·3	73·3	72·6	71·7	70·2	73	70·9	70·3	72·2	72·1	71·1
Pre-Auricular—Total Horizontal	41·1	38·9	44·6	43·8	50·6	43·5	51·9	—	—	—	—	—	—	—	—	—	—	—	51·6	41	46·3	41·2	44·6	44·2	42·2	44·2	43·1	41·5	41·9	39·8	43·1	46·1	46·5	47	41·2	45·6	44·6	45·1	46·9
Frontal—Total Sagittal	34·6	33·3	34·2	34·7	33·3	32·5	34	33	34·4	35·8	31	—	—	—	33·1	33·1	34·1	34·7	34	32·5	33·6	—	36·2	33·2	33·4	35·3	37·1	32·3	—	34·1	32·6	32·4	33·7	35·5	31·4	34·6	35·2	34·9	32·6
Parietal—Total Sagittal	34·3	31·9	30·3	30·1	33·6	36·9	31·7	29	34·1	33·5	31·2	—	—	—	32	35·2	34·6	32·8	34	33	34·2	—	33·4	34·9	31·8	33	31·6	33·4	—	32·1	31·7	36·6	32·5	33·4	35·4	33	29·4	34·4	37·2
Occipital—Total Sagittal	31·3	34·9	33·3	35·8	33·1	30·6	34·2	38	31·4	30·7	37·7	—	—	—	34·9	31·7	31·3	32·5	32	34·5	32·2	—	30·4	31·8	34·8	31·6	31·3	34·2	—	33·8	35·8	31	33·7	31·1	31·1	32·4	35·5	30·7	30·2

Monday, 25 *May*, 1896.

Professor J. J. Thomson, President, in the Chair.

At a Meeting held at the Observatory the following communications were made:

(1) *On the spectroscope used in connexion with the 25-inch refractor.* By H. F. Newall, M.A., Trinity College.

Mr Newall exhibited the spectroscope and shewed some of the photographs of stellar spectra with terrestrial comparison spectra taken with the instrument. A general description of the spectroscope has been published in the Monthly Notices of the Royal Astronomical Society, 1896, vol. lvi. p. 98.

(2) *On a suggestion for a form of spectroheliograph.* By H. F. Newall, M.A., Trinity College.

The results obtained by Mr Hale in photographing the sun's disc with the light of some definite wave-length emitted by it are so full of interest and of promise of further development that it seems pardonable to make an untried suggestion which may lead to increased facility in taking such photographs.

In a paper entitled "The Spectroheliograph" (*Astronomy and Astrophysics*, 1893, p. 241) Mr Hale has given a *résumé* of the earlier attempts to photograph prominences and of his own work in developing the spectroheliograph. (See also *Astronomy and Astrophysics*, 1892, pp. 407, 603.)

M. Deslandres has in a paper entitled "Appareils enregistreurs de l'atmosphère solaire" (*L'Astronomie*, June 1894), considered various forms of apparatus for researches similar to those carried out by himself and by Mr Hale. (See also *Bulletin astronomique de l'Observatoire de Paris*, Feb. 1894).

The general principle of the method is as follows:

An image of the sun is produced and allowed to fall on a slit; the illuminated part of the slit is a chord of the sun's disc. If the proper relative motion of the slit and the sun's image is brought about, the slit can be illuminated successively by parallel chords of the sun's disc.

The slit is connected with some spectroscopic (or rather *spectrogenic*) apparatus which produces a monochromatic image of the slit, and this image is isolated from the neighbouring images by

letting it fall on a second slit. We can thus produce monochromatic images of successive parallel chords of the sun's disc.

Photographic apparatus is then arranged so that the monochromatic images are laid side by side, and a complete photograph of the sun's disc can thus be reconstructed.

It is clear that there are many ways of bringing about the proper relative motions of the slits, the image of the sun and the photographic plate, but practical experience points to the desirability of reducing as far as possible the number of parts between which there is relative motion.

Mr Hale has in the case of the spectroheliograph of the Kenwood Observatory attached a spectrogenic apparatus, consisting of collimator, diffraction grating and camera, to an equatorial; and by driving the equatorial at the proper rate he keeps the sun's image stationary relative to the spectrogenic apparatus, the whole being moved relatively to the earth. Then he moves the slit relatively to the collimator, and the second slit relatively to the camera, in which is a fixed photographic plate; the two slits move at different rates but are connected by carefully designed link-work.

M. Deslandres has used a heliostat, which does away with the motion of the sun's image relatively to the earth; and he advocates the relative fixity of slits and spectrogenic apparatus.

Mr Hale, in using on Mount Etna a spectroheliograph designed by him for the late Mr Cowper Ranyard (*Astronomy and Astrophysics*, 1894, p. 662), made an advance in arranging that the camera and collimator should be parallel, so that the image of the sun and the photographic plate should be in parallel planes. This method has been adopted by Mr Hale for the spectroheliograph designed for use in connection with the 40-inch Yerkes equatorial. The slits and the spectrogenic apparatus are relatively fixed, and the whole arrangement is bodily moved relatively to the sun's image and the photographic plate.

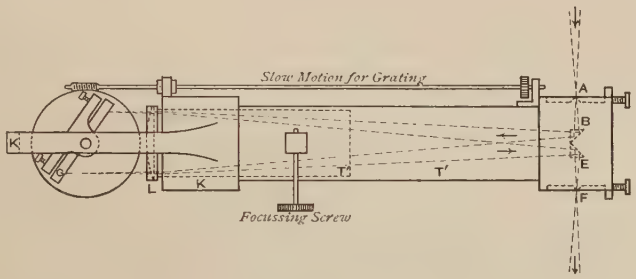
In this arrangement the resulting disc of the sun is round and undistorted, and there is no need for the rectification of the distortion which Mr Hale has had to deal with in his earlier investigations. It is perhaps worthy of note that there appears in Mr Hale's photographs of detail on the sun's disc much less perspective foreshortening of detail near the sun's limb than one would expect to find.

In another arrangement, namely the "coronagraph" used by Mr Hale at Kenwood, 1893 (*Astronomy and Astrophysics*, 1894, p. 673), a modification of the Littrow spectroscope was used. But the spectroscope was fixed relatively to the sun's image and to the photographic plate, and the slits were moved, the nature of the case being such that the slits move in opposite directions.

Some modification of Littrow's spectroscope *with fixed slits* would seem obviously the simplest one to use in these investigations. I am under the impression that Mr Hale has recorded his rejection of this form, but I cannot find such a reference. Recent work with a modification of Littrow's spectroscope leads me to believe that the difficulty arising from reflexions from the lens-surfaces can be overcome by the use of suitable stops. The spectrogenic apparatus here suggested is accordingly a modification of Littrow's spectroscope.

Modification of Littrow's Spectroscope.

The essence of this form is that one tube and lens serve both for collimator and for camera or observing telescope.



The tube T serves as collimator and camera; an object-glass L is fitted in a tube T' which slides in the main tube T for focussing purposes.

A slit A is fitted at one side of the tube T, its length being perpendicular to the axis of the collimator; a reflector B placed near the axis reflects light normally incident on the slit so that it passes along the axis of the collimator and so through the object-glass.

A diffraction grating (or a train of prisms with a reflector at the end) is attached properly to the tube T by means of a framework K. Light, after falling on the grating (or after a double passage through the prisms), falls on the object-glass again and passes along the tube T till it falls on a second reflector E, set so as to reflect the light out through the slit F, which is fixed opposite the first slit. The arrangement is in a certain sense direct vision; and the line joining the middle points of the two slits may be called *the line of direct vision*; it is of course perpendicular to the axis of the collimator. (Such a form of instrument would be convenient for ordinary spectroscopic

purposes, an eyepiece being attached in place of the slit *F*. It might be preferable in such a case to have the slits parallel to the axis of the collimator, a corresponding change being made in the position of the grating.) For spectroscopic purposes an achromatic object-glass would be convenient, but for the use of the instrument in a spectroheliograph a single lens is all that is needed. A suitable small stop must be set on the inner surface of the lens near its centre to cut off stray reflected light.

So far as I am aware, it has been the custom in all forms of spectroheliograph to *move the slit slowly across the sun's disc.*

Accurately equable motion is needed in order to avoid streaks in the resulting photographs—the streaks here referred to are those parallel to the slit, not those which are perpendicular to the slit and are due to dust on the slit—and even when the motion is equable, the effect of a cloud passing in front of the sun will be to diminish the density of the photograph only in that part of the disc over which the slit moves during the passage of the cloud. It often happens that the best definition in solar observations is obtained when there is considerable haze and even fog, which is necessarily of varying opacity.

The necessary exposure for the whole disc may be attained either by moving the slit slowly once across the disc or by letting the slit pass comparatively quickly backwards and forwards over it often enough to get the necessary total exposure, the image of the sun being kept stationary whichever method is adopted. Mr Hale has adopted the first method, and I am not aware that the second method has been either tried or suggested. The oscillatory motion might be given to the slits and connected spectrogenic apparatus by properly suspending the apparatus and allowing it to swing like a pendulum. This method involves the use of a fixed beam of sunlight.

The suggestion which I have to make may be summarized as follows:—

1. A cœlostat is to be used to throw a horizontal beam of sunlight.

2. An object-glass is to be inserted in the beam so as to form a fixed image of the sun—'*fixed*' in the sense connected with the cœlostat as opposed to the heliostat.

3. The spectrogenic apparatus with fixed slits described and illustrated above is to be suspended like the bob of some forms of ballistic pendulums, and in such manner that (*a*) the slit *A* receives the image of the sun upon it, and bisects the circular disc of the sun when the apparatus is at rest; (*b*) the line of

direct vision as defined on page 181 is horizontal and coincides in direction with the axis of the horizontal beam of sunlight; (*c*) the slits are vertical. This pendulum is to be set swinging with an amplitude slightly greater than the diameter of the sun's image.

4. A photographic plate is to be set either all but in contact with the second (monochromatic) slit, or else considerably further back, with a lens placed between it and the second slit in such a way as to form an enlarged image of the second slit on the plate. The lens and photographic plate are to be fixed relatively to the earth and consequently also to the image of the sun.

The resulting photographs will shew the average appearance of the disc during the whole exposure. The central parts of the sun's disc would get less exposure in a given interval of swinging than those parts of the borders of the disc at the ends of a horizontal diameter of the image: but this would be no disadvantage if one considers the difference in brightness between the centre and the limb. The relation between the exposure at the centre and near the limb might also be controlled in their small limits by altering the amplitude of the swing. Loss of definition would arise either because of atmospheric disturbance, which would affect the whole disc, or because of violent motions on the sun's surface, which would affect only restricted regions on the disc. The motions appear however to be seldom so great as to be likely to affect photographs taken with comparatively short exposures, and on such occasions local loss of definition would indicate the disturbed regions.

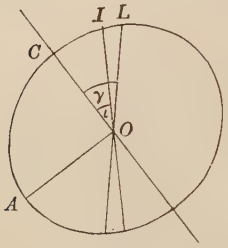
I have hesitated to commit this suggestion to paper without giving it a practical trial; my only excuse is that I am in hopes that some one may be induced to give the method a trial earlier than I see any prospect of being able to do myself.

(3) *On the Period of the Earth's Free Eulerian Precession.* By J. Larmor, M.A., F.R.S., St John's College.

1. The main object of this note is to state a principle which allows us to estimate the effect of elastic yielding of a rotating solid on the period and character of the free precession of its axis of rotation. It has been suggested by the recent papers of Professor Newcomb and Mr Hough[1], in which the discussion is chiefly confined to the case of an incompressible homogeneous spheroid.

[1] S. Newcomb, *Monthly Notices R.A.S.*, 1892; S. S. Hough, "The Rotation of an Elastic Spheroid," *Phil. Trans.*, 1896.

2. In order to obtain a clear notion of the action of the internal forcive which causes the free precession, let us examine in a geometrical manner the well-known case of a perfectly rigid solid rotating round its axis OC of greatest moment of inertia. When this motion is disturbed, let OI represent the instantaneous axis of the rotation ω of the solid, and OL the axis of resultant angular momentum which is fixed in direction in space. In the case of a symmetrical solid OC, OI, OL lie in one plane. With reference to the solid itself, OI and OL will describe right cones round OC with common angular velocity Ω, that of the free precession in the solid body of the instantaneous axis of rotation. The line OL being fixed in space, the point L of the body will thus move away from the point L fixed in space with velocity $-\Omega \,.\, OL \sin\gamma$, where γ represents the angle COL; but as the body is rotating at the instant round the axis OI with angular velocity ω, the velocity of its point L must also be $\omega \,.\, OL \sin(\gamma - \iota)$, where $\gamma - \iota$ represents the angle IOL. Thus we have the geometrical relation



$$-\Omega \sin\gamma = \omega \sin(\gamma - \iota) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

The angular momentum of the rotating solid with respect to its centre of gravity O is at time t made up of

$$C\omega \cos\iota \text{ round } OC \text{ and } -A\omega \sin\iota \text{ round } OA.$$

After an infinitesimal lapse of time δt, OC and OA have turned round the axis OL fixed in space through an angle $\omega'\delta t$ say; thus OC has moved through an angle $\omega' \sin\gamma \,.\, \delta t$, and OA through an angle $\omega' \cos\gamma \,.\, \delta t$. These changes in the axes of the constant component angular momenta will introduce two reacting couples, acting round the same axis perpendicular to the plane COI and equal to

$$C\omega \cos\iota \,.\, \omega' \sin\gamma \text{ and } -A\omega \sin\iota \,.\, \omega' \cos\gamma:$$

and in the steady precessional motion these couples must equilibrate each other, so that

$$C \cos\iota \sin\gamma + A \sin\iota \cos\gamma = 0;$$

that is,

$$C \tan\gamma = A \tan\iota \quad \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Combining equations (1) and (2),

$$\Omega = \omega(\cot\gamma \sin\iota - \cos\iota)$$

$$= \omega \frac{C - A}{A} \cos\iota \quad \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

Thus the angular velocity of free precession, as seen by an observer partaking of the motion of the solid[1], bears the ratio $(C-A)/A \sec \iota$ to the angular velocity of the rotation: when the amplitude of the precession is small, the ratio reduces to $(C-A)/A$.

3. We have now to examine in what respects this argument is modified when there is elastic yielding to the centrifugal force of rotation, instead of absolute rigidity. As the period of relaxation of a strain in the Earth is a fraction of a day, while that of the Eulerian precession is measured in hundreds of days, it follows that the distortion due to centrifugal force will follow with great exactness the movements of the instantaneous axis of rotation: and the same will usually hold good in other problems of slow and small free precessional motion. Thus if C' and A' represent the moments of inertia, round OC and OA, of the configuration which the body would assume when the strain due to centrifugal force is removed,—or, what is the same thing, of the configuration it would assume under the action of an applied bodily forcive equal and opposite to the centrifugal force of the rotation—the angular momentum will now be made up of the components,

$$\begin{aligned} C'\omega \cos \iota &\text{ round } OC, \\ -A'\omega \sin \iota &\text{ round } OA, \\ I\omega &\text{ round } OI, \end{aligned}$$

where I is the increase of moment of inertia round OI due to the centrifugal bulging.

Thus the reacting couples, in the plane COI, are

$$\begin{aligned} C'\omega \cos \iota \,.\, \omega' \sin \gamma, \\ -A'\omega \sin \iota \,.\, \omega' \cos \gamma, \\ I\omega \,.\, \omega' \sin (\gamma - \iota); \end{aligned}$$

and these must balance. When the axis OI does not deviate far from OC, the moment of inertia round it is equal, to the second order of small quantities, to that round OC; thus $I = C - C'$, and the condition for steady precession is therefore

$$C' \tan \gamma + (C - C')(\tan \gamma - \tan \iota) = A' \tan \iota,$$

that is,
$$C \tan \gamma = \{C - (C' - A_1)\} \tan \iota;$$

so that, by the geometrical relation (1)

$$\Omega = \frac{C' - A'}{C - (C' - A')} \cos \iota \ldots\ldots\ldots\ldots\ldots\ldots (4).$$

[1] This is different from the precession of OC or OI in space, which is round the fixed axis OL, and of angular velocity Ω', where $\Omega' \sin \gamma = \omega \sin \iota$.

The validity of this result is also, as above stated, confined to the case in which $2\pi/\Omega \sin \iota$ is large compared with the time of relaxation of elastic strain in the solid of revolution to which it belongs.

Since $C - C'$ is small compared with C and A, as the elastic deformation only slightly alters the shape of the body, the third of these balancing reacting couples can be neglected compared with the other two. Thus *the free precession, under the influence of elastic yielding, will be the same as would belong to an absolutely rigid body, whose configuration is the one that the actual solid would assume were the centrifugal force removed, its elasticity being supposed unimpaired and the same as actually exists for smaller distortions.*

4. In order that the principle stated in this form may hold good, it is not necessary that the solid should be symmetrical round the axis of rotation. The couple due to the elastic deformation by the centrifugal force can be similarly neglected when the moments of inertia of the solid are all unequal, provided, as above, the oscillation of its axis of rotation remains small, as will be the case if it is spinning round the axis of greatest moment and is but slightly disturbed. Thus, in accordance with Poinsot's theorem, the motion may be represented by the rolling of the modified momental ellipsoid of the solid on a fixed plane; so that the axis of instantaneous rotation will trace out a small ellipse on the surface of the rotating body and the precession will therefore be exactly periodic[1]. It follows easily from the Eulerian equations of motion of a solid free from external forcive, that the period of the free precessional motion of the axis of rotation round this ellipse is to that of the rotation of the body in the ratio $\left\{\frac{A'B'}{(C'-A')(C'-B')}\right\}^{\frac{1}{2}}$, where A', B', C' are the effective moments of inertia, viz. those that would exist when the strain due to the centrifugal force is supposed removed as above; agreeing with the result already given where A' and B' are equal.

5. If when the centrifugal force is thus taken off, the rotating solid became dynamically symmetrical like a homogeneous sphere, there would be no free precession at all: if it became effectively prolate, the precession would be in the negative direction. In the actual case of the Earth, the precession is in the positive

[1] It follows from Poinsot's kinematic representation of the Eulerian motion, that for any solid rotating with no couples acting on it, the motion of the axis of rotation in the body itself is in all cases strictly periodic. It is not difficult to extend this result to the more general case in which there are fly-wheels or other sources of gyrostatic momentum attached to the solid.

direction; which is in agreement with the accepted doctrine that the Earth assumed originally a form of fluid equilibrium, in which it has been gradually solidifying, so that were the centrifugal force *now* removed it would still retain an oblate form.

6. The practical outcome, as regards Astronomy, is that, as the precessional constant $(C-A)/C$ of the *actual Earth* is determined precisely by the periods of the ordinary astronomical precessions and nutations, so also the modified precessional constant $(C'-A')/A'$ *of the Earth as it would be* were the centrifugal force of the axial rotation removed, can be estimated from the period of the free Eulerian precession,—in so far, that is, as this period can be disentangled from the actual observations of changes of latitude, which are also affected by large irregular variations due to meteorological causes, and more or less of an annual character. As the value of $C-A$ is determined by a knowledge of the variation of gravity at the Earth's surface, the values of C and A are separately known; and therefore the value of $C'-A'$ is approximately known in terms of the Eulerian period, thus affording an additional *datum* of somewhat precise character for investigations relating to the physical condition of the Earth's interior.

On the other hand, the forced nutations due to extraneous astronomical forcives, even those of short period, are of course not sensibly affected by the circumstance that the centrifugal part of the Earth's ellipticity follows the movements of the axis of rotation.

7. The conclusions above stated are restricted to the case of a solid body, that is, one in which parts which were initially near together cannot wander far apart during the motion. If a large portion of the interior is fluid, other considerations will influence the result[1]. If the interior of the Earth were all fluid except an enclosing shell of moderate thickness, say a few hundred miles, there could be no sensible free steady precession at all, unless the rigidity of this shell were far beyond any that is actually known.

The slight change of configuration due to transfer of the centrifugal force to a new axis of rotation, which is here contemplated, need not be purely elastic, in order to make the conclusions valid. It may be in part viscous: but the wide discrepancy between the actual period of the Earth's free precession and the one that would apply to a perfectly rigid Earth is evidence that this deformation is really in large part elastic. For if elasticity proportional to, or a function of, strain were entirely absent, then

[1] Cf. Kelvin, *Math. and Phys. Papers*, Vol. III. p. 132; Hough, *Phil. Trans.* 1895.

viscosity proportional to, or a function of, rate of alteration of strain would (unless its coefficient were excessively large) be ineffective in preventing the assumption of the configuration of fluid equilibrium in the slow motions here considered, and free precession of periodic character would therefore be non-existent.

8. The influence of the mobility of the surface waters on the period of the free precession has however yet to be considered. For them also, if they are sufficiently deep, the time of subsidence of a disturbance will be small compared with the period of the precession: so that the principle already employed will still have an application. The simplest case to consider first is that in which the waters cover the whole surface. When the axis of rotation changes, the centrifugal strain will be shifted to the new axis; thus the ellipticity of the underlying solid Earth will be altered owing to its elastic yielding, while that of the ocean surface will be altered by a different amount easily calculated from the conditions of fluid equilibrium.

If on the other hand the ocean consisted only of a thin layer, its law of depth would be changed considerably, and this may involve sensible currents in it. Taking the angular momentum arising from the velocity of these currents (which will diminish the correction due to the surface waters) to be negligible in the actual case, the precessional couple will be as before that of the configuration the Earth would assume if the centrifugal force were removed; it will be that of a spheroid of water with the free surface that would exist on the removal of that force, together with that of the underlying solid Earth with its centrifugal force removed, but with its specific gravity reduced by unity.

If the underlying Earth were absolutely rigid, the effect of the layer of water on the precessional couple would be to diminish it by the couple arising from a spheroid of the same density as water, with ellipticity equal to that part of the ellipticity of the ocean surface which is due to centrifugal force alone: if the underlying Earth is yielding, the ellipticity of this effective water-spheroid must be that of the ocean surface less that of the solid Earth, both due to centrifugal force. In any case, it is clear that under these circumstances the free ocean will cause an increase in the precessional period. This, as above stated, is on the supposition that the ocean is on the average deep enough for the angular momentum of the currents created in the alteration of its free surface to be neglected: if for instance it were very shallow it could obviously have no appreciable effect, and that would be in part because the momentum of the currents would compensate the effect of altered ellipticity, in part because its surface would not then have time to attain the equilibrium form.

9. The effect of the transference of masses of water by ordinary ocean currents and other causes has been considered long ago by Lord Kelvin and Prof. G. H. Darwin, and more recently by Prof. Newcomb, with the result that such disturbances are amply sufficient to originate displacements of the Earth's axis comparable with the amplitudes of the observed changes of latitude.

10. If the Earth were absolutely rigid but covered throughout by surface waters, the momental difference $C'-A'$ which gives the free precessional velocity would thus be less than the one $C-A$ which gives the forced astronomical precession, by that of a spheroid of water of the ellipticity ϵ_1 $(=\frac{1}{517})$[1] which is due to centrifugal force alone. This latter spheroid, of semi-axes $a(1-\frac{2}{3}\epsilon_1)$ and $a(1+\frac{1}{3}\epsilon_1)$, has the moments of inertia

$$C_1=\tfrac{2}{5}a^2E_1(1+\tfrac{2}{3}\epsilon_1),\quad A_1=\tfrac{2}{5}a^2E_1(1-\tfrac{1}{3}\epsilon_1);$$

so that for it $C_1-A_1=\frac{2}{5}a^2E_1\epsilon_1$, where E_1 is the mass of a sphere of water of the dimensions of the Earth.

11. The potential of the attraction of the actual Earth at distant points is, by Laplace's formula,

$$V=\gamma\left(\frac{E}{r}+\frac{A+B+C-3I}{2r^3}+\ldots\right),$$

where E is the Earth's mass, a its radius, and $\gamma E/a^2=g$. In the case of symmetry when $A=B$, we have thus

$$\frac{V}{\gamma}=\frac{E}{r}-\frac{C-A}{2r^3}(3\cos^2\theta-1)+\ldots$$

θ being the co-latitude.

The potential of the centrifugal force of rotation is

$$V_1=\tfrac{1}{2}\omega^2r^2\sin^2\theta.$$

Over the surface of the ocean of ellipticity ϵ, given by

$$r=a(1+\epsilon\sin^2\theta),$$

the total potential $V+V_1$ must be constant: thus

$$\frac{E}{a}(1-\epsilon\sin^2\theta)-\frac{C-A}{2a^3}(3\cos^2\theta-1)+\tfrac{1}{2}\omega^2a^2\gamma\sin^2\theta=\text{const.},$$

so that

$$\frac{E}{a}\epsilon-\frac{3}{2}\frac{C-A}{a^3}-\tfrac{1}{2}\omega^2a^2\gamma=0,$$

which gives

$$C-A=\tfrac{2}{3}a^2E(\epsilon-\tfrac{1}{2}m);$$

where $m=\omega^2a/g$, is the ratio of centrifugal force to gravity at the

[1] Thomson and Tait, § 821.

equator, viz. is $\frac{1}{289}$. The value of ϵ, the ellipticity of the sea-level, is about $\frac{1}{294}$[1]: so that the value of $C-A$ is just one half of that which would belong to the actual free surface in the absence of centrifugal force.

[We observe, incidentally, that

$$\frac{V}{\gamma} = \frac{E}{r} - \tfrac{2}{5}a^2 E(\epsilon - \tfrac{1}{2}m)\frac{3\cos^2\theta - 1}{2r^3};$$

and $$g = -\frac{dV}{dr} - \omega^2 r\sin^2\theta$$

$$= \gamma\frac{E}{a^2}(1 - 2\epsilon\sin^2\theta) - \gamma\frac{E}{a^2}(\epsilon - \tfrac{1}{2}m)(3\cos^2\theta - 1) - \omega^2 a\sin^2\theta$$

$$= g_0\{1 - (\tfrac{5}{2}m - \epsilon)\sin^2\theta\},$$

where g_0 is the value of g at the Pole; which is Clairaut's formula for gravity.

Again, the value of $(C-A)/C$, derived from the precession of the equinoxes, is $\cdot00327$[2]: thus

$$C = \tfrac{2}{5}a^2 E(\epsilon - \tfrac{1}{2}m)/\cdot00327 = \cdot35a^2 E.$$

If the Earth were homogeneous of density $5\frac{1}{2}$, we should have $C = 0\cdot40a^2E$: thus the increase of density towards the centre diminishes the axial moment of inertia by 12 per cent.[3]]

Thus for the actual Earth, $C - A = \frac{1}{3}Ea^2 \cdot \frac{1}{289}$; from which the value for the centrifugal water spheroid, that is

$$C_1 - A_1 = \tfrac{2}{5}\frac{E}{5\frac{1}{2}}a^2 \cdot \tfrac{1}{577},$$

is to be deducted, leaving the effective moment reduced in the ratio of 72 to 82.

In this case therefore of a rigid Earth covered by a fluid ocean, the period of the free precession would be increased in the ratio of 82 to 72 or 1·14 to unity by the presence of the surface waters.

In the actual case the surface waters do not cover the whole Earth, so that this is only a superior limit. Moreover the distribution of surface waters is not symmetrical, so that the result of deducting the centrifugal spheroid of water will be to leave an effective solid with moments of inertia all unequal, and the path of the instantaneous axis of rotation on the Earth will be an ellipse, not a circle. But one axis of the ellipse cannot possibly

[1] Col. Clarke's *Geodesy*, p. 305.

[2] As quoted from Leverrier in *Thomson and Tait*, § 828.

[3] As to how this fits with Laplace's hypothetical law of internal density, cf. G. H. Darwin, in *Thomson and Tait*, § 828.

exceed the other from this cause by more than 7 per cent., even if the Earth were rigid.

Actually, the diminution of precessional moment due to the surface waters will be much reduced by the elastic yielding of the core, for when the centrifugal force is removed not only the surface but also the bed of the ocean will diminish in ellipticity, so that the differential effect will be smaller. It follows therefore that 6 to 8 per cent. is an outside limit to the actual increase of period of the precession that can be due to mobility of the surface waters.

12. On the equilibrium-theory here advanced, the portion of the ellipticity of the surface waters which arises from centrifugal force alone, shifts in company with the axis of rotation, thus causing a tide of the same period as the free precession, as was first pointed out by Lord Kelvin[1]. The equation of this ellipticity is $r = a(1 + \epsilon_1 \sin^2\theta)$, where $\epsilon_1 = \frac{1}{577}$. Thus a shift $\delta\theta$ in the axis of rotation involves a change of level $dr/d\theta \,.\, \delta\theta$; which is at the rate of $0{\cdot}2 \sin 2\theta$ of a foot for each second of arc. The reductions of tidal observations in Holland by Bakhuysen, and on the Pacific and Atlantic coasts of North America by Christie, made with this object in view, actually gives a tidal component, of the period of the Eulerian precession, with amplitudes respectively about ·03, ·04, and ·05 of a foot, thus corresponding to a precessional radius of the order of a quarter of a second of arc, which is the observed order of magnitude.

The somewhat close agreement which has been found between the phase of this tide and the phase of the precession tends towards verifying both that the equilibrium theory here worked out is sufficient and that the free precession of period 427 days is fairly constant in amplitude. The amount of the tide as here computed will however be diminished by the centrifugal yielding of the solid Earth; though, this diminution being regular, we should still find that if the precession were quite regular in amplitude, the tide would at each station be proportional to the amplitude of the precession multiplied by the sine of twice the latitude. But it would appear from the observed latitude variations that the amplitude of the precession is altered so irregularly that the harmonic analysis of a long series of observations can hardly bring out anything more definite than a sort of mean amplitude of tide. The actual tide thus produced follows, on the equilibrium theory, the movement of the Pole exactly; but it can only be disentangled from the observations of aggregate tide by an assumption that it is of a periodic harmonic character.

[1] *Collected Papers*, Vol. III. p. 332.

Were it possible to pick out this tide with fair approximation, we should be able to derive information as to how much the surface of the solid Earth yields to the centrifugal force.

13. In all the foregoing, the relaxation of strain in the Earth that would follow the removal of the centrifugal force means such change of strain as would occur if the materials retained their existing moduli of elasticity and were of unlimited strength. As a matter of fact, a removal of the centrifugal force of the Earth's rotation would result in strains so great that the solid constitution of the interior would break down, and the materials would flow into new configurations in which the strains would be eased. In so far in fact as the configuration actually existing is due to consolidation from a fluid form, there can be in it no internal stresses other than simple hydrostatic pressure. In all the previous argument, what has been really considered has been the very slight change of configuration arising from a very slight shift of the axis of rotation: and we can treat of that as the difference of the centrifugal strains that would belong to the two positions of that axis, when we assume the elastic constants to be the actual ones that belong to small strains, and any limitation of elastic strength is ignored. The question of imperfect elasticity thus hardly comes up at all in such a case.

14. The question as to whether the interior of the Earth is in the main solid or fluid, is, when precisely expressed, really a question as to what intensity of deforming stress will make the materials in its interior begin to flow. Every actual solid substance has its own elastic limit: and when subject to deforming forces sufficiently beyond this limit, the substance will flow like oil. The result here arrived at is that for the small stresses involved in the slight changes of the axis of rotation that accompany the free precession,—stresses that are of the order of a moderate number of grammes per square centimetre—the materials of the Earth's interior are solid and elastic, and as will appear below (§ 16) of very high rigidity. Beyond a certain limit they must give way and flow: the stresses due to the astronomical precessional forces are not however in the main beyond this limit.

As the Earth slowly alters its form owing to change in the distribution of surface materials and loss of heat, strains will arise in its interior and slowly increase until the material gives way somewhere, with earthquake-like effects. At such a weak place there may be a great deal of strain-energy degraded into heat, sufficient it may be to melt the material. The region under a range of volcanic mountains may have a plane of weakness of this kind, up which the molten material is forced to the

surface; which finally heals up after the surrounding stress is reduced in the main to hydrostatic pressure.

15. It has been estimated by Mr Hough[1] that, were the Earth a homogeneous solid with the actual surface ellipticity, the Chandler period for the free precession would require that it should have the rigidity of steel. The order of magnitude of this result will not be entirely altered by actual heterogeneity: so that, if we can assume the correctness of this period, we may conclude that, as regards the slight stresses here involved, the Earth is an elastic solid of about the same order of rigidity as steel. This is in fair accord with the observations of seismologists[2], who find that earthquake shocks are propagated to distant parts of the Earth not only by the ordinary surface waves, but also by minute tremors which enormously outrun the earthquake proper, arriving in a time that would correspond to propagation in a straight line across the interior of the Earth, provided it possessed an average rigidity about $\frac{5}{3}$ of that of steel.

(4) *Note on a point in theoretical dynamics.* By Sir Robert Ball.

Let α be a screw about which a free rigid body is made to twist in consequence of an impulsive wrench administered on some other screw η. Except in the case where α and η are reciprocal it will always be possible (in many different ways) to design and place a rigid body so that two arbitrarily chosen screws α and η will possess the required relation.

Let now β and ξ be two other screws (not reciprocal); we may consider the question as to whether a rigid body can be designed and placed so that α shall be the instantaneous screw corresponding to η as an impulsive screw while β bears the same relation to ξ.

It is easy to see that it will not generally be possible for α, β, η, ξ to stand in the required relations. For taking α and β as given there are five disposable quantities in the choice of η and five more in the choice of ξ. We ought therefore to have ten disposable coordinates for the designing and the placing of the rigid body. But there are not so many. We have three for the coordinates of its centre of gravity, three for the directions of its principal axes, and three more for the radii of gyration. The mass of the body is not disposable. The other nine quantities being assigned the relations of the impulsive and instantaneous screws, as

[1] Loc. cit. *Phil. Trans.*, 1896.
[2] Cf. Prof. Milne, *Brit. Assoc. Report*, 1896, "On Earthquake Phenomena."

geometrical entities, are not affected by alterations in the mass. This produces its effect in the twist velocity acquired and not in the position of the screw.

It thus appears that if the four screws had been chosen arbitrarily we should have ten conditions to satisfy and only nine disposable coordinates. It is hence plain that the four screws cannot be chosen quite arbitrarily. They must be in some way restricted. The object of the present note is to shew that these restrictions are two in number and to set down what they are.

Draw a cylindroid A through α, β, and another cylindroid P through η, ξ. Then an impulsive wrench about any screw ω on P will make the body twist about some screw θ on A. As ω moves over P so will its correspondent θ travel over A. It is easy to shew that any four screws on P will be equi-anharmonic with their five correspondents on A, and that consequently the two systems will be homographic.

In general to establish the homography of two cylindroids three corresponding pairs of screws must be given. And of course there could be a triply infinite variety in the possible homographies. It is however a somewhat remarkable fact that in the particular homography with which we are concerned there is no arbitrary element. Given two cylindroids A and P, then without any other considerations whatever all the corresponding pairs are determined. This is first to be proved.

If the mass be one unit and the intensity of the impulsive wrench on ω be one unit then the twist velocity acquired by θ is

$$\frac{\cos(\theta\omega)}{p_\theta},$$

where $\cos(\theta\omega)$ denotes the cosine of the angle between the two screws θ and ω and where p_θ is the pitch of θ. If therefore p_θ be zero then $\cos(\theta\omega)$ must be zero. In other words, the two impulsive screws ω_1, ω_2 on P which correspond to the two screws of zero pitch θ_1, θ_2 on A must be at right angles to them respectively. This will at once identify the correspondents on P to two known screws on A.

We have thus determined two pairs of correspondents, and we can now find a third pair. For if ω_3 be a screw on P reciprocal to θ_2 then its correspondent θ_3 will be reciprocal to ω_2. Thus we have three pairs θ_1, θ_2, θ_3 on A, and their three correspondents ω_1, ω_2, ω_3 on P. This establishes the homography, and the correspondent θ to any other screw ω is assigned by the condition that the anharmonic ratio of ω_1, ω_2, ω_3, ω is the same as that of θ_1, θ_2, θ_3, θ.

Reverting to our original screws α and η, β and ξ, we now see that they must fulfil the conditions

$$(\omega_1\omega_2\omega_3\eta) = (\theta_1\theta_2\theta_3\alpha),$$
$$(\omega_1\omega_2\omega_3\xi) = (\theta_1\theta_2\theta_3\beta),$$

where the quantities in the brackets denote the anharmonic ratios.

On working out the relations they assume the simple forms

$$\frac{p_\alpha}{\cos(\alpha\eta)}\cos(\beta\eta) + \frac{p_\beta}{\cos(\beta\xi)}\cos(\alpha\xi) = 2\varpi_{\alpha\beta},$$

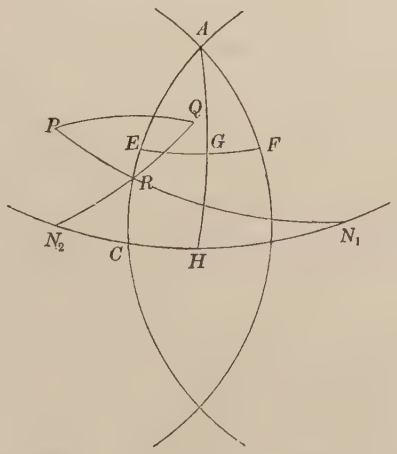
$$\frac{p_\alpha}{\cos(\alpha\eta)}\varpi_{\beta\eta} = \frac{p_\beta}{\cos(\beta\xi)}\varpi_{\alpha\xi},$$

where $\varpi_{\alpha\beta}$ denotes as usual the virtual coefficient of the two screws α and β.

I had already given the second of these equations in the Tenth Memoir on the Theory of Screws, *Trans. Royal Irish Academy*, Vol. xxx., p. 571. I did not then know that there must be the further condition represented by the first. Nor was I then aware of the definite homography between the screws on two cylindroids which expressed the geometrical conditions under which alone one of these cylindroids could be the locus of impulsive screws corresponding to the screws of the other as instantaneous screws.

(5) *On the maximum deviation of a ray of light by a prism.* By Prof. A. Anderson, M.A., Sidney Sussex College.

Let the great circle N_1N_2 be the principal plane of the prism, N_1 and N_2 the normals to the faces. With N_1 and N_2 as centres



describe small circles with radii equal to the critical angle. These circles must intersect if it be possible for a ray to get through, and the point R representing the direction of the ray in the prism lies on the part of the sphere common to the areas of the circles.

Draw a small circle whose distance from N_1N_2 is η' the angle which any ray in the prism makes with the principal plane, and let it intersect the two small circles already drawn in E and F. Then for positions of R on EF, G the middle point is that which corresponds to minimum deviation and E, F are the points corresponding to maximum deviation[1]. Hence to find the absolute maximum deviation we must examine the values of the deviation for positions of R on the small circle CA.

Let R be one of these positions. Join N_1R and take N_1P a right angle. Join N_2R and produce so that $\sin \overset{\wedge}{N_2Q} = \mu \sin \overset{\wedge}{N_2R}$. Then P is the direction of the incident ray and Q that of the emergent ray, PQ being the deviation.

Let $PQ = \delta$, $N_2R = \phi$, $RQ = \theta$; then if A be the angle of the prism it is easy to shew that

$$\cos \delta = (\mu^2 - 1)^{\frac{1}{2}} \cos A \frac{\sin \theta}{\sin \phi} - \mu^2 + 1 + \mu \cos \theta.$$

The deviation is therefore stationary when

$$\cos \gamma \cos A \frac{\sin \theta}{\sin \phi} + \cos \theta$$

is stationary, which may be shewn to hold when

$$\cos A \sin \phi = \tan \gamma \sin \theta,$$

a condition equivalent to

$$\cos \phi = \cos \gamma \frac{1 + \cos^2 A}{2 \cos A}.$$

Since $1 + \cos^2 A > 2 \cos A$, the necessary condition $\phi < \gamma$ is satisfied.

We may easily find for what values of the angle of the prism a stationary value of δ is possible. Since $\cos \phi$ is positive, $A < \frac{\pi}{2}$. Again, the least value of the variable ϕ is $A - \gamma$ or $\gamma - A$ and consequently

$$\cos \gamma \cdot \frac{1 + \cos^2 A}{2 \cos A} < \cos (A - \gamma),$$

which reduces to $\tan A < 2 \tan \gamma$.

[1] *Proc. Camb. Phil. Soc.* ix. p. 109.

Supposing these conditions to hold, we must find whether the value of ϕ given by

$$\cos\phi = \cos\gamma \cdot \frac{1+\cos^2 A}{2\cos A}$$

corresponds to a minimum or a maximum value of the maximum deviations. Denoting the expression

$$\cos\gamma\cos A\frac{\sin\theta}{\sin\phi}+\cos\theta$$

by Ψ, we have

$$\frac{d\Psi}{d\phi} = \frac{\sin\theta}{\cos(\theta+\phi)\sin\gamma\cos\gamma\cos A}[\cos\gamma\cos A\ \sin\phi - \sin\theta\sin\gamma].$$

The quantity in brackets is zero when $\cos\phi = \cos\gamma \cdot \frac{1+\cos^2 A}{2\cos A}$, positive when ϕ is less than the value given by this equation, and negative for greater values. The value of ϕ given by the above equation corresponds then to a maximum value of Ψ and consequently to a minimum value of the maximum deviations. The absolute maximum deviation corresponds therefore to that one of the points C and A for which the deviation is greatest.

If $A > \frac{\pi}{2}$, $\frac{d\Psi}{d\phi}$ is negative for all values of ϕ and therefore Ψ diminishes from C to A. The absolute maximum deviation corresponds therefore to the point A and is given by

$$\sin\frac{D}{2} = \sqrt{\mu^2-1}\,\sin\frac{A}{2}.$$

If $A > \tan^{-1}(2\tan\gamma)$ the quantity in brackets is always negative and the absolute maximum deviation again corresponds to A with the same value as for the last case.

As examples of the first case, which is of greater interest than the others, take two glass prisms one of 20° and the other of 50°, and assume the critical angle to be approximately 42°. With this value of the critical angle the angle of the prism must be $< 60°\ 57'$ for a stationary value of the maximum deviations to exist. For the first prism the deviation corresponding to the point A is found to be $22°\ 24'$ and for the point C $35°\ 50'$. The maximum deviation is accordingly $35°\ 50'$. For the second prism, on the other hand, the deviation corresponding to A is greater than that corresponding to C, the values being respectively $56°\ 23'$ and $46°\ 3'$. It is easy to write down an inequality involving the angle of the prism and μ, separating cases where the absolute maximum deviation corresponds to the point A from those in which it corresponds to C, but it does not seem capable of concise expression.

(6) *On a method of discussing the plane sections of surfaces.* By A. C. Dixon, M.A., Trinity College.

The plane sections of surfaces are generally treated by examining their projections upon one of the co-ordinate planes. It is possible however to find at once the equation to any section of any surface referred to any axes, or indeed to any triangle, in its plane.

Let $(x_1, y_1, z_1)(x_2, y_2, z_2)(x_3, y_3, z_3)$ be the co-ordinates of the three vertices of the proposed triangle of reference referred to the original axes. Let (u, v, w) be the areal co-ordinates of any point in the plane of the triangle referred to that triangle. Then the co-ordinates of the same point referred to the original axes are

$$(ux_1 + vx_2 + wx_3,\ uy_1 + vy_2 + wy_3,\ uz_1 + vz_2 + wz_3).$$

By substituting these for x, y, z in the equation to any surface we have the equation, in areal co-ordinates, to the section of the surface made by the plane of the triangle.

As an application—if we write down the condition that this curve should have a double point and in this condition suppose x_3, y_3, z_3 to be current co-ordinates we have an equation which will represent the tangent planes that can be drawn to the surface through the straight line joining (x_1, y_1, z_1) to (x_2, y_2, z_2).

Thus for a conicoid we have the equation to a pair of tangent planes in the form

$$\begin{vmatrix} S, & S_1, & S_2 \\ S_1, & S_{11}, & S_{12} \\ S_2, & S_{12}, & S_{22} \end{vmatrix} = 0,$$

$S = 0$ being the equation to the surface itself, and

$$(S, S_{11}, S_{22}, S_{12}, S_2, S_1 \mathcal{X} w, u, v)^2 = 0$$

the areal equation to the section.

If we wish to have Cartesian instead of areal co-ordinates in the plane, suppose (ξ, η, ζ) to be the origin and $(l_1, m_1, n_1)(l_2, m_2, n_2)$ the direction-cosines of the axes, rectangular or oblique. Denote the co-ordinates in the plane by (u, v) and the co-ordinates of the same point referred to the old axes are

$$(\xi + l_1 u + l_2 v,\ \eta + m_1 u + m_2 v,\ \zeta + n_1 u + n_2 v).$$

For instance, let us consider a plane section of the surface $ax^2 + by^2 + cz^2 = 1$.

Let (λ, μ, ν) be the direction-cosines of the normal to the plane of the section.

The equation is

$$a(\xi + l_1 u + l_2 v)^2 + b(\eta + m_1 u + m_2 v)^2 + c(\zeta + n_1 u + n_2 v)^2 = 1.$$

As ξ, η, ζ do not enter into the highest terms, all parallel sections must be similar and similarly situated conics.

Suppose the origin to be the centre; then

$$a\xi l_1 + b\eta m_1 + c\zeta n_1 = 0,$$
$$a\xi l_2 + b\eta m_2 + c\zeta n_2 = 0,$$

so that
$$\frac{a\xi}{\lambda} = \frac{b\eta}{\mu} = \frac{c\zeta}{\nu}.$$

Thus the locus of the centre of a section parallel to a given one is a straight line through the centre of the surface.

If the axes of the section are the axes of u and v, we have

$$al_1 l_2 + bm_1 m_2 + cn_1 n_2 = 0,$$
$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0,$$

or
$$\frac{l_1 l_2}{b-c} = \frac{m_1 m_2}{c-a} = \frac{n_1 n_2}{a-b}.$$

Substitute for (l_2, m_2, n_2) in the equation $\lambda l_2 + \mu m_2 + \nu n_2 = 0$, and we find

$$\lambda \frac{b-c}{l_1} + \mu \frac{c-a}{m_1} + \nu \frac{a-b}{n_1} = 0$$

as the equation that determines the directions of the axes of the section in terms of λ, μ, ν.

Or we may eliminate (l_2, m_2, n_2) by means of multipliers and write

$$al_1 = \rho l_1 + \sigma\lambda,\ bm_1 = \rho m_1 + \sigma\mu,\ cn_1 = \rho n_1 + \sigma\nu.$$

Thus
$$\rho = al_1^2 + bm_1^2 + cn_1^2,$$

so that $(1 - a\xi^2 - b\eta^2 - c\zeta^2)/\rho$ is the square of the semi-axis in the direction (l_1, m_1, n_1); l_1, m_1, n_1 are proportional to

$$\frac{\lambda}{a-\rho}, \frac{\mu}{b-\rho}, \frac{\nu}{c-\rho},$$

and
$$\frac{\lambda^2}{a-\rho} + \frac{\mu^2}{b-\rho} + \frac{\nu^2}{c-\rho} = 0.$$

If the section is a circle we have

$$a(l_1^2 - l_2^2) + b(m_1^2 - m_2^2) + c(n_1^2 - n_2^2) = 0,$$
$$al_1 l_2 + bm_1 m_2 + cn_1 n_2 = 0.$$

From these eliminate c. Thus

$$a(n_1l_2 - n_2l_1)(n_1l_1 + n_2l_2) = b(m_1n_2 - m_2n_1)(m_1n_1 + m_2n_2),$$

or
$$a\lambda\mu\nu = b\lambda\mu\nu.$$

By symmetry each $= c\lambda\mu\nu$.

Thus, unless the surface is a sphere, λ or μ or $\nu = 0$.

These examples will perhaps be enough to illustrate the method, which is an extension of that of Joachimsthal, so freely used by Dr Salmon.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

ANNUAL GENERAL MEETING.

Monday, 26 *October*, 1896.

PROFESSOR HUGHES, VICE-PRESIDENT, IN THE CHAIR.

The following officers and new members of Council were elected:—

President:

Mr F. Darwin.

Vice-Presidents:

Prof. G. D. Liveing, Prof. Newton, Prof. J. J. Thomson.

Treasurer:

Mr Glazebrook.

Secretaries:

Mr Newall, Mr Bateson, Mr Baker.

New Members of Council:

Dr Gaskell, Mr Marr, Mr Larmor, Dr Marshall Ward, Mr Shipley.

Mr F. DARWIN, President, then took the Chair.

The names of the Benefactors were read.

The following Communications were made to the Society:

(1) *On the casts of* Iguanodon bernissartensis, *Boulenger, recently presented to the Museum of Zoology by H.M. the King of the Belgians.* By S. F. Harmer, M.A., Superintendent of the Museum of Zoology.

Mr S. F. Harmer exhibited the casts graciously presented to the Museum of Zoology by His Majesty the King of the Belgians. The casts, which are reproductions of some of the famous specimens preserved in the "Musée Royal d'Histoire Naturelle de Belgique," at Brussels, were specially prepared for the University under the supervision of M. Dupont, the Director of that Museum; and they constitute an entire skeleton, together with supplementary casts of the skull, lower jaw, ilium and some other bones. The gift included the elaborate iron-work necessary for the support of the skeleton, which is a copy of the specimen figured in the *Bull. Mus. Roy. d'Hist. Nat. de Belgique*, Tome II. 1883, Pl. v. (See also p. 120 of the same Volume.)

A letter announcing the receipt of this magnificent donation was written to the Vice-Chancellor on August 12, 1896; and is published in the *Cambridge University Reporter*, 1896—97, p. 20.

The originals were discovered in April, 1878, in the colliery of Bernissart, a village situated between Mons and Tournai, close to the French frontier. The bones, which are of Wealden age, were found at a depth of 356 metres (322 metres below sea-level). A notice of their discovery was communicated to the Belgian Academy[1] on May 7, 1878, by Professor Van Beneden, who had recognized them as probably belonging to *Iguanodon*. A fuller account was communicated to the Academy, later in the same year, by M. Dupont[2], Director of the Brussels Museum. M. Dupont announces the discovery of five skeletons, a number which has since then been increased to nearly thirty, all of which appear to have been full-grown. The majority of the specimens belonged to a new species, *I. bernissartensis*, Boulenger[3], the specific distinctness of which has been later maintained by Dollo[4]; while the others belonged to *I. mantelli*, well known in England through the labours of Mantell, Owen and others.

The bones were in a very friable state; and it was necessary to subdivide the clay in which they were lying, into blocks, which were then surrounded by plaster of Paris. In this condition the

[1] Van Beneden, P. J.: "Sur la découverte de reptiles fossiles gigantesques dans le charbonnage de Bernissart"; *Bull. Ac. Roy. Belgique*, 2 Sér., XLV. 1878, p. 578.

[2] Dupont, E.: "Sur la découverte d'ossements d'Iguanodon," *Ibid.* 2 Sér., XLVI. 1878, p. 387.

[3] *Ibid.* 3 Sér., I. 1881, p. 606.

[4] *Bull. Mus. d'Hist. Nat. Belgique*, I. 1882, p. 161.

blocks were brought to the surface, having previously been numbered so that they could afterwards be rearranged in their proper relative positions. On removing the plaster the bones were exposed, and treated in the manner necessary for their preservation. The whole process was carried out with the greatest care and skill; so that, instead of obtaining merely a few of the best preserved bones, the Museum at Brussels is now in possession of the entire series of more or less complete skeletons, five of which are actually mounted in one of their public galleries.

The genus *Iguanodon* was founded in 1825, by G. Mantell[1], who described the teeth of this animal from Wealden formations in Sussex; and suggested the generic name in consequence of the resemblance of the teeth to those of the Iguana. Our knowledge of the genus was largely increased in later years by the discovery of numerous remains of *I. mantelli* and probably other species[2] in the Wealden strata of Kent, Sussex and the Isle of Wight. These fossils were for the most part described by Owen, and their description occupies no inconsiderable part of the Monographs published for that author by the Palaeontographical Society[3]. The same formations were known to contain the three-toed footprints of gigantic bipedal animals. These footprints, long suspected to have been formed by *Iguanodon*, may now be definitely considered to have belonged to that animal.

Although much had been done, in England and elsewhere, towards understanding the structure of *Iguanodon*, no skeleton which was more than fragmentary had been described before the Belgian discoveries were made. The magnificent series which has been made available for scientific study, owing to the well-directed energy of the Brussels Museum, has resulted in making the bony structure of *Iguanodon* almost as well known as if it had been a recent species[4]. The Museum of Zoology is highly favoured in having received this splendid present. The group of the Dinosauria were hitherto entirely unrepresented in the Museum; and *Iguanodon* is of the greatest importance as an educational object, not merely from its immense size, but because of the striking resemblances which it shows to Birds in certain parts of its structure.

[1] *Phil. Trans.* 1825, Part I. p. 184.

[2] Cf. Lydekker, R.: "Cat. of the Fossil Reptilia and Amphibia in the British Museum," Part I. 1888, p. 195.

[3] "Monograph on the Fossil Reptilia of the Cretaceous Formations," Part I. 1851, p. 105; *ibid.* Suppl. II. 1861, p. 27; "Monograph on the Fossil Reptilia of the Wealden and Purbeck Formations," Part II. 1855; *ibid.* Suppl. I. 1858; *ibid.* Suppl. II. 1859 (as *Streptospondylus*); *ibid.* Suppl. III. 1864; *ibid.* Suppl. IV. 1872; *ibid.* Suppl. V. 1874.

[4] The specimens found at Bernissart have been described by Dr L. Dollo in a series of papers published in the *Bull. Mus. d'Hist. Nat. Belgique* (Vol. I. 1882, pp. 161, 205; Vol. II. 1883, pp. 13, 85, 223; Vol. III. 1885, p. 129); also in *Arch. de Biol.* VII. 1887, p. 249; *Bull. Sci. France et Belgique* XIX. (3 Sér. I.), 1888, p. 215; and elsewhere.

(2) *Remarks on the supposed relationship of Birds and Dinosaurs.* By H. Gadow, M.A., Ph.D., F.R.S.

It is least objectionable to divide the enormous order of Dinosauri into the following four groups.

I. *Sauropoda.* Premaxilla with teeth. Pubis single, with symphysis.

Fore- and hindlimbs pentadactyle and of the walking type, plantigrade.

Tarsalia free.

Hereto belong: *Atlantosaurus, Brontosaurus, Morosaurus, Diplodocus,* mostly gigantic beasts from the Upper Jurassic of Wyoming. *Ornithopsis* from the English Wealden.—These genera of Sauropoda are not the ancestors of the other Dinosaurs.

II. *Theropoda.* Carnivorous. Premaxilla with teeth. Pubis simple, with symphysis.

Forelimbs shorter than the hindlimbs; digitigrade, with frequently pronounced upright gait. The first and fifth metatarsals and toes are often reduced. Proximal tarsalia with the tendency to fuse with the tibia; the astragalus sometimes with an ascending process. Metatarsals elongated and sometimes fused with each other.

Hereto belong: *Anchisaurus,* from the Upper Triassic of Connecticut; in the same level of which are the *Ornithichnites* (Brontozoum giganteum). *Zanclodon* from the Keuper of Wuerttemberg. *Megalosaurus* from the Oolite of Europe and Colorado. *Allosaurus, Ceratosaurus, Coelurus, Hallopus* from the Upper Jurassic of North America.—*Compsognathus* from the Bavarian Oolite: long neck, with slender ribs. Forelimbs much shortened, but only three clawed fingers are known. The whole hindlimb is almost typically avine; the astragalus has an ascending process and is fused with the tibia; the distal tarsalia are fused with the metatarsalia. The 2nd, 3rd and 4th metatarsalia are much elongated and fused with each other, the 5th is very vestigial, and the 1st is much reduced proximally and carries the hallux which is placed backwards. The four toes have 2, 3, 4 and 4 (not 5) phalanges. The fibula is slender. The femur mostly with an inner (fourth) trochanter.

III. *Orthopoda.* Herbivorous. Premaxilla without teeth. With a premandibular edentulous piece, the so-called predentary bone, which is probably a calcified horny pad. Each pubis consists of an anterior and a posterior arm, the so-called prepubis (homologous with the "pubis" of most recent Reptiles and the pectineal process of Birds) and the postpubis (homologous with the processus lateralis pubis of Chelonians and Saurians, and with the

"pubis" of Birds). The right and left pubes form no symphysis but remain widely asunder.

A. Stegosauri. Plantigrade. Tarsals and metatarsals free. Sometimes three-toed. Forelimbs massive, not much shortened. —With a dermal armour.

Scelidosaurus from the Lias to the Chalk, in England. *Stegosaurus* from the Jurassic of North America.

B. Ornithopoda. Forelimbs much shorter than the hindlimbs, but stout and pentadactyle. Hindlimbs elongated; digitigrade. Femur with a fourth trochanter.

Camptosaurus and *Laosaurus* from the Upper Jurassic of North America. Tarsals and metatarsals free. Fifth toe lost, first not functional; number of phalanges of the first to fourth toes 2, 3, 4, 5.

Hypsilophodon, from the English Wealden. Metacarpals short, the fifth vestigial; the four fingers with 2, 3, 4, 2 phalanges; first and fourth fingers much reduced. Astragalus and calcaneus separate. Metatarsals elongated, free; fifth vestigial. Toes with 2, 3, 4, 5 phalanges.

Iguanodon. Wealden of Europe. Astragalus, calcaneus and distal tarsals free. Metatarsals free, first styliform, fifth lost. Toes with 3, 4, 5 phalanges.

Hadrosaurus and *Diclonius,* from the Upper Cretaceous of North America; allied forms in Europe. The most differentiated of all Ornithopoda, as shown by its bill.

Ornithomimus. Upper Cretaceous of Colorado. The systematic position of this creature must remain doubtful, only the fore- and hindlimbs, without their girdles, being known. The metatarsals, of which only the second, third and fourth are developed, are much elongated; the third is, with its proximal half, pushed back between the second and fourth, exactly as in Birds. The astragalus has an ascending process and is fused with the tibia. The fibula is very slender.

IV. *Ceratopsia.* Marsh. Pubis simple, with symphysis. Femur without inner or fourth trochanter. Forelimbs very little shorter than the hindlimbs. Pentadactyle, quadrupedous.—Mandible with a toothless "predental" and premaxilla, likewise with a toothless separate element, the so-called os rostrale.—With dermal armour. Skull "horned."

The Ceratopsia consequently combine characters of the Sauropoda and Orthopoda; their occurrence in the Cretaceous formations of Europe and of North America, justifies us in treating them as a separate group of the Dinosauri.

Ceratops and *Triceratops* from Europe and North America.

It is easy to select a considerable number of characters amongst the various Dinosaurs, which also reoccur in Birds, and some of

which had until a recent date been considered as peculiar to Birds.

The double two-armed pubic bones: all the Orthopoda.

The increased number of vertebrae to which the long ilia are attached: most of the Dinosaurs with an upright gait.

The upright gait itself: Orthopoda and most Theropoda.

The possession of ascending process of the so-called astragalus and its fusion with the tibia: Compsognathus and Ceratosaurus among Theropoda; and in Ornithomimus.

Fusion of the proximal tarsalia with the tibia and fibula: Anchisaurus.

Attachment of the distal tarsalia to the metatarsalia: Compsognathus.

Vestigial condition, or absence of the fifth metatarsal: Anchisaurus, Compsognathus, Scelidosaurus, Iguanodon.

Backward position of the hallux, and proximal reduction of its metatarsal: Compsognathus.

The proximal half of the third metatarsal is pressed back between the second and fourth: Ornithomimus.

The functional metatarsals are much elongated and more or less fused with each other: Compsognathus, Ceratosaurus.

The phalangeal numbers of the second to fourth toes, namely 2, 3, 4, 5: Camptosaurus and Laosaurus; 3, 4, 5 in Iguanodon.

It would be a very unscientific, but unfortunately not unprecedented, procedure to take all these numerous resemblances together, and to conclude that Birds have descended from Dinosaurs. Huxley was the first to show clearly that Birds have sprung from a Reptilian stock, and he said of the Dinosaurs, especially his "Ornithoscelidae" (Iguanodon, Scelidosaurus, Hadrosaurus, Megalosaurus, Compsognathus, among others) that they "present a large series of modifications intermediate in structure between existing Reptilia and Aves" (*Manual of Anatomy of Vertebrated Animals*, London, 1871, p. 223). Since that time our knowledge of the Dinosaurs has been enormously increased. Baur (*Morphol. Jahrbuch*, Vol. VIII. 1882; "Der Tarsus der Voegel und Dinosaurier") proved to his own satisfaction "dass die Dinosaurier in Wirklichkeit die Stammeltern der Voegel sind, glaube ich nach meinen Untersuchungen als sicher hinstellen zu duerfen," and he stuck to the same view in two shorter papers in *Morphol. Jahrb.* x. 1885, in answer to Professor Dames' criticisms (Archaeopteryx, in *Palaeontolog. Abhandl.*, Vol. II. 1885; and *Morphol. Jahrb.* x.). In *Morph. Jahrb.* x. p. 615, Baur said "In den herbivoren Dinosauriern, speciell in den ornithopodenaehnlichen Formen, haben wir die Stammeltern der Voegel zu suchen, und zwar die Stammeltern der Ratiten, waehrend wir die Carinaten als von den Ratiten abstammend betrachten." He has since given up this

view. Nor is the descent of Birds from any known group of Dinosaurs favoured by K. v. Zittel in his *Handbuch der Palaeontologie*, or by Haeckel in his recent work *Systematische Phylogenie der Wirbelthiere*.

Nevertheless this notion of the Dinosaurian ancestry of, and especially Iguanodont relationship of Birds has got hold of many people's minds, thanks to various popular and not overcritical writers. The picture of the Iguanodon, side by side with that of a Dinorius or Moa, makes a powerful appeal.

The oldest bird known is Archaeopteryx, from the Oolite of Bavaria. Consequently all those Dinosaurs which are of Oolitic and later date have to be excluded from the supposed ancestry, and they happen to be those in which (as in Ceratosaurus, Compsognathus, Iguanodon, Ornithomimus) the resemblances are greatest. There remain only *Anchisaurus* of the Rhaetic, the contemporary of the unknown Brontozoum which left its three-toed footprints, and Anchisaurus disagrees from birds in its pelvis. *Zanclodon* of the Keuper is also one of the Theropoda. Moreover the most bird-like foot is either that of one of the Theropoda (which are excluded by their pelvis) or of some of the latest Ornithopoda. The upright walk, which has been assumed and improved upon independently by members of both Theropoda and Orthopoda, has produced the same, or nearly the same modifications in them as in the birds. A splendid lesson in isomorphism, homoplasy, or convergent analogies. The common substratum from which all the Dinosaurs and the Birds have sprung, have been of course some pentadactyle, plantigrade, four-footed Reptiles. The movable quadrate of birds, firmly fixed in Chelonia, Crocodilia, Sphenodon, Dinosaurs and Theromorpha, hardly points in their direction.

Lastly, they who entertain the Dinosaurian ancestry of Birds seem to forget, or at least they do not fully appreciate the enormous differences between the forelimb of Archaeopteryx and that of any Dinosaur with avine resemblances in the hindlimbs. The forelimbs of these Dinosaurs are already developed in a direction diametrically opposed to that from which a bird-like wing could be developed. To reconcile the wing of a Bat with the foreleg of a Cow, we have to go back to the indifferent pentadactyle type.

The skull presents another difficulty, and here again Compsognathus comes perhaps nearest to that of a generalised bird's skull; and that the Jurassic Archaeopteryx and the Cretaceous Hesperornis have teeth in the anterior portion of the mandible, seems to exclude the Orthopoda in which the toothless, padded, front portion of the underjaw is such a universal feature.

It can of course be assumed, for argument's sake, that the ancestors of Birds were a group of Reptiles with an upright gait,

with tarsal and metatarsal bones still free, with a quadrate bone not yet fixed, without upper and lateral separate temporal-parietal fossae, with a bifurcated pubis, with a complete alveolar dentition, —but would we call such creatures Dinosaurs, and if so, where are they?

(3) *Notes on Cyclostomatous Polyzoa.* By S. F. Harmer, M.A., King's College.

(Received Nov. 5, 1896.)

On a former occasion I announced to this Society the occurrence of embryonic fission as a normal feature in the development of *Crisia*[1]. Since that time I have accumulated fresh evidence[2] tending to show that the process then described is, with various modifications, characteristic of the Cyclostomata in general. My most recent observations, on *Idmonea serpens*, L., although by no means completed, have led to results which appear to me not unworthy of the acceptance of the Society.

The ovicell of *I. serpens* belongs to a type which is, in many respects, intermediate between that of *Crisia* and that of *Lichenopora*. The colony grows as a lobed encrustation on other objects, and each lobe usually develops an ovicell, which in its fully-developed form appears as a punctate calcareous film extending between the bases of the more distal zoœcia. The tubular aperture does not develop until the ovicell has reached an advanced stage of development. The morphology of this type of ovicell has not hitherto been sufficiently studied.

As in the case of *Crisia* and of *Lichenopora* the ovicell begins to develop before it can be recognized externally, since it is at first in no way distinguishable from a zoœcium. There can in fact be little doubt that the entire ovicell is a much modified zoœcium, its tubular aperture corresponding with the orifice of an ordinary zoœcium.

Sections through the growing edges of vigorous colonies, in the breeding season, commonly reveal the presence of egg-cells in connexion with the caecum of the stomach of many of the younger polypides and buds. These eggs are precisely similar to those which I have observed in other Cyclostomes, or to those which had previously been described, by Claparède[3], for instance, in the young buds of *Bugula*. One, two or three eggs are present in each fertile polypide, and each is enclosed in a distinct follicle. The eggs appear in the very young polypide-buds, and in all cases observed, those which are in connexion with the younger polypides show no trace of nuclear division. In the eggs of recently mature

[1] *Proc. Cambridge Phil. Soc.* VII. 1892, p. 48 [read Feb. 10, 1890].
[2] *Quart. J. Micr. Sci.* XXXIV. 1893, p. 199; XXXIX. Part I. 1896, p. 71.
[3] *Zeitsch. f. wiss. Zool.* XXI. 1871, p. 166.

polypides, whose rectum already contains the remains of Diatoms, I have found evidence of the division of the nucleus of one of the eggs, as well as some indication that the other eggs (if any) of the same individual are degenerating. The development of an egg takes place, however, in a very small proportion of the ovigerous polypides, as is shown by the small number of ovicells and by the fact that each ovicell contains but a single "embryophore[1]."

The next stage is the degeneration of the fertile polypide, as in *Lichenopora*. Up to this time, there has been no external difference between the young ovicell and an ordinary zoœcium; but after this stage the ovicell becomes easily recognizable as such. The punctate, calcareous film is the outer wall of a cavity continuous with the cavity of the fertile zoœcium; and this structure, usually described as the "ovicell" or "oœcium," is thus merely a dilated part of that zoœcium. The young ovicell contains a small brown body, the result of the degeneration of the fertile polypide, with which are connected various structures, together composing the embryophore. The latter differs considerably in detail from that of either *Crisia* or *Lichenopora*, but parts corresponding with the "tentacle-sheath" and the "follicle" of the former and with the "suspensor" of the latter can be recognized. In a late stage the embryophore contains a very large number of embryos in various stages of development. Its proximal part is cylindrical and lies in the body-cavity of that part of the fertile individual which was developed before the more distal part of the same individual took on the external characters of an ovicell. The more distal part of the embryophore divides into several more or less cylindrical lobes, which radiate out into the parts of the ovicell lying between the oblique series of zoœcia. I have convinced myself that the "secondary embryos" arise by a process of embryonic fission; but I propose to withhold the details of this process for a later paper, in which I hope also to discuss certain brown vesicles, occurring in the ovicells, polypide-buds, tentacles and elsewhere, which retain their colour even in paraffin-sections, and are hence an excessively conspicuous feature in preparations of this species.

The general results of this series of observations are as follows:—

(1) the normal occurrence of embryonic fission in Cyclostomata is confirmed by the examination of a new case;

(2) the ovicell of *Idmonea serpens* is shown to be a modified zoœcium, and its early history is indicated;

(3) certain considerations suggest themselves with regard to the classification of the Cyclostomata.

[1] See *Quart. J. Micr. Sci.* xxxix. p. 94.

I am the more inclined to consider the last point because of the views which have just been expressed by Dr Gregory[1], who reluctantly accepts the conclusions that "there are no true genera among Cyclostomata, but only certain convenient, but artificial, groups of species"; that "there is no hope in this order of ever establishing divisions with the same absolute diagnoses as in most other groups of Invertebrates"; and that "Zoarial characters are the only ones available for systematic work among the Cyclostomata." Dr Gregory's classification is based mainly on the characters of the entire colony and of the zoœcia. It appears to me that he has omitted to notice sufficiently a most important adjunct to classification, namely the ovicells, whose value in classification has previously been insisted on by Waters[2] and myself.

I fully admit the difficulty of distinguishing genera and species among the Cyclostomata, and I by no means assert that it is always possible to determine the species to which a fragment of a Cyclostome, or even a single colony belongs. The ovicells, which appear to me to form the surest diagnostic features, are unfortunately not always available; and on the assumption that they are modified zoœcia there was probably a period in the evolution of the group during which the fertile zoœcia were not to be distinguished externally from the others. But it appears to me that in dealing with recent species at least, the study of a large series of specimens, in various stages of their existence, and particularly during their reproductive period, enables us to establish "absolute diagnoses" which are in no way different from those which distinguish the members of other groups of Invertebrata. It may not always be possible to apply these diagnoses in a particular case, but I think that it would be going too far to assert that the differences do not exist.

In spite of the agreement of *Crisia*, *Idmonea* and *Lichenopora*, the three genera which I have specially studied, in the fundamental fact of the occurrence of embryonic fission, the details of their development and particularly the structure of the ovicells and of the embryophores are widely different in the three cases. This I shall hope to show, in greater detail, in a future paper. It is true that these are extreme cases; but the differences are not confined to extreme cases, as the following instances will show.

I have on a previous occasion attempted to show that the British species of *Crisia* can be distinguished from one another by their ovicells[3]. Another genus in which the diagnostic value of the

[1] J. W. Gregory, "Catalogue of the Fossil Bryozoa in the Department of Geology, British Museum." 8vo. 1896, p. 21.

[2] Mr Waters has returned to this point, in a review of Dr Gregory's Catalogue published in the November number of *Natural Science* (Vol. IX. 1896, p. 334).

[3] *Quart. J. Micr. Sci.* XXXII. 1891, p. 127.

characters of the ovicells is very apparent is *Hornera*; but without going further into this case, I may be permitted to compare three recent species of *Diastopora* (*Berenicea* in Dr Gregory's sense) which I have examined with the special object of testing Dr Gregory's views. In each case I have succeeded in observing the continuity of the ovicell with a cylindrical portion situated in the ordinary series of zoœcia and representing the condition in which the ovicell occurred before it took on the external characters of an ovicell. This condition was in *Idmonea serpens* shown to be associated with the fact that the ovicell is a modified zoœcium, and I have little doubt that the same conclusion may be drawn with regard to *Diastopora*. I wish to insist on this point because Dr Gregory has figured ovicells, in various species of *Berenicea*, which from my examination of recent genera I have no hesitation in regarding as modified zoœcia (cf. Pl. III., fig. 3; Pl. IV., figs. 1, 2, 5, 6 of his Catalogue). These he terms "gonocysts," and on p. 12 he defines a gonocyst as "A form of marsupial chamber produced by expansions within the zoarium, and not by the modification of a single zoœcium." I believe, on the contrary, that the ovicell is probably a modified zoœcium in all Cyclostomata. This is certainly true of *Crisia*, *Idmonea serpens* and *Lichenopora*.

In *Diastopora suborbicularis*, Hincks, the fertile zoœcium is but slightly expanded distally, as in Dr Gregory's Pl. III., fig. 3. In *D. obelia*, Johnst., the dilated part of the ovicell is very much more developed, while in *D. patina*, Lamk., the dilated part grows out laterally on each side in a direction concentric with the margin of the colony. The lateral growth may be so marked that the dilated part of the ovicell grows completely round the discoidal colony, and with the secondary union of the two lobes of the fan-shaped colony, the two lobes of the ovicell may also meet one another and form a complete ring-shaped ovicell. Should a second ovicell be developed as a modification of a younger zoœcium than that which produced the first one, it is developed concentrically with the first and of course outside it.

The specific difference between *D. suborbicularis* and *D. patina* is further expressed in the slight development and vertical position of the "terminal membrane"[1] in the former, and its large development, and oblique position (owing to the great extent of the basal lamina) in the latter; and in the fact that the fan-shaped form of the colony is developed into a secondarily discoidal form much earlier in *D. patina* than in *D. suborbicularis*.

These examples will serve to show that recent species of Cyclostomata may be distinguished by means of their ovicells. It will further be apparent that the ovicells themselves belong to

[1] *Quart. Journ. Micr. Sci.* XXXIX., p. 93.

several distinct types. Thus the ovicell of *Crisia* is practically a zoœcium which has become pear-shaped. Its morphological value as a modified zoœcium is apparent at the first glance owing to the fact that it replaces an ordinary zoœcium in the alternate, biserial arrangement characteristic of this genus. There is no sudden constriction between the proximal and the distal part of the ovicell; and this condition is probably due to the fact that as the fertile polypide is never more than a mere bud, the proximal part of the ovicell at no period functions as an ordinary zoœcium.

In other Cyclostomes which I have examined, there is a sudden enlargement of the ovicell which corresponds with the point at which it becomes, so to speak, disengaged from the series of zoœcia. This enlargement no doubt corresponds with the time at which the embryophore begins to increase rapidly in size.

In the Tubuliporidae, the ovicell in other respects resembles that of *Crisia*, except that it projects less beyond the ordinary zoœcia. In the simpler cases, as in *Diastopora suborbicularis*, it is still more or less pear-shaped, but in other species of the same genus and in *Idmonea serpens* it enlarges so as to grow round the neighbouring zoœcia. The lobes of the ovicell then unite with one another on the distal side of the zoœcium round which they have grown, and the zoœcium is left as a column passing freely through the cavity of the ovicell.

The ovicells of *Hornera* probably form a third type; while those of *Lichenopora* are the most complicated which I have so far examined. Here the ovicell no longer develops as a continuous cavity; but, although at its commencement it is merely an ordinary zoœcium, its extent is afterwards increased by the fusion with it of the "alveoli" which are developed at the growing margin of the colony.

I hope to have the opportunity of making a more extended study of the ovicells in this group of Polyzoa; but in the meantime I wish to emphasize my opinion that it will probably not be found impossible to give diagnoses of recent Cyclostomatous genera with much less precision than those which are given for the genera belonging to other groups of Invertebrates.

Certain considerations with regard to the general architecture of the Cyclostome colony have suggested themselves to me during my recent observations[1].

The colony in *Lichenopora* is a modified funnel, whose mouth

[1] The varieties of form assumed by the colony, in various genera and species of Cyclostomata, strikingly resemble those which have been described by Mr H. M. Bernard in the Coral *Turbinaria* ("Notes, Morphological and Systematic, on the Madreporarian Genus *Turbinaria*," *Ann. Mag. Nat. Hist.* (6) xv. 1895, p. 499; "Catalogue of the Madreporarian Corals in the British Museum" (4to.) Vol. II. 1896). It is not unlikely that the growth of a colony of Polyzoa may be regulated by laws which resemble those that govern the growth of a colony of Zoantharia.

has expanded widely so as to cover the proximal part of the colony and thus to give rise to the discoidal form characteristic of the adult[1]. The mouth of the young funnel is closed by the uncalcified "terminal membrane." This, during the growth of the colony, gives rise to zoœcia in such a way that a part of it, capable of forming new zoœcia, is left round the entire circumference of the colony, in addition to the part which appears to cover the entire upper surface, and from which the "cancelli" are developed. In *Lichenopora* the adult colony assumes the form of a somewhat modified funnel, but in *Flosculipora pygmaea*, MacGill., an Australian species, the funnel-shaped form persists with practically no modification.

The peculiarities of *Lichenopora* are thus partly dependent on the mode of development of the terminal membrane, which forms an uninterrupted, annular margin round the mature zoœcia and ovicell.

In another series of forms, including *Idmonea serpens*, *Tubulipora* and *Diastopora*, the proximal part of the terminal membrane is used up completely in the formation of the proximal zoœcia. The terminal membrane thus becomes incomplete on the proximal side of the colony, but it remains as a more or less semicircular margin, capable of giving rise to new zoœcia, on the distal side. Growth thus ceases on the proximal side, and the colony assumes a fan-like instead of a circular shape. The terminal membrane is commonly used up in the formation of zoœcia at a point opposite the primitive disc of the colony, and the growing margin is thus divided into two. By further growth of these parts, accompanied by a cessation of growth in the middle of the distal side of the colony, the fan becomes bilobed; and by further subdivisions of the parts of the terminal membrane, each lobe becomes further lobed. This interruption and subdivision of the terminal membrane is one of the essential differences between *Lichenopora* and *Idmonea*, and it is a noteworthy fact that this generic distinction is already marked at the commencement of the life of the colony. I do not at the present moment include the branching species of *Idmonea* in this comparison; and it is possible that the affinities of *I. serpens* are really with species of *Tubulipora*, as many authors have assumed by placing it in that genus, rather than with forms like *I. atlantica*, *I. radians* and other similar species.

In recent species of *Diastopora*, the colony early acquires a fan-like shape. The terminal membrane does not, however, divide, and the colony does not assume a lobed form. It is well known that in this genus the two lateral edges of the fan grow towards the proximal end of the colony, where they unite secondarily with

[1] *Quart. Journ. Micr. Sci.* xxxix., pp. 81, 93.

one another. In other words, the two free ends of the interrupted terminal membrane grow towards one another above the proximal end of the colony. When they have united, complete fusion takes place, and the two halves of the membrane become completely confluent. This results in the assumption of a secondarily discoidal condition, which differs fundamentally in origin from the discoidal colony of *Lichenopora*, where the entire rim of the colony is a direct derivative of the rim of the young funnel-shaped stage.

In *Crisia* again, the branching colony is produced by the repeated division of the terminal membrane, accompanied by great elongation of the lobes or branches to which the growing-points, or segments of the terminal membrane, give rise. The branches at the same time become jointed. In other words, a Crisia-colony is comparable with a funnel whose distal end has become repeatedly divided in a longitudinal direction, the divisions extending a varying distance towards the base of the colony. The basal lamina of *Lichenopora* and *Idmonea* is thus represented by the "backs" of the branches.

The above instances will serve to show that the zoarial form of several types of apparently dissimilar Cyclostomes is regulated by a law of growth which is probably common to the majority of recent species, if not to all. The unilaminate condition of *Hornera*, of *Frondipora verrucosa*, and of erect species of *Idmonea* is readily intelligible on the supposition that the "backs" of the branches correspond with the basal lamina of encrusting species.

Monday, 9 November, 1896.

Mr F. Darwin, President, in the Chair.

The following were elected Associates of the Society:—

Mr A. E. Smith, Cambridge.
Dr Novak, Prag.
Mr J. Henry, Trinity College.
Mr W. C. Henderson, Trinity College.
Mr G. B. Bryan, St John's College.

The following Communications were made to the Society:—

(1) *On the Nature of the Röntgen Rays.* By Professor Sir G. G. Stokes.

In this communication the author explained the views he had been led to entertain as to the nature of the Röntgen rays, and to a certain extent the considerations which had led him to those conclusions. As Röntgen himself pointed out, the X rays have their origin in the portion of the wall of the Crookes tube on which the so-called cathodic rays fall, and it is natural that our notions as to the nature of the X rays should be intimately bound up with those we entertain as to the nature of the cathodic rays. Two different views have been adopted on this question. Several eminent German physicists hold that the cathodic rays are essentially a process going on in the ether, the nature of which nobody has been able to explain; and that if any propulsion of molecules from the cathode accompanies them, it is merely a secondary phenomenon. The other view is that the cathodic rays are not proper rays at all, but that they are essentially streams of molecules. The latter view is that which, so far as the author knows, is universally adopted in this country. The author expressed the fullest conviction that the cathodic rays are no mere process going on in the ether, but that the propulsion of molecules is of the very essence of the phenomenon; only it is to be remembered that the molecules are not to be thought of as acting merely dynamically, by virtue of their mass and velocity; they are carriers of electricity; and it would seem to be mainly to this circumstance that some at least of their effects are due. He indicated what he believed to be the true answers to the objections of those who regard the cathodic rays as processes in the ether; and adopting the theory that they are streams of molecules explained how, in his opinion, this theory, taken in connection with the more salient features of the X rays to which the cathodic rays give birth, leads us to a theory of the nature of the X rays.

Everything leads us to regard the X rays as being, like rays of light, some process going on in the ether, and sufficient indications of their polarization appear to have been obtained to lead us, at least when those indications are taken along with the undoubted polarization of the Becquerel rays with which they have so many properties in common, to refer the Röntgen as well as the Becquerel rays to a disturbance transverse to the direction of propagation. The absence of refraction, which is so remarkable a feature of the X rays, leads us to regard their progress through ponderable matter as taking place by vibrations in the ether existing in the interstices between the ponderable molecules; a view which if correct leads us incidentally to a somewhat novel view as to the mechanism of the refraction of light. The absence, or almost complete absence, of diffraction and interference of the X rays leads to one of two alternatives:—either that they are of excessively short wave length, or that they are non-periodic or only very slightly periodic, the X light being on the latter supposition regarded as a vast succession of independent pulses analogous to the "hedge-fire" of a regiment of soldiers. According to the author's view, each electrically charged molecule on arrival at the target gives rise to an independent pulse, and the vastness of the number of pulses depends on the vastness of the number of molecules in even a minute portion of ponderable matter.

(2) *On the Contact Relations of certain Systems of Circles and Conics.* By Mr W. McF. Orr.

[This paper will appear in *Transactions*, Vol. xvi. Part II.]

(3) *On certain cases of Discharge in Vacuo and on the zigzag path of Lightning.* By J. Monckman, D.Sc., Downing College.

Prof. J. J. Thomson, when conducting a series of experiments on the striation of sparks in vacuum tubes, made use of various forms of tubes, among which was one which may be called a multiple arc. It consisted of a rectangular tube with two short ones containing the electrodes (Fig. 1). The two sides *CAE* and *GFD* were exactly equal in length. When a discharge took place through the tube, it was observed that at a low pressure the spark passed down one side and on changing the direction of the current, the spark passed down the other side, continuing to change sides on each reversal.

In order to avoid repetition I will speak of this changing of the spark from one limb to another, when the direction of the current is changed, as the 'Reversal of the Spark.'

The multiple arc tube A was joined to a mercury gauge B and onward through the clip H to the receiver of a mercury pump. By this arrangement the pressure may be regulated to the

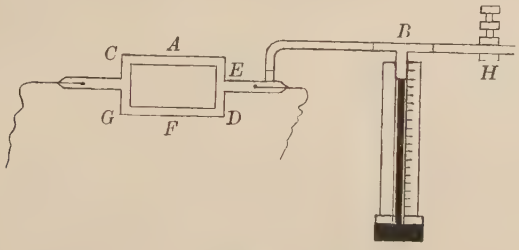


Fig. 1.

greatest nicety by keeping the receiver exhausted and carefully opening H, a little air may be drawn out. And with a little practice there is no difficulty in going through a whole series of readings varying two or three millimetres at a time.

The first series of experiments were tried with ordinary air, and it was found that when the pressure was reduced to 19 mm. the reversal of the spark commenced, but it ceased when the pressure fell to 11 mm., beyond which the spark passed down one side only.

Change of gases. I next tried the effect of changing the gas, and found that only one refused to produce any reversal while another acted very poorly, all the other gases giving reversals at pressures varying with the nature of the gas used.

Very great care was exercised in preparing pure and dry N, H, and O, the object being to test whether a pure dry elementary gas would produce the phenomenon; also pure N was mixed with HCl (pure) with what result will appear in the table following.

It is interesting to notice that the two gases that produce the least effect are those that were found by Faraday to give the worst examples of brush discharge.

Density of spark. There is a particular density of spark more favourable than others for producing the reversal. The most convenient adjustment of the vibrating contact breaker of the coil can be found only by trial, as it varies with the coil used and with the tube. I found that a position about half-way between the position of brightest spark and that at which the hammer vibrated with frequent stoppages, answered very well with the particular instruments used. The proper density appears to be as small a discharge as can be got to pass steadily, since a dense one bursts through and often passes down both sides at once.

Variation of the electrodes. In the experiments described the electrodes were platinum wires, on the ends of which small knobs

had been fused by exposing them to the heat of the electric arc. With some of the gases other electrodes were used and it was found that Pt, in all forms, Pd, and C, were effective, while Al was only slightly so. The former bodies occlude gases, and when the pressure is reduced, drive off small particles of their substance into the surrounding space, while Aluminium does not do so.

I have collected into one table the results obtained by the previously described experiments.

Table shewing the changes produced by variation of electrodes, of gases, and of pressure.

Electrodes	Gas	Pressure in mm. at which the spark reverses	Pressure at which it goes down both sides	Pressure at which it passes down one side only	
Platinum wire with small knobs at the ends	Air (ordinary)	19 to 11 17 to 9 19 to 11	18 with very strong currents	10 8 10	
	Carbonic dioxide	18 to 11 23 to 11 28 to 10		10	
	Ammonia gas	18 to 8† 25 to 11 22 to 8 30		7 10 7	
	Ether	38		?	
	Coal gas	30 to 11		10	
	Hydrogen	30 to 8		8	
	Oxygen	27 to 14	14 to 5	4	Reversals poor
	Hydrochloric acid gas	none	none	all pressures	
	Nitrogen	14 to 8		7	
	Mixed Gas HCl and N		38 to 4	3	*
Platinum foil	Air	36 to 16		6	
Paladium	Air	35 to 20		10	
	Nitrogen	30 to 25		10	
Carbon	Ammonia gas	50 to 20	19		
Aluminium		very little indeed			

† When the sparks were not passed until the pressure was reduced to 30 m., they were greenish and reversals were obtained. When the sparks were passed as the pressure varied, the greenish colour soon disappeared and reversals occurred at from 18 to 25 m. pressure.

* After using HCl and N the electrodes became covered with a brown substance and the reversals ceased to appear although trial was made with dry and damp air and alcohol vapour, but they were again obtained after heating the platinum in the blow-pipe flame.

Further experiments showed this to arise from the Hydrochloric acid gas which not only gave no reversal itself, but so acted upon the platinum as to prevent reversal with other gases. The same action took place in a small degree with Oxygen, which caused the reversal to fail in Hydrogen the first time after Oxygen had been used.

Position of the electrodes, Angle of the short tubes and Occluded Gases. When the short tubes containing the electrodes were fixed obliquely, it was found that the spark passed along the limb which allowed the negative current the more direct course into the main tube.

This took place with all gases, including Oxygen and Hydrochloric acid, which had no effect in the preceding experiments.

When however the knobs were pushed forward, the old form of reversal was obtained, Oxygen giving poor results and Hydrochloric acid none at all.

Since reversals were obtained with electrodes that occluded gases and not with Aluminium, it was of interest to try by experiment whether such gases produced any part of the effect. Three platinum wires with knobs at the ends were fixed to an ebonite rod. The distance between the first and second was exactly the same as that between the second and third. The first and third were joined together, the whole placed under a receiver and exhausted. Afterwards Hydrogen was liberated electrolytically upon No. 1 and Oxygen upon No. 3, but on replacing and exhausting no difference could be detected.

Charging of the tube. Supposing that it was possible that the glass tube might become charged and so influence the spark, I tried the effect of increasing such charge by placing strips of metal under one limb of the tube and found that the discharge was drawn down the side on which the metal was placed.

By using two pieces of metal, or two small saddles of lead joined by a wire, and placing one at C (Fig. 1) and the other at D, or one at E and the other at G, I found that the spark could be made to pass down either side and to reverse when the current was changed. The direction being from the negative electrode past the nearest saddle. Thus if the negative were on the right hand and the saddles at E and G, the spark passed along the limb A and on reversing along the limb F, but if the saddles were at D and C it passed through F and reversed through A.

We could thus produce the reversals as we wished and part of the original phenomenon was explained. To determine how much, two other pieces of apparatus were tried, but as both methods gave the same results I will describe one only.

A jar 6 inches in diameter and 8 inches in height was placed upon a metal plate and made tight with soft wax. The two lower electrodes A and B were in metallic communication with the plate, the upper ones C and D were connected together outside of the jar. D was fused into a glass tube which could be moved, backward or forward, in order to make the distances exactly equal (3 inches).

Reversals were observed between the pressures 45 mm. and

25 mm. beyond which the current always took one path and could not be changed.

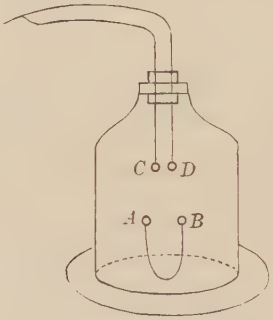


Fig. 2.

With this jar reversals were obtained not only when the current was reversed, but also on restarting after a short break. The latter appeared to be caused by an agency about half as energetic as the former.

After allowing air to enter the receiver and again exhausting, the force that previously acted was found to have decreased, and the next discharge might or might not take the same path as the last before admitting the air.

When using a powerful Wimshurst machine, in a dark room, during the performance of an experiment to be described later, I observed that when the knobs were opened beyond the position of taking an ordinary spark, there began to be a glow around the negative which illuminated a somewhat globular space around that pole. The positive spark passed into this space, but constantly changed its course, sweeping the negative charge away from its path, it then took another path and having cleared that of negative changed again and so on through a kind of cycle.

If there were two paths open as Fig. 2 and a spark passed from C to A, removing part of the glow from the space around A, there would (after cooling) be a greater tendency for the next spark to pass between D and B.

On the other hand if after a spark had passed between C and A the direction of the spark were reversed, it would most probably pass from B to D, there being a larger quantity of negative around D than around A.

This is probably the explanation of the previous reversals and also why the introduction of air into the receiver decreased the power of producing it.

Underlying the whole matter is the cause of the production of the glow at the negative and why some gases produce it better

than others; on these points my experiments do not enable me to offer any opinion.

Zigzag form of spark. When the path of a discharge is long, the spark passes in the well-known zigzag course, often giving off at the angles a series of side flashes or ramifications. This is usually explained by supposing that "The spark finds this form of path easier owing to the presence of irregularly distributed particles of dust which act as stepping-stones, giving the discharge a path of less resistance although longer."

The action of the strips of metal in the previously described experiments led me to think that the irregular form of the path was due to particles charged by induction, attracting (not conducting) the spark away from the straight line and the ramifications were discharges to these particles, after which the spark was free to return to its natural course.

In order to test the correctness of this idea I first fixed a number of small pieces of tinfoil upon a piece of mica by means of shellac varnish, and this upon an ebonite rod. When the stream of electricity was passing between the poles of a Wimshurst Machine, the mica disc was placed a little to one side of the path. The sparks were drawn out towards the disc.

In order to obtain the ramifications at the angles I placed a large number of small brass beads upon silk threads about one millimetre apart. These threads were then fixed to an ebonite stand in parallel lines and as close together as could be managed without allowing them to touch. When this was placed on one side of a straight spark, the path immediately changed, forming an angle towards the beads, and giving off the peculiar ramifications that are so often seen in photographs of lightning.

The action of charged dust, &c. passing away from the negatively charged conductor will no doubt produce part of this, especially when the spark is short, but in the case of lightning I believe the inductive action to be much more important.

Monday, 23 *November*, 1896.

Mr F. Darwin, President, in the Chair.

The following were elected Fellows of the Society:

H. M. Macdonald, M.A., Fellow of Clare College.
G. H. J. Hurst, M.A., Fellow of King's College.

The following Communications were made to the Society:—

(1) *On the Superficial Colour of a Silver-Zinc Alloy.* By C. T. Heycock, M.A., King's College, and F. H. Neville, M.A., Sidney Sussex College.

Whilst granulating by pouring into water an alloy of silver and zinc whose composition was very nearly AgZn (62·25 % Ag) we found that it was superficially coloured a red tint closely resembling that of copper although before melting it was silvery white. The only reference we can find in the text books is in Percy's *Metallurgy* where it is stated that some alloys of silver and zinc acquire a pale pink tinge on exposure to air. A variation in the composition of the alloy to the extent of 5 per cent. of silver more or less does not much impair the colour, but with a deviation from AgZn of 10 per cent. of silver the effect is no longer noticed.

The present communication contains the results of some experiments we have made on this coloration. There appear to be two methods probably not radically different of turning the silver white alloy into the red state. The best way is to heat the alloy up to a temperature near 300° C., 285° is the lowest effective temperature, and suddenly to chill it. In this way it acquires a fine red colour, provided that the cooling has been rapid. The necessity for rapid cooling is connected with the fact that if the red alloy is heated to 150° C. it turns white. It thus appears that the red colour is due to a substance which cannot exist above 150°, and which cannot come into existence unless the body be heated to 300° C.

These peculiar conditions distinguish the phenomenon from all superficial colour changes of metals known to us.

We found that the cooling could be effected by letting filings of the alloy fall through the air or by throwing the hot metal into water, alcohol, benzene or ragosine. As the last two liquids contain no oxygen it became doubtful whether the presence of oxygen was indispensable for the effect. We made numerous experiments to settle this point, the most conclusive being the formation of

the red colour in an atmosphere of pure hydrogen. To effect this some filings of the alloy were placed in a glass tube about 18″ long. Pure dry hydrogen was then passed through the tube for an hour or more, the tube being heated throughout its length to at least 300° C. The tube was finally sealed by the blow-pipe at both ends, the arrangements throughout being such that no air could possibly enter the tube containing the alloy. To avoid the danger of the hydrogen containing oxygen it was prepared, in one experiment from palladium which had been charged at a lower temperature, and in another experiment, the hydrogen, from zinc and sulphuric acid, was passed through a foot of red-hot copper gauze before entering the alloy tube. All important joints were made by sealing the glass tubes to one another with the blow-pipe.

We thus obtained two closed tubes containing a little of the alloy in an atmosphere presumably free from oxygen. The alloy in the tube was then brought to one end and that end heated to 300° C. while the other end was cold. We then threw the hot alloy to the cold end of the tube and shook it there to promote rapid cooling. In both cases it turned red. We have also obtained the red in an atmosphere of carbon dioxide.

Hence the formation of the red colour does not depend on the chemical nature of the surrounding atmosphere: *it is a change for which the alloy is itself sufficient.* We find that in a low vacuum of anything above a millimetre the red comes as easily as in air, but Mr Skinner, who was kind enough to get a good vacuum, say $\frac{1}{50}$ mm. in a tube of the alloy, found that it did not change colour. We have since verified this repeatedly but we find that if such a high vacuum-tube be heated above the temperature at which it was sealed the red can be obtained in it. The behaviour of these vacuous tubes is explained we think by Mr Skinner's suggestion that it is due to variations in the power of cooling by convection. In low vacua the convection cooling of the particles of alloy by the gases in the tube will be almost as rapid as at the atmospheric pressure, but in high vacua this mode of cooling will be absent and hence the alloy cools too slowly to turn red. If we heat such a tube to a higher temperature gas is given off from the walls and the vacuum falls, hence the red can be obtained.

The other way of getting the red colour is to turn up in a lathe or to file the white alloy. The fine particles thus obtained have a pink tinge, but in the course of a few days or weeks they become a bright pink. This gradual change is not as one might think due to the air but to a latent power communicated to the turnings as they are separated from the main mass of metal—the temperature of each fragment being very high at the moment of its separation.

That the air has nothing to do with this gradual change of colour can be proved by heating some of the turnings to 150° C. and then exposing them to the air. A month's exposure gives no trace of red. In fact, provided the alloy has been annealed at 150°, the white colour is quite stable in air, in moisture or in oxygen.

The red colour also seems to be quite stable at ordinary temperatures. The red resists dilute sulphuric and hydrochloric acids and the alkalies, and what is more remarkable, it is not affected by sulphuretted hydrogen or by ammonium sulphide reagents which blacken silver. The action of ozone on the red is very slow but it does finally blacken the alloy.

These are the main facts we have observed. We do not know the chemical change associated with the red colour, and on account of its being a surface action involving minute quantities of matter it will probably be very difficult to settle this. Oxygen may play a part in the reaction, for there is almost certainly some zinc oxide in the alloy, and possibly some oxygen occluded by the silver. The upper limit of temperature 300° is that at which Ag_2O is completely decomposed under atmospheric pressure.

We hope however to be able to replace speculation by facts in a further communication.

(Note added Dec. 31, 1896.)

Since reading the above paper before the Society, we have prepared some of the alloy from metals that had previously been scrupulously freed from oxygen or oxides, the silver by heating to a red heat for some hours in an atmosphere of hydrogen, and the zinc by distillation in a hydrogen vacuum. The alloy made in hydrogen from metals thus prepared turns a fine red when heated and suddenly cooled in the unopened sealed tube in which it was made. The colour change cannot therefore be due to oxygen or to the formation or decomposition of oxides.

We have also found that a silver-cadmium alloy AgCd undergoes somewhat similar colour changes.

(2) *Note on Thermometric "Fixed Points."* By E. H. Griffiths, M.A., Sidney Sussex College.

It is commonly assumed that our present definitions of the thermometric fixed points are theoretically sound and in practice satisfactory. This may possibly be the case, but, at the same time, there are certain ambiguities having their origin in experimental difficulties which may render profitable a consideration of the whole matter; especially as recent advances in the science of exact thermometry make it impossible to tolerate any doubt as to the fundamental definitions on which that science depends.

Section I. *Freezing Point.*

I assume that the ordinary definition of freezing point is as follows: "the temperature of a mixture of ice and water when the temperature condition is one of statical equilibrium."

From the theoretical point of view there appear to be no objections to offer to this definition; practically, however, it is found unsatisfactory.

As Nernst has pointed out, no such statical condition is possible if the exterior temperature differs from that of the mixture, or if heat is supplied by the stirring, &c. Now, under the ordinary conditions of a laboratory the surrounding temperature is very different from that of the mixture, and although it may be urged that the convergence temperature t_g closely approximates to the true freezing point t_0 when the masses of ice and water are considerable, the difference is nevertheless a measurable one, and when such is the case we cannot be said to possess an ideal standard. In other words, in practice the system is a dynamical, not a statical one.

The method of under-cooling and super-fusion does not, as Nernst has shown, suffice to overcome the difficulty, and some recent observations of my own lead to the conclusion that very accurate results can only be looked for when the condition is truly statical.

Another practical difficulty is the inevitable question as to the purity of the material. I take it that the definition assumes absolute purity, otherwise the temperature would not be fixed. The work already accomplished by Loomis, Jones, Abegg, Wildermann, Harker and others has sufficiently proved how considerable are the effects of small impurities; on the other hand, we certainly do not as yet know (and it appears to me improbable that we ever shall know) the true freezing point of pure water, thus, if we are compelled to wait for that knowledge before we decide upon the value of our lower fixed point there is little prospect of finality.

As an illustration. At the Liverpool meeting of the British Association I communicated the results of some recent experiments which, to my regret, forced upon me the conclusion that the depression of the freezing point due to the solubility of glass was greater than had hitherto been supposed. It is probable that the difficulties which have been encountered by enquirers into the freezing point of dilute solutions have been partly due to want of attention to this matter, for I can find no record of experiments in which the water used has been from the time of distillation uncontaminated by contact with glass.

This would be of small consequence if the depression thus

caused was always constant; but glass varies greatly in composition and its solubility probably depends so largely on the nature of the substances previously in solution that the effects may be very variable. We should thus be compelled to use platinum or gold vessels, and the heavy expense entailed would alone suffice to put the freezing point of "pure water" out of court for all practical purposes. Granted that these difficulties could be overcome there arises a question as to the influence of the contained gases. Assuming the accuracy of the ordinarily-accepted constants we find that the freezing point of water if saturated with oxygen and nitrogen would be lowered about 0°·002 C., and it is difficult to see how this cause of error is to be eliminated. Under such circumstances the experimenter who diminishes the causes of error only departs the further from what we may call the practically accepted standard, and additional refinements may but introduce greater confusion unless some general agreement is arrived at.

An observer who is in the habit of standardising thermometers usually repeats his experiments under similar conditions, and finding his results consistent in themselves is apt to conclude that his standard is identical with that of others.

The following is an illustration of the danger arising from such a conclusion.

I have been in the habit of standardising my own platinum thermometers by immersing them in a large copper vessel containing about 15 lbs. of finely-powdered ice and ordinary distilled water. This vessel is separated from a second copper one by a layer of Norwegian felt nearly 2 inches in thickness. The thermometers are immersed to a depth of 8 or 10 inches, and, where they pass into the vessel, are surrounded by felt. The indications of these thermometers are unaffected by the stem temperature, and under these circumstances it appeared probable that the temperature indicated could not be appreciably affected by the temperature of the laboratory. I have recently been using a platinum thermometer presenting certain novel features which I believe tend to greater accuracy, also the coils of the resistance-box consist of naked wires immersed in rapidly-stirred oil, and thus *the* great impediment to accurate work, viz. uncertainty as to the temperature of the box coils, is eliminated. On four different occasions this thermometer was placed in the ice, as above described, and its resistance on these respective occasions was as follows:

1293·968
·966
·963
·966

Now in this thermometer the value of $\frac{dR}{dt}$ (at 0) = 5·073, and

thus a difference of 5 in the last figure represents a change of 0°·001, hence the greatest difference from the mean on these occasions was considerably less than 1/1000°. This at first sight appeared satisfactory.

At the commencement of October I was enabled to make a couple of determinations of the freezing point with the same thermometer in a new apparatus which has been constructed for the purposes of an investigation into the freezing point of dilute solutions. This apparatus (the particulars of which I hope subsequently to publish) has been so devised that the temperature of the air space surrounding the entire vessel (as also that of the walls surrounding that air space) is at the temperature of the mixture in the inner vessel. I have only as yet been able to make two determinations; the results are as follows:

1293·947
·948

Hence it would appear that in spite of the agreement in the previous results, the influence of the room temperature was sufficient to cause a rise exceeding 0°·003 in the temperature. The similarity of the conditions throughout the first series accounts for the agreement amongst their results.

It may be objected that such accuracy is unnecessary. To this I would reply that it is evidently desirable that the limit of error of a "fixed point" should certainly be less than the limit of experimental error due to the instruments by which temperature can be measured.

I do not feel in a position to make any satisfactory suggestion; at the same time I think we should be moving in the right direction if we more strictly defined the *conditions* of a determination. For example, we should obtain more constant results than are possible under existing circumstances if we adopted, as a temporary standard, water distilled at a certain rate from alkaline permanganate within some definite but convenient limit of time, which, having been collected in either copper or Jena glass vessels was then mixed in certain definite proportions with ice, crystallised from it, and placed in a vessel of definite size when the temperature of the laboratory was within certain limits*. The ideal standard being experimentally unobtainable, we require a practical one if uniformity and accuracy are to be attained.

* As a difference in pressure of 100 mm. causes a change of about 0°·001 in the freezing point, a statement as to the standard pressure should (in anticipation of further advances in thermometry) be added to any definition such as that suggested *supra*.

Section II. *Boiling Point.*

The following is, I believe, our definition of standard boiling point: "the temperature of the steam issuing from water boiling under a pressure of 760 mm. of mercury at temperature 0° and at sea-level 45°."

From a practical point of view this definition is, I now believe, sufficient. Until recently I have been doubtful as to the rapidity with which, if the barometer changes, the steam acquires the temperature corresponding to the new pressure, and I anticipated that experiment would show there was a considerable lag. I had hitherto failed to satisfy myself on this point since the uncertainty arising from want of knowledge as to the actual temperature of the box coils was greater than the temperature changes consequent on the changes of pressure usually occurring during an experiment, and results obtained from observation on different dates would convey no information concerning the lag, if any. As previously indicated, I now can measure the changes in temperature of the box coils (I do not here refer to the absolute temperature) certainly to within 0°·01 C., and the resulting observations are sufficient to settle this lag question, at all events to my own satisfaction. On October 7th Mr Whetham and I were engaged in standardising a thermometer for the purposes of a joint investigation. The pressure on this occasion proved to be so variable that it appeared hopeless to attempt an accurate determination of the resistance at 100°. We, however, continued the observations for a period of nearly 3 hours in the hope of throwing some light on the matter to which I have referred. The results are shown in the following table, but are much better exhibited when plotted. The actual numbers, as also the variations, are so dissimilar that I have adopted the following method for showing the relative movements. The mean pressure throughout the observations was 755·00 mm.; the mean resistance (after correction for temperature of coils) was 1792·546 of my box unit (1 box unit = ·01 ohm approximately). In columns 2 and 3 I give the differences from these means respectively at the given times*, the barometer differences are then plotted on 10 times the scale of the resistance differences, and thus the paths of the two irregular curves are brought comparatively near together. It is evident that hardly any signs of lag are observable; in fact, if there is any lag, it would appear to be rather in the barometer than in the temperature of the steam. In column 4 I give the boiling point reduced from the pressure according to Regnault's

* The times were only recorded to the nearest minute, further accuracy being unnecessary.

experimental values as revised by Moritz. (The barometer readings are corrected for temperature and for g to sea-level latitude 45°.)

The mercury standard barometer used has been recently compared with a Tonnelot barometer in the possession of Mr Heycock which was standardised by Dr Guillaume in the spring of this year, and I may remark that a discrepancy of as much as ·2 mm. would appear to exist between the Sèvres and Kew standards if the comparisons of my $\frac{3}{4}$-inch barometer are to be trusted. The conditions of the comparison with Mr Heycock's barometer, however, were not entirely satisfactory, and I hope to have an opportunity of repeating them under more favourable circumstances. I cannot therefore speak with certainty as to the *absolute* value of the pressure which, however, is immaterial for our present purpose.

An error of ·03 mm. in a barometer reading corresponds to 0°·001 in the boiling point, and great precautions, therefore, are necessary to ensure sufficient accuracy. If the barometer is continually handled when resetting during a series of observations, the thermometer attached to the instrument does not truly indicate the temperature of the mercury in the barometer tube. The compensated sulphuric acid barometer of Professor Callendar and the writer's does not require handling, its scale is nearly six times as open as the mercury scale, and its readings are independent of temperature changes. It cannot, however, be relied on for absolute readings, one reason being the permanent shrinking of the glass bulb which is similar in nature to that characteristic of mercury thermometers. The plan I have adopted, therefore, is as follows. At regular intervals of about 20 minutes a careful reading of both barometers is taken, the mercury barometer being left untouched with the case shut throughout the interval, while at intermediate times the sulphuric acid barometer is read through a telescope and the absolute values deduced by interpellation from the comparisons at each end of the interval.

The following arrangement ensures great accuracy in the setting of the ivory point, which is a frequent cause of error in mercury barometer readings. The cistern is entirely surrounded by a covering of black paper, with the exception of a small window in front and another behind the ivory point. The point of view is thus always the same. Across the further window is placed a very thin black line whose image is seen in the mercury across that of the ivory point. This image is viewed through a powerful magnifying glass and the curvature of the line can be seen before the dimple is apparent. I would strongly recommend this method, for I find that great accuracy results from its adoption.

The numbers in Col. v. are obtained directly from the platinum thermometer readings whose constants are as follows:

$$R_1 = 1793{\cdot}434, \quad \delta = 1{\cdot}5105,$$

$$\frac{R_0 = 1293{\cdot}947}{FI = \ 499{\cdot}487}, \text{ hence } \frac{dR}{dt} \text{ at } 100^\circ = 4{\cdot}9195;$$

$$\therefore \text{ b. p.} = 100 + \frac{R - 1793{\cdot}434^*}{4{\cdot}9195};$$

and as the numbers given in Col. III. are the differences from 1792·546, by substitution we get

$$\text{b. p.} = 100 + \frac{d - {\cdot}888}{4{\cdot}9195},$$

where d is the difference as given in Col. III.

The last figure of the numbers in Col. III. was read directly from the box-vernier without any estimation of intervals—the galvanometer showing a distinct deflection for a difference of ·001.

Since $\frac{dR}{dt}$ at $100^\circ = 4{\cdot}92$ approximately,

it follows that a difference of 5 in the last figure corresponds to 0°·001, hence the scale even as regards pressure is nearly twice as open as that of a barometer whose vernier reads to ·01 mm. and thus, provided there is no lag, a more accurate determination of the pressure can be made by this thermometer than by observing the barometer itself.

The barometer observations were taken by one observer at the same instant as the other read the resistance indicated on the box.

An inspection of this table will I think convince the most sceptical that the temperature lag of the steam may be disregarded. It appears astonishing that the temperature adjustment should be so rapid, especially when a sudden fall such as that shown from 2.30 to 3 p.m. *supra* takes place. Here the cooling can only be caused by excess of evaporation, and when one reflects on the mass of copper, &c., bounding the enclosure, the rapidity of adjustment is surprising.

I should mention that the hypsometer is of a somewhat unusual form and that special screens are used both below and above the thermometer bulb.

* These values of the constants were obtained subsequently, and are quite independent of the observations on Oct. 7th.

Results of Boiling Point determinations, Oct. 7, 1896.

I Time	II Bar. − 755·00 (corrected)	III $R - 1792{\cdot}546$	IV B. P. from bar	V B. P. from therm. readings
	m.m.		°	°
1.35	+ ·85		99·848	
1.46½	+ 1·03	+ ·177	·855	99·855
1.48	+ 1·03	+ ·179	·855	·856
2.23	+ 1·10	+ ·183	·857	·857
2.29	+ ·99	+ ·167	·854	·852
2.33	+ ·81	+ ·142	·847	·848
2.44	+ ·60		·839	
2.51	+ ·44	+ ·075	·833	·835
2.53	+ ·29	+ ·047	·828	·829
2.58	− ·20	− ·042	·810	·811
3.0	− ·39	− ·073	·803	·805
3.2	− ·54	− ·109	·797	·797
3.9	− ·94	− ·180	·782	·783
3.21	− ·96	− ·185	·781	·782
3.23	− ·93	− ·174	·783	·784
3.27	− ·85	− ·163	·786	·786
3.29	− ·80	− ·152	·788	·789
3.30	− ·77	− ·146	·789	·790
3.33	− ·78	− ·150	·788	·788

It thus appears that our present definition is sufficient from a practical standpoint for, since there is no lag, the same conditions can be reproduced by different observers with sufficient accuracy and, further, Regnault's values of $\frac{dt}{dp}$ are, for pressures near 760, sufficiently accurate. From a theoretical point of view, however, the conditions are by no means perfect. We require a *standard absolute pressure*, the value of which can be stated without our present arbitrary method of reference to a mercury column, and it seems fitting that the attention of physicists should now be directed to this question. The effects of any change, however, would be so far-reaching that I doubt if I should have ventured to express my own views on this matter had it not been for the powerful support afforded by a letter I have recently received from Professor Hicks, F.R.S., of Sheffield. He has been so kind as to give me permission to make what use I please of that letter, and I cannot close this communication better than by transcribing it almost in full.

".....It is time to come to some determination as to the value of the normal pressure—on which indeed all thermal units of measurement ultimately depend. At present we define standard pressure as that of 76 cm. of mercury at 0° in Paris—a complicated and roundabout method which depends on a local value of gravity, a temperature, and a density.....

"The natural standard of pressure—10^6 dynes per sq. cm.—is, as is well known, very nearly the same as the present standard of 76 cm. I find that a column of mercury 75 cm. (three-quarters of a metre) high at 0° at sea-level in latitude 45° exerts a pressure of (1 − ·00007) megadyne/cm.^{2}, for most purposes identical with a megadyne (in lat. 45° 1′ identical).

"I would propose then that the megadyne per sq. cm. be formally adopted for the standard pressure to which to refer properties of gases. The fact that it is $\frac{3}{4}$ the pressure of a metre of mercury at 0° in lat. 45° is convenient and may be rendered accurate either by altering infinitesimally the height, the temperature, or the latitude. *Or*, the term "atmosphere" might be kept for it.

"In this method instead of speaking of a vacuum of so many millionths of an atmosphere we should say so many dynes per sq. cm. The advantages are that the unit is an absolute one, that it is nearly the same as that now in general use and that its value is probably much nearer the time-mean of barometric height in the majority of laboratories than 76 cm.

"Supposing that the absolute unit of pressure be adopted, the 100° on the new thermometer would be the temperature of steam under one megadyne per sq. cm., which—using data given in Everett's "Units"—works out to 99·62° C. The new degrees would therefore be ·9962 times the present ones.

"Taking again the number given in the B.A. Report for diminution of specific heat of water per 1° as ·00036, and supposing it to hold backward from 10° to 0° (which, however, can scarcely be the case) the unit of heat (4·2 joules) would be measured by the heat necessary to raise 1 gram of water from 0° to 1°. This last result, if really true, is a curious coincidence and would be an argument in favour of the change here proposed. It would define the heat unit as at present, and at a temperature at which accuracy is in general most easily attained.

. . . . . .

"It may be objected that the alteration of the thermometric scale puts the proposal outside the limits of what is practical. To this I would observe that there are two classes of people to be considered, the general public and the scientific world. To the first it would make no difference, the thermometers used by them would scarcely show the difference and as a rule they do actually err by amounts much larger than the change proposed. In

accurate scientific thermometry it would only mean an alteration in the table of corrections for each instrument, and in translating old results it would simply mean multiplying by a factor $1 \pm \cdot 0038$....."

I may say that I am in general in hearty accord with the propositions advanced by Professor Hicks although, at the same time, we differ on one or two minor points.

The conclusions I have arrived at may be summed up as follows:—Our present definition of the freezing point is theoretically sound but practically impossible; our present definition of standard boiling point is practically satisfactory while theoretically capable of improvement.

THE COUNCIL.

(*Elected Oct.* 26, 1896.)

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PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Monday, January 25, 1897.

Mr F. Darwin, President, in the Chair.

The following Communications were made:

(1) *Some results obtained by staining the Brain with the Chrome-silver method* (illustrated by photo-micrographs). By Dr A. Hill.

Dr Hill showed a granule of the olfactory bulb with a looped axis-cylinder, and also certain forms of granule of the cerebellum not hitherto described. He also exhibited sections and photographs showing the variations in the form of the "thorns" on the dendrites of nerve-cells which can be produced by varying the hardening process. (1) The thorns may be absent. (2) They may be long or short. (3) They may have the typical form of a minute rod with a dot at the end, or the dots may be divided and lie on the course of the rod. (4) They may be replaced by long filaments.

Dr Hill submitted the theory that the conducting filaments of nerve-cells are continued from one neuron to another. He regards the "thorns" as the cell-ends of the non-staining nerve-fibrils which are invested with a film of cell-plasm. He supposes that the extension of the cell-plasm along the course of the fibril is

necessary for the conduction of impulses and thinks that the amount of this film varies, resulting in the variations in the state of "openness" or inhibition of the reflex path.

(2) *A possible explanation of the quinqueloculine arrangement of the chambers in the young of the microspheric forms of* Triloculina *and* Biloculina. By J. J. Lister, M.A., St John's College.

In the later stages of growth of the shells belonging to the genus *Biloculina* the arrangement of the chambers is very simple (Figs. 1—3). They are disposed on either side of an axis, the mouth being directed alternately to one and the other end of this axis, and the median plane which divides any single chamber symmetrically, is also the median plane of the whole series of the chambers added during the later stages of growth. In the megalospheric

Fig. 1. *Biloculina depressa* d'Orb. Megalospheric form, longitudinal section. × 28.

Fig. 2. Transverse section of a specimen with a smaller megalosphere of the same species. × 50.

form (Figs. 1 and 2) this arrangement is followed from the beginning, that is to say, the two chambers succeeding the megalosphere and all subsequently formed chambers are so disposed.

In the microspheric form of this genus however, as has been admirably shown by M. Schlumberger, the arrangement of the chambers is at first on a different plan (Fig. 3), namely, that

Fig. 3. Transverse section of the microspheric form of the same species. × 90 (Two external chambers have been omitted from the figure.)

which characterises the genus *Quinqueloculina* during its whole growth. In the shells of this genus (Fig. 4) the chambers are

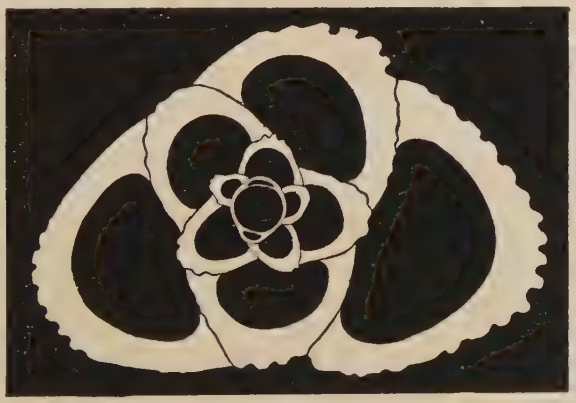


Fig. 4. Transverse section of the megalospheric form of *Quinqueloculina undulata* d'Orb. × 100.

Figs. 1—3 are from Schlumberger, "Revision des Biloculines des grands fonds." *Mém. d. l. Soc. Zoologique de France*, Tome IV. année 1891, p. 155. Figs. 1, 2, and 4. Fig. 4 from "Monographie des Miliolidées du Golfe de Marseille," by the same Author. *Mém. d. l. Soc. Zoologique de France*, Tome VI. année 1893, p. 57. Fig. 23.

disposed on the sides of a long axis, and the mouth is directed alternately towards one and the other end of the axis, but the

plane dividing any single chamber symmetrically, is not identical with the corresponding plane of the preceding chamber but directed at a definite angle to it. It is as though in a *Biloculina* shell, while the plane in which the new chambers were formed remained constant, the long axis were rotated through a definite angle in the interval between the formation of each chamber and the completion of its predecessor. The result is that the walls of several chambers take part in forming the external contour of the shell. In *Quinqueloculina* five chambers take part, at any given stage of growth in forming the contour. In *Triloculina* where the "rotation of the axis" is through a different angle the contour presented by the shell is formed by three chambers.

In the *microspheric* form of the genus *Biloculina* the arrangement of the chambers which succeed the microsphere is at first on the quinqueloculine plan, then on the triloculine, and it is not till several chambers have been formed that the biloculine plan is attained.

Similarly M. Schlumberger has shown that in the genus *Triloculina* the chambers are at first arranged on the quinqueloculine plan in the microspheric form, the triloculine plan being attained later.

What is the explanation of this remarkable change in the mode of growth in the microspheric form of *Biloculina* and *Triloculina*? And why should it be characteristic of the microspheric form, while the megalospheric form attains the arrangement of chambers characteristic of the genus to which it belongs from the beginning of its growth?

It appears possible that the explanation is to be found from the consideration of the different ways in which the two forms are produced.

In dealing with the life history of *Polystomella* I have shown that there is good reason to regard the megalospheric and microspheric forms of the Foraminifera as belonging to different generations. The megalospheric form is produced by an asexual process—the protoplasm of the microspheric parent emerging from the shell and breaking up into a number of spheres, the megalospheres, the initial chambers of the megalospheric form.

The precise origin of the microspheric form is not yet clearly made out, but there is some evidence in support of the view that it results from the conjugation of the zoospores produced by the megalospheric form, that is, that it is produced by a sexual process.

The relative sizes of the zoospore and microsphere, and the scarcity of the microspheric form as compared with the megalospheric are in favour of this view and Schaudinn's observation of

the conjugation of zoospores in *Hyalopus**, a form however in which dimorphism has not been shown to exist, yields it considerable support.

If we apply this probable, though still insecure result to the problem presented by the Miliolidæ, we find that while the asexually-produced megalospheric form has a direct development, the sexually-produced microspheric form goes out of its way to repeat the arrangement characteristic of allied forms before it attains the arrangement proper to its own genus.

Is not this a particular instance of a phenomenon widely met with in higher forms of animals, in which the individuals produced by budding attain the adult structure by a direct development, while those produced from the egg often develope by an indirect course, going out of their way to repeat lost features characteristic of the archaic forms of their race? To give an instance, the ascidians belonging to the genus *Clavellina* develope asexually by buds which grow out from the stolons at the base of a colony and are fashioned directly into the adult form, while the result of the development of the fertilised egg of *Clavellina* is to produce a tailed larva wholly unlike the parent and recalling the free swimming *Appendiculariæ*.

In the case of higher animals the larval stages are lost, the body of the larva being fashioned into that of the adult, but in this group of the Protozoa, the *Miliolidæ*, the peculiar structure of the young is permanently recorded, being built in and retained in the centre of the chambers subsequently added.

While many of the biloculine and triloculine forms of the Miliolidæ present the contrast in the mode of growth of their megalospheric and microspheric forms, above described, in a very marked manner, I feel that the case in favour of the explanation here offered is to some extent weakened by the existence of exceptions presented by certain species belonging to the trematophorine Miliolidæ, and especially by *Idalina antiqua* (d'Orb. sp.), whose structure has been elucidated by M. M. Munier-Chalmas and Schlumberger†.

In this species, whose growth in its later stages is mainly on the biloculine type, the first formed chambers of the megalospheric form are in some cases, though not in all, disposed as in the microspheric form, on the quinqueloculine plan. Here then both megalospheric and microspheric forms of the species may, in the early stages of growth go out of their way to repeat the mode of growth characteristic of the genus *Quinqueloculina*. The authors point out that the megalospheric form does in some cases grow on

* *Sitz. Ber. Gesellsch. Naturforsch. Freunde Berlin*, Jahrg. 1894.

† Note sur les Miliolidées trématophorées. *Bull. de la Soc. Géologique de France.* 3e Ser. t. XIII. (1885), p. 273.

the biloculine plan throughout, and when it does not, the biloculine arrangement is sooner reached, that is after the formation of a smaller number of chambers, than in the case of the microspheric form. The exception does however appear to detract in some measure from the sharpness of the contrast in the mode of growth of the two forms, and, though I think without altogether annulling it, from the force of the explanation which I have offered.

(3) *On the theory of Osmotic Pressure.* By J. Larmor, M.A., F.R.S., St John's College.

As osmotic theory is now attracting general attention in this country, it seems desirable that all the positions that are maintained in regard to it should be clearly set forth. The excuse for offering the following remarks is that for some time I have paid attention to the subject in its relation to general molecular theory, both in the thermal and the electrical aspects. I fail to recognize how the validity of the thermodynamic basis of the law of osmotic pressure can be shaken: and though the idea of ionic dissociation in solutions is an additional hypothesis which must be judged separately by the extent of its agreement with the facts, it appears to me that in some form—possibly not at all in the chemical imagery with which it is at present often expounded—it holds the field. It is difficult in fact to see how the hypothesis that the same chemical element can have different valencies in different series of compounds, which is now usually accepted, is fundamentally any whit less paradoxical than the hypothesis of ionic dissociation; anything that throws light on the real nature of the one must also illuminate the other.

In his recent note on this subject*, Lord Kelvin appears to allow, within certain limits, the cogency of the argument which bases the law of osmotic pressure on Henry's empirical law of solubility for gases, an argument which has recently been carefully restated by Lord Rayleigh, having previously been employed, as he remarks, in forms more or less explicit, by van't Hoff, Nernst and other investigators. The connexion thus established however hardly amounts to a physical demonstration, because it only deduces one empirical relation from another. Yet it seems desirable to draw attention to the fact, which I have not seen anywhere remarked, that this method had been employed by von Helmholtz in 1883, two years before van't Hoff announced his theory of the correlation between osmotic and gaseous pressures; and that the principles given by him on an investigation of the work-equivalent of gaseous solution, made in connexion with the theory of galvanic

* *Proc. R. S. Edin.*, Jan. 1897; *Nature*, Jan. 21, 1897, p. 272.

polarization *, involve in fact an implicit prediction of the osmotic law. This circumstance, that the law of osmotic pressure, as regards dissolved gases, is tacitly involved in von Helmholtz's equations, does not of course confer on him a position in the actual development of the subject.

But the theory of osmotic pressure can I think be placed on a purely abstract basis independently of the law of solubility of gases, which would then assume the form of a deduction from it. The broad principles on which this is to be done have been in fact laid down in a precise and very general manner, but without special application, by Willard Gibbs as early as 1875, in his fundamental development of the laws of mechanical availability of energy†. The following position is, I believe, sound. Each molecule of the dissolved substance forms for itself a *nidus* in the solvent, that is, it sensibly influences the molecules around it up to a certain minute distance so as to form a loosely connected complex, in the sense not of chemical union but of physical influence. The laws of this mutual molecular influence are unknown, possibly unknowable; but provided the solution is so dilute that each such complex is, for very much the greater part of the time, out of range of the influence of the other complexes, as for instance are the separate molecules of a free gas, then the principles of thermodynamics necessitate the osmotic laws. It does not matter whether the nucleus of the complex is a single molecule, or a group of molecules, or the entity that is called an ion: the pressure phenomena are determined merely by the number of complexes per unit volume. To determine the osmotic forces, we must know the change in *available* energy that is involved in dilution of the solution by further transpiration of the pure solvent into it. In finding that change, the laws of mutual action between molecules of the dissolved substance are not required: for there is actually no action between them, and as soon as the solution becomes so concentrated that such mutual action between the complexes comes in, the theory is no longer exact. Nor are the laws of mutual action between the molecules of the dissolved substance and those of the solvent required, because the effect of transpiration of more of the solvent into the solution is not in any way to alter the individual complexes. The change in available energy of the system, on dilution, thus solely arises from the expansion of the complexes into a larger volume; and it can be traced into exact correlation with the change

* H. von Helmholtz, "Zur Thermodynamik chemischer Vorgänge III"; in collected papers Vol. III. pp. 105—114, especially his equation (4), and the theory of diffusion at the end. [The law had also been formulated explicitly in this manner by Willard Gibbs as early as 1876, *loc. cit. infra*, p. 227.]

† *Trans. Connecticut Academy*, Nov. 1875, p. 138, "Effect of a Diaphragm (Equilibrium of Osmotic forces)."

of available energy that occurs in the expansion of a gas. This argument meets the objection that a true theory should involve a knowledge of the molecular actions between the various molecules. It would seem that with just the same cogency it might be argued that a real investigation of the connexion of the alteration of the freezing point of a liquid by pressure and its change of volume on freezing should involve a knowledge of the individual molecular actions in the liquid: and so it would, had we not the means of evading considerations of molecular constitution that is afforded by Lord Kelvin's great principle of dissipation, which is for this very reason at the basis of all physical theory.

There is however one point to be remembered, namely, that the theoretical osmotic pressure is a limiting value which may not be reached by an actual arrangement, unless we can be certain that it works reversibly and so without heating effects.

The remark has been made by Lord Kelvin, that the connexion between Henry's law and the osmotic law must break down when the solution of the gas is accompanied by change in its state of molecular aggregation. It is also probable from the fundamental ideas as to dissociation and aggregation, that such change would usually be partial, and not uniform over all the dissolved molecules; so that it is not to be expected that Henry's law would in such circumstances hold good. The point in which the argument, as set forth in precise form by Lord Rayleigh*, becomes then inapplicable, is that the gas expelled from solution by the osmotic process must be considered as emerging in the actual state of aggregation differing from that of its free condition, and its return to the latter state involves further change of available energy.

If the considerations above stated, which will be most suitably developed in detail in another connexion, are valid, it follows that Professor Poynting's† recent suggestion with a view to evading the necessity of the ionic dissociation hypothesis cannot avail, as it would not lead to the desired value for the osmotic pressure: that pressure depends on the number of molecular complexes involving the dissolved substance that exist on the dilute solution, but not on their individual degrees of complexity.

* *Nature, loc. cit.*, p. 254.

† *Phil. Mag.* Oct. 1896.

Monday, February 8, 1897.

Mr F. Darwin, President, in the Chair.

Dr A. C. Haddon, Christ's College, was elected a Fellow of the Society.

The following Communications were made:

(1) *On the Cathode Rays.* By Professor J. J. Thomson.

The experiments described in this paper are of two kinds: the first set are on the electric charges carried along the rays, the second on the deflection produced in these rays when they traverse a uniform magnetic field.

In the experiments on the electrical effects produced by the rays, the cathode, a plane disc, was placed in a small side tube fused on to a large bulb; between this tube and the bulb there was a thick earth-connected metal disc with a slit in it; a pencil of cathode rays shot through this slit into the bulb. In the bulb on the side opposite to the slit there was an arrangement similar to that used by Perrin in his experiments on the charges carried by the cathode rays; it consisted of two cylinders, one inside the other; the outer cylinder was connected with the earth and the inner cylinder (which was insulated from the outer) was connected with one pair of quadrants of an electrometer. Slits were cut in the cylinder so that the cathode rays could pass through the slits into the inside of the inner cylinder. The cylinders were placed at a considerable distance from the direct line of the rays, so that unless the rays were deflected by a magnet they did not enter the cylinder. The charge in the cylinder produced by each make and break of the coil was investigated. A slight charge was found to pass into the cylinder even when it was not in the direct line of the rays, due probably to a diffused charge sent out from the tube through the slit into the bulb at each discharge of the coil; this charge was small; it was generally negative but at high exhaustions was frequently positive. When the rays were deflected by a magnet so as to pass inside the cylinder the cylinder received a strong negative charge; the charge was large as long as the phosphorescent patch was stopped by the cylinder, small when by the motion of the magnet the patch was removed to one side or another of the cylinder. This experiment seems conclusively to show that there is a flow of negative electricity

along the cathode rays. The following experiments show however that there must be something besides a stream of negatively electrified particles along the cathode rays. If the coil is kept running the negative charge in the cylinder does not increase indefinitely, it reaches a certain limit and then remains constant, though the cathode rays keep pouring into the cylinder, and further if the inner cylinder be charged negatively to begin with, then if this charge exceeds a certain amount, though the insulation is perfect when the rays are not playing upon the cylinder, yet as soon as the rays fall upon it some of the *negative* charge escapes.

In the experiments on the magnetic deflection of the rays, the rays were produced in a side tube and sent into a large bell jar through a slit in a metallic plate. The bell jar was placed between two coils arranged as in a Helmholtz galvanometer so as to produce a uniform magnetic field. The rays in their course through the bell passed in front of a glass plate ruled into squares. A large number of photographs of the rays were taken in different gases and at various degrees of exhaustion. The following were some of the results obtained.

The magnetic deflection of the cathode rays in air, hydrogen, carbonic acid gas and methyl iodide is the same provided the mean potential difference between the cathode and the anode is the same.

Coming through the slit there are certain 'rays' which are not deflected by a magnet: these have little if any power of producing phosphorescence. The path of the rays for the first part of their course was very approximately circular.

(2) *On Electricity in Gases and the formation of Clouds in Charged Gases.* By J. S. Townsend (M.A. Dub.).

1. Of all the electrical phenomena with which we are acquainted there are few more interesting from a scientific point of view than those which are to be found in gases. The branch of the subject which is treated of in this paper is that which is related to the properties of gases having an electrostatic charge. No attempt is made here to give an account of the different methods that have been employed to charge gases, as it is my intention merely to enumerate some of the more important results arrived at while investigating this subject.

2. Even with their rough methods of detecting electrification Laplace and Lavoisier knew that the hydrogen evolved when a metal dissolves in an acid carries with it a charge. It is therefore all the more remarkable that with the present delicate methods of detecting electric charges the question of electrification by natural processes, in contradistinction to artificial methods should have

attracted so little attention. The greater part of this paper is devoted to an account of the electrification of gases which are given off when a liquid is decomposed by an electric current, being the most noteworthy result which I have arrived at while examining the charges on newly prepared gases. Under certain circumstances gases can be obtained by this method which have an electrification, ten or twenty times greater than that which can be given them by artificial processes. I find also that these gases possess the remarkable property of forming a dense cloud when they come into contact with moisture. The density of the cloud increases with the density of the charge, and so distinctly does this property manifest itself that a dry gas, charged even with a density of electrification one-twentieth of that which can be obtained, forms a distinctly visible cloud when it comes into the moist atmosphere of the room.

3. It is necessary here to describe some principles which will be frequently used in the course of the following experiments. In order to distinguish between cases when a gas, or the dust or spray it may contain, carries the charge it is sufficient to send the gas through a series of fine gauzes or through a tube containing glass wool; by so doing not more than 20 or 30 per cent. of the charge will be lost if the gas is electrified, but if the electricity resides on the dust or the spray it will be completely removed by filtering through the wool. In the case of a gas, the discharging power, or the fraction of the charge left behind on any apparatus, varies greatly according to the rate at which it is sent through, being less for the greater velocities. There is another very striking difference between the behaviour of a charged gas and that of a collection of small charged particles when left in a vessel, namely: the gas retains its charge for a considerable time, while a collection of small charged particles rapidly discharges against the sides of the vessel. This difference will also serve to distinguish between the two cases.

4. Let I, fig. 1, be an insulated inductor, the outer surface being made a conductor by covering it with tinfoil, and when it is connected to the insulated quadrants of the electrometer the motion of the spot of light (reflected from the mirror attached to the needle), on the scale will register the quantity of electricity entering or leaving I. If the air from the room be blown into I from a blow-pipe bellows through a plug of glass wool the position of the spot of light will not be affected, but if the tube containing the wool be removed and the air be blown directly in from the indiarubber tubing it will be found that the electrometer shows a large positive charge on I. This charge was unable to pass

through the tube containing the glass wool, and if this be now replaced on the end of the indiarubber tubing it will be found impossible by blowing unelectrified air into I to displace any of the charge that had been put in after it had been there for the space of half a minute.

We have a simple explanation of this result by supposing the charge to be distributed over a small number of carriers, each having a relatively large charge, but in the case of a gas with an equal electrification per c.c. the number of carriers is very much larger and the charge on each proportionally smaller, being probably the atomic charge, so that the force due to their mutual repulsion which acts on each carrier would be small, and consequently a long time would elapse before they would reach the sides of the vessel. This property of a charged gas can be illustrated by the following experiment.

5. The electrification of hydrogen which is given off when a metal is dissolving in an acid can be seen by pouring a 50 per cent. solution of sulphuric acid, at a temperature of about 80°, over the iron wire contained in the vessel A, fig. 1. The hydrogen as it is given off is made to bubble through permanganate of potash in B,

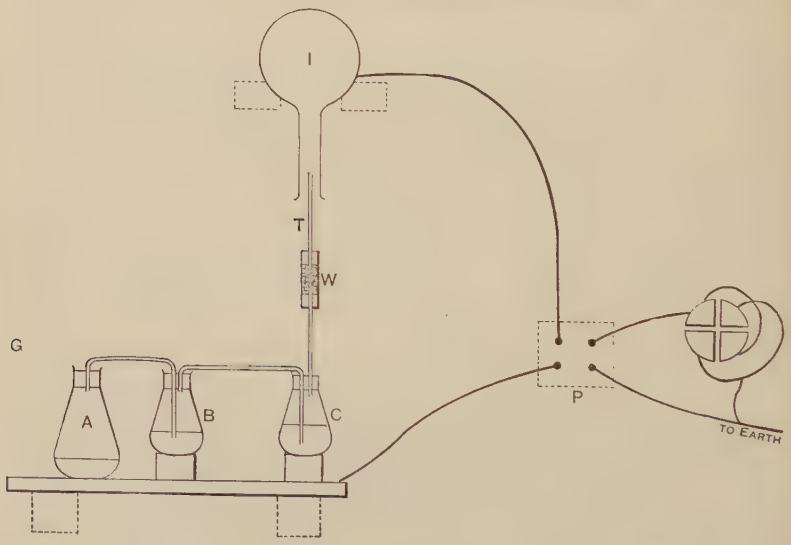


Fig. 1.
(The dotted lines represent insulators.)

and through sulphuric acid in C, and then to pass through a tube of glass wool W before entering I, which is connected to the

insulated quadrants of the electrometer, the generating apparatus G being connected to earth. These connections can be easily made by means of the mercury cups in the paraffin block P. The spot of light on the electrometer scale shows a large deflection in the positive direction as the hydrogen enters I. If G be now removed and the hydrogen left in I for the space of five or ten minutes it will be found that if it be then displaced by blowing in unelectrified air that the spot of light will return almost to the place from which it started showing that the gas retained more than 95 per cent. of its original charge. Other experiments similar to this showed that when a gas is left in an inductor for the space of two hours that it still retains from $\frac{1}{5}$th to $\frac{1}{4}$th of its original charge.

6. With regard to the fact that the hydrogen, given off when iron is dissolving in sulphuric acid, carries with it a positive charge, I find that this result had previously been obtained by Enright, who examined the electrical properties of several newly prepared gases, and although his investigations were incomplete, and his results capable of different interpretations, nevertheless they have not been accorded the credit they deserve. None of his experiments were directed to finding out whether the charge was on the gas or on the spray, but he unhesitatingly assumed that the electricity was in all cases carried by the gas. Under these circumstances it is easy to see that some of his conclusions may have been erroneous. (Enright: 'Electromotive force of contact between a liquid and a gas,' *Phil. Mag.* 1890; *Nature*, vol. 36.)

Enright's experiments were repeated by Brown (Brown: 'Electrification of the Effluvia from Chemical or from Voltaic Reactions'), who found that all the experimental results were quite correct. No new light was thrown on the matter; the conclusion which the experimenter himself arrived at was that it was impossible to decide whether the gas or "effluvium," which it carries with it from the acid in which a metal is dissolving, is charged, the balance of evidence being in favour of the latter. Professor Lodge (*Phil. Mag.* 1890; *Nature*, vol. 36) attributed the cause of the electrification which Enright observed to the well-known electrification of spray, but the following experiment will show that his interpretation was wrong.

7. The experiment which Enright performed and from which he drew his conclusions with regard to a metal dissolving in an acid is very easily performed by insulating the apparatus G, fig. 1, and connecting it to the quadrants of the electrometer. If the vessel A be left open so that the hydrogen may escape directly into the air it will be found that when a 50 per cent. solution of

sulphuric acid at about 80° is poured over two or three grammes of iron wire that the electrometer first shows that a large positive charge is escaping from A, and, as the process of dissolving goes on for a few minutes, that the escape of positive electricity gets less and less till finally a neutral stage is reached, and immediately after this negative electricity in large quantities begins to escape and continues to do so till all the iron is dissolved. From this Enright concluded that at one time positive and at another time negative electricity was carried by the gas. Further examination however does not justify this conclusion, for if instead of letting the gas escape into the air from A it be made to pass through the liquids in B and C and escape by the tube T into the air it will be now found that G loses positive electricity as before, reduced in quantity, whereas none of the negative which in Enright's experiment is the larger of the two charges will now escape with the gas. We see therefore that there is no evidence here that the hydrogen carries the negative electricity, and that it is more probable that this charge was carried by the spray which it acquired by an inductive effect.

8. The gases which are given off by electrolysis have always been assumed to be completely cleared of electricity. It does not require a very delicate electrometer to show that this is not the case; for if they are examined by any of the ordinary methods it will be found that under certain circumstances these gases carry a large charge which remains on the gas after it has been passed through wool or bubbled through a liquid, and owing to the regularity with which they can be produced, by keeping the current through the electrolyte constant, the method affords an extremely simple means of studying the properties of charged gases.

9. As far as experiments which I have already made on this subject show, it is possible to obtain gases with any electric density up to 5×10^{-3} electrostatic units. This has been found to be the case with regard to positively charged oxygen and hydrogen which are given off when dilute sulphuric acid is electrolysed, and also with regard to the negative oxygen which is given off from a caustic potash cell. The hydrogen from the latter also has a negative charge, but it is small compared with that which is on the oxygen, whereas in the case of the sulphuric acid cell the hydrogen as a rule has the larger charge. The effects are in all cases increased by raising the temperature of the cell. The charge and sometimes even the sign is altered by impurities, but it is not proposed here to enter into details concerning the electrolytes. Suffice it to say that the positive gases are got from sulphuric acid, and the negative from a caustic potash solution when a

current of about 12 or 14 ampères is sent through them. They have also been found to be electrified when so small a current as two ampères is sent through them, but the rate of evolution is so small that the larger currents have been generally used.

10. When these charged gases are bubbled through water, they become clouded. The cloud which is thus formed can be completely removed by bubbling the gases through sulphuric acid or sending them through a U tube of $CaCl_2$, but when they get into the atmosphere of the room they condense the moisture and form a stable cloud in an unsaturated atmosphere. The process of forming the cloud in positive or negative oxygen by bubbling through water, and removing it again by bubbling through sulphuric acid, can be gone through without losing more than 20 or 25 per cent. of the original charge on the gas. Clouds having similar properties are formed by the hydrogen coming off a rapidly dissolving metal.

11. Fig. 2 represents a convenient form of apparatus for examining the electrical properties of these gases. The electrolyte is contained in the beaker *F*, and the gas which is given off from

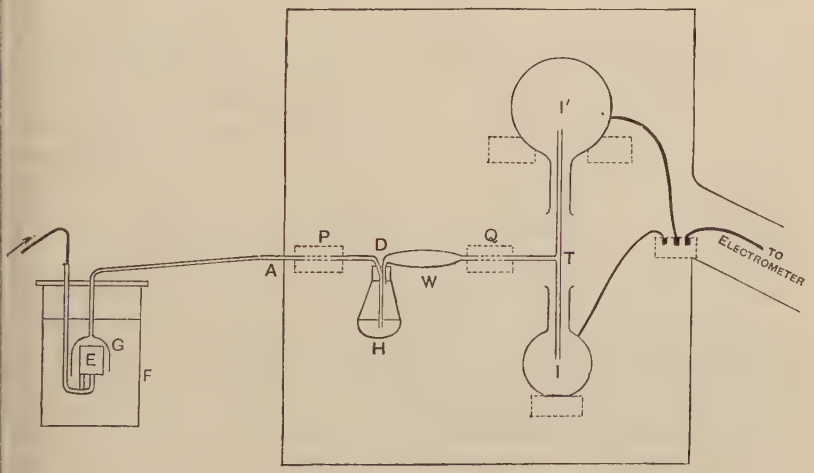


Fig. 2.
(The dotted lines indicate insulators.)

the platinum electrode *E* rises into the cup *C* and is brought by glass tubing to the paraffin tunnel *P* inside a large screen *S*, which contains all the apparatus which it is desirable to connect to the electrometer in order to measure charges. Between the two tunnels *P* and *Q* can be put any apparatus *D* whose discharging

power it is required to find. The gases after passing through D are led into an inductor by means of the tube T, the inductor I being used for oxygen and I' for hydrogen. By connecting I to the electrometer while a fixed current is going through the cell the quantity of electricity Q_1 which the gas carries with it through D per minute can be found, and then, with the same current through the cell, D can be connected to the quadrants and the quantity of electricity Q_2 which the gas leaves behind per minute can be obtained. When D consists of a sulphuric acid bottle H and a tube of glass wool W, it is found, when the gases from the sulphuric acid and caustic potash cells are examined, that they have a large charge when they enter an inductor. This charge depends very little on the liquid which is in H, but is considerably reduced by packing the wool very tight in W. The discharging power $\frac{Q_2}{Q_1+Q_2}$ varies according to the rate at which the gas is sent through the apparatus; this can be seen by increasing the current through the cell. But as this is almost a self-evident fact we will pass on from it and describe a series of more important experiments.

12. In the following experiments the discharger D between P and Q consisted of a bottle H into which various liquids could be placed and also a tube of glass wool W'. Instead of leading the gas from the cell directly into P, it was passed first through a bottle H' outside the screen so that on putting sulphuric acid into H' the gas was led into D perfectly dry, and by substituting water for the sulphuric acid in H' the gas entered P carrying with it a cloud. The liquid whose effect upon the charge on the gas is to be tested is placed in H, and sulphuric acid in H', and after the current has been running for a sufficiently long time to drive out the uncharged air from the apparatus, D is connected to the quadrants and the number of divisions N_2 which the spot of light on the electrometer scale passes over per minute is observed. The inductor I is then connected to the quadrants and D put to earth, and the number of divisions N_1 that the spot passes over per minute, due to the charged gas entering I, is then found. It is necessary to blow all electrified gas out of I before beginning to find N_1, as this number would be diminished if any electrified gas were to be escaping from I while the observations were being made. The sulphuric acid is then removed from H' and water put in its stead, and two similar numbers N_2' and N_1' are then obtained, the same current flowing through the cell as when N_2 and N_1 were being determined.

The following table contains the numbers thus obtained for sulphuric acid, water, and potassium iodide solution, in H.

These numbers give accurately the effect, on the discharging power due to the presence of a cloud, as no change was made in D

	N_1	N_2	N_1'	N_2'
Sulphuric acid in H	27·5	18	30	17
Water in H.....	26	22	28·5	18
KI solution in H	27	17	32	15
	Gas entering the liquid dry		Gas entering the liquid carrying with it a cloud	

while water was being substituted for sulphuric acid in H'. In order that the relative discharging powers of sulphuric acid, water and KI solution should be obtained to the same accuracy, the depth to which the tube in H dips into the liquid should be very accurately adjusted. But seeing that in these experiments a large part of the discharging power is due to the glass wool no particular precautions were taken to ensure this, but the tube was dipped under the liquids in each case to equal depths as accurately as the eye could judge.

13. The above numbers were obtained from experiments with negative oxygen, and as far as they and several similar experiments show, there is no increase, but rather a diminution in the discharging power of D, both in the case of positive and negative oxygen, when potassium iodide solution is substituted for sulphuric acid or water in H. So that the question of the ozone being the carrier of electricity is inadmissible, as is also the suggestion that the cloud consists of hydrogen peroxide, for not only is the charge not removed by being sent through the potassium iodide solution, but when the dry gas is led into this solution the cloud appears in the space in H above the solution.

14. When we consider the first series of numbers, which show that the removal of the cloud by sulphuric acid does not discharge the gas even to the same extent as bubbling the dry gas through the same acid, it might at first sight appear that the cloud and the charge had nothing to do with one another. It must however be remembered that only a very few of the drops forming the cloud come into contact with the acid, and the disappearance of the greater number of the drops is due to evaporation caused by the lowering of the vapour pressure by the sulphuric acid, and we have thus a good example of vapour coming off a charged sphere without carrying with it any of the charge. (See J. J. Thomson, *Recent Researches in Electricity and Magnetism*, page 54.)

15. This evidence showing that the carriers of the electricity

must be the gas is supported by a series of experiments of quite a different kind which were done with the negative oxygen from the caustic potash cell. A small inductor containing some sulphuric acid in the bottom was insulated and connected to the quadrants, and the charged oxygen then admitted, the spot of light having gone n_1 divisions negative. The inductor was then corked and the acid shaken up with the gas for $1\frac{1}{2}$ minutes and then again insulated and connected to the quadrants, when on removing the cork and blowing out the oxygen it was found that the spot of light went in the positive direction, giving say n_2 divisions. This showed that the gas retained n_2/n_1 of its original charge. The actual values obtained in two experiments of this sort were $n_1 = 56$, $n_2 = 36$ and $n_1 = 66$, $n_2 = 39$. This experiment was repeated with strong pyrogallic acid instead of sulphuric acid and it was found that a considerable fraction, about 45 per cent., of the charge remained on the gas, and could be blown out. The pyrogallic acid was left in the inductor and some strong caustic potash added, and a fresh quantity of charged oxygen was then put in and shaken up with the potassium pyrogallate for exactly the same time, $1\frac{1}{2}$ minutes, as in the case of pyrogallic acid, and it was now found that only about 8 per cent. of the charge could be blown out, showing that, of the 45 per cent. which would have remained if no absorption of oxygen had taken place, more than eight parts in ten have now been removed from the air in the inductor. The numbers which were obtained in three experiments of this kind are given in the following table; n_1 the charge (in divisions of the scale) put into I when it contained pyrogallic acid, and n_2 the charge which was blown out after shaking up with the acid for $1\frac{1}{2}$ minutes, n_1' the charge put in after KOH had been added, and n_2' the charge blown out after shaking up with the pyrogallate for $1\frac{1}{2}$ minutes.

n_1	n_2	n_1'	n_2'
86	37	71	3·5
86	43	73	8·5
85	39	65	4

16. If the electrification resides on the drops when the cloud is formed, it ought to be possible to diselectrify the gas by forming a cloud in it and then letting the cloud fall in a vessel. This is in fact what does occur, for if the gas be thoroughly dried by passing it through two sulphuric acid bottles and then let into an inductor with sulphuric acid in the bottom only a very slight

cloud is formed, and after leaving the gas in the inductor for five minutes almost the whole charge is carried out when unelectrified air is blown in; but if instead of having sulphuric acid in the bottom of the inductor some water be put into it, so that when the gas is admitted a dense cloud forms, it will now be found that after the lapse of five minutes only about half of the original charge can be blown out—the remainder having been carried to the surface of the water by the falling of the cloud.

The Tables I. and II. contain the numbers, which were obtained in experiments of this kind, N_1 denoting the charge put into the inductor (in divisions of the scale) and N_2 the charge blown out after five minutes. Table I. refers to the case where sulphuric acid is in the bottom of the inductor, and Table II. to the case where water was in the bottom of the inductor and a cloud formed, T the number of minutes taken to put in the gas.

TABLE I.

T	N_1	N_2
4	$34\frac{1}{2}$	33
$1\frac{1}{4}$	70	60
3	36	34

TABLE II.

T	N_1	N_2
4	42	21
$1\frac{1}{2}$	87	40
3	31·5	16·5

17. In order to weigh the cloud and at the same time find the electrification two methods were adopted. In the first method

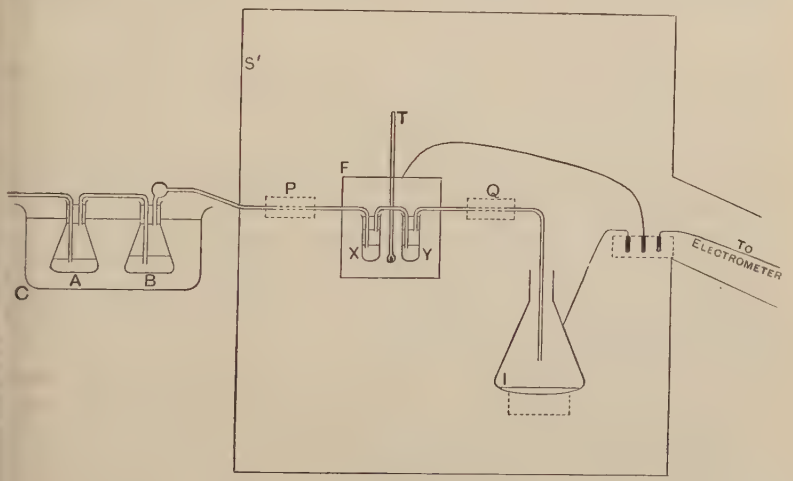


Fig. 3.
(The dotted lines indicate insulators.)

the gas was bubbled through KI solution in the bottle A, and then dried by the sulphuric acid in B (Fig. 3), and led into the paraffin block P. Between P and Q there are two small test-tubes fitted up as shown, the first X containing water and the second Y sulphuric acid; they were covered with tinfoil F and a thermometer T' gave the temperature of the tubes. When the thermometer came to a fixed temperature the bath C in which A and B were immersed was adjusted to the same temperature. The gas as it entered P being dry removed from X not only a quantity of vapour to saturate the gas, but also a dense cloud, both the vapour and cloud being removed by the sulphuric acid in Y, so that the gas escaped from Q in the same state as it entered X. The tubes X and Y were weighed before being put into position, and when the temperatures of A and B were made the same as T, a stream of electrified gas was passed through the apparatus for about 10 minutes, during which time the inductor I and the discharger D were alternately connected with the electrometer for periods of two minutes to find the rate at which each was gaining a charge. The tubes X and Y being weighed a second time it was generally found that X lost more weight than Y gained, so that it was found necessary to put wool in the upper parts of the tubes, that in X being soaked with water, and that in Y with sulphuric acid. The results of these experiments were not quite satisfactory on account of the difference of weight, which occurred chiefly when the stream of gas was irregular, as it sometimes rushed too fast for the sulphuric acid to remove its moisture. On this account the method is only suitable for oxygen, when about 12 or 14 ampères are running through the cell. The results obtained for the negative oxygen were consistent, and the numbers shown in the table of observations show that in the process of forming the cloud in X and removing it in Y only 21 per cent. of the electrification is lost when the gas is going through at the rate of ·85 c.c. per sec.

In the following table, which gives the result of these experiments, θ is the temperature of the water that the cloud is formed in, T the time in minutes that the stream of gas is allowed to run through X and Y, C the current in ampères through the cell, N_1 and N_2 the total charges in divisions of the scale acquired by I and D during the time T, W_1 the loss of weight of X, and W_2 the gain of weight of Y in grammes.

θ	T	C	N_1	N_2	W_1	W_2
10·8	12	14	332	84	·0174	·0174
11·5	10	14·5	305	80	·0161	·0155

When I was connected with the electrometer one division of the scale denoted ·00405 electrostatic units of electricity, and when D was connected each division denoted ·00429 absolute units.

From these observations we get the electrification per c.c. and the corresponding weight of cloud—it being assumed that the electricity passing from X to Y is $Q_1 + \frac{Q_2}{2}$,

Q_1 being the quantity of electricity that enters I,

Q_2 being the quantity of electricity left on D.

18. Positively electrified oxygen and hydrogen were also sent through this apparatus in order to be able to compare the discharging powers, and it was found that for positive oxygen given off by 14 ampères that the discharging power is ·21, exactly the same as in the case of negative oxygen when going at the same rate, ·85 c.c. per second. But in the case of the hydrogen the discharging power was greater although it was going faster; thus when there were 11·8 ampères going through the cell, the gas going at the rate of 1·44 c.c. through the apparatus, the discharging power was ·33 and with 13 ampères the discharging power was ·29.

19. The second method that was adopted to find the weight of the cloud was arranged with a view to ensure the complete saturation of the gas, and the complete removal of the moisture by bubbling through a set of three bulbs containing sulphuric acid, and then weighing the latter. The sulphuric acid bulbs were placed between P and Q and their discharging power found by the methods already described while a current equal to that which it is desirable to use to form the cloud is going through the electrolyte,—the sulphuric acid having been removed from B (fig. 3), and water substituted in its stead. In finding the discharging power it is necessary to have the paraffin tunnel P perfectly dry inside so as to ensure insulation, and this can be relied upon for about 15 minutes while the moist gas is being carried through P, but it was found that a considerable quantity of moisture was left in P, and in the small tubes which were embedded in the paraffin, so that an error would be introduced in the weighing. If however the discharging power be first found P can be removed and the sulphuric acid bulbs can be connected to the tube leading from B which has a bulb blown on it so as to prevent any spray being carried along it. The current should be allowed to run for about three or four minutes before putting on the bulbs to ensure that this delivery tube from B gets thoroughly moistened. Having found the discharging power of the bulbs it is now only necessary to observe the charge acquired

per minute by the inductor into which the gas enters after leaving the acid, and the total charge crossing from B to the sulphuric acid can be calculated.

20. This method gave very consistent results, and in the case of oxygen agreed fairly well with those obtained by the first method. The following is a list of the observations made: C the current through the cell in ampères, θ the temperature of the bath in which the bottles A and B were immersed, T the time the current was on, δ the discharging power, and W the increase of weight of the sulphuric acid bulbs, N the number which gives the charge in electrostatic units per degree of the scale, I the charge per minute on the inductor in divisions of the scale.

	C	θ	T	I	W	N	δ
Positive Oxygen	14	9·7	15	22	·0185	·00402	·45
	14	11·6	20	28·5	·0316	·00402	·45
Positive Hydrogen	12·5	11	15	23·5	·0285	·00480	·54
	12·5	11·6	15	21·5	·0279	·00480	·54
Negative Oxygen	14	11·8	20	20·8	·0330	·00402	·46

When the weight of the moisture, carried across due to the gas being saturated, is subtracted from the increase of weight of the sulphuric acid bulbs we get the following numbers, which give the connection between the charge per c.c. and the weight of the cloud per c.c.

Positive oxygen.

Charge per c.c. in electrostatic units	Weight per c.c. in grammes
$3{\cdot}13\times10^{-3}$	15×10^{-6}
$4{\cdot}06\times10^{-3}$	$20{\cdot}4\times10^{-6}$

Negative oxygen.

$2{\cdot}42\times10^{-3}$	$18{\cdot}4\times10^{-6}$	By first Method
$2{\cdot}62\times10^{-3}$	$19{\cdot}4\times10^{-6}$	
$3{\cdot}0\times10^{-3}$	$21{\cdot}5\times10^{-6}$	

Positive hydrogen.

$2{\cdot}7\times10^{-3}$	$10{\cdot}8\times10^{-6}$
$2{\cdot}4\times10^{-3}$	$9{\cdot}9\times10^{-6}$

These tables show that the negative oxygen has a denser cloud for equal electrification than the positive gases. It is also manifest that the weight of the cloud in each case is proportional to the electrification, so that there can now be no doubt that the formation of the drops is an electrical phenomenon.

21. Rough approximations can be made to the size of the drops in the case of the electrified oxygen as the outline of the cloud is sometimes very definite and the rate at which the cloud falls can be found. It is convenient for this purpose to photograph the same cloud twice, allowing three or four minutes to elapse between the two exposures. By this means it is found that the cloud in the positive oxygen falls at the rate of 10 mms. in three minutes, and that in the negative at the rate of 14 mms. in three minutes. From these observations we get the radius a of the drop from the equation $V = \frac{2}{q}\frac{ga^2}{\mu}$ (Lamb, *Motion of Fluids*, p. 229). Taking $\mu = \cdot 000185$ we get for the radius of the drop in the negative oxygen $8 \cdot 1 \times 10^{-5}$, and in the positive $6 \cdot 8 \times 10^{-5}$. Dividing the weight of the drop into the weight of the cloud per c.c. the number of the drops is obtained, and dividing this number into the charge per c.c. the charge on each is got. Thus for positive oxygen the charge on each carrier is $2 \cdot 8 \times 10^{-10}$, and for the negative $3 \cdot 1 \times 10^{-10}$.

22. The weight of the drops forming the cloud as found by the photographic method is not accurate to more than 8 per cent. The tables giving the "relative" values of the weight of the cloud per c.c. to the electrification per c.c. are accurate to 4 per cent., although a slight general discrepancy exists in the charges, as the condenser, two concentric cylinders, which was used to standardise the electrometer had no guard ring. Hence the numbers given above as the charges on each carrier are not more unequal than the errors of the experiments would account for, so that there is nothing in this result to disprove that the oxygen carriers have the actual atomic charges. The drops in the hydrogen cloud are more numerous than those in the oxygen clouds for the same electrification, but I must reserve for some future occasion the exact estimation of the charge on the hydrogen carrier, as at present it has not been found to the degree of accuracy that is desirable.

23. The clouds which are formed by electrified gases are perfectly stable even in an unsaturated atmosphere, and in this respect bear a close resemblance to atmospheric clouds. There are other points in favour of the supposition that the condensation of the atmospheric clouds is due to electrical causes: for instance, it has been amply shown in the foregoing experiments that

negatively electrified oxygen forms round it a cloud of larger drops than the positive for the same density of electrification. The negative cloud does on this account fall more rapidly than the positive, so that eventually the upper layers of the atmosphere would have a light positive charge. This is exactly what does exist in the atmosphere, namely, that its normal state is positive, as Lord Kelvin points out in a pamphlet on "Electrification of Air." (*Philosophical Society of Glasgow*, 1894–5.) He also refers to experiments of Elster and Geitel's, which show that when rain or snow falls into a basin it generally shows a large electrical effect whether positive or negative, even in the space of a quarter of a minute, with however on the whole a preponderance of negative electrification. When we come to seek a theoretical explanation of the formation of these clouds the way is perfectly clear up to a certain point. It can be shown (J. J. Thomson, *Applications of Dynamics to Physics and Chemistry*, Chap. XI.) that if a drop having an electrification E and radius a be in a saturated atmosphere near a flat surface, then when $a^3 = \frac{E^2}{16\pi T}$, T being the surface tension of water, a particular size of drop is obtained such that if a is less than this value, moisture will be condensed on the drop, and when a is greater than this value, the drop will evaporate and become smaller. So that on these considerations only we get a stable nucleus formed, which in the case of the oxygen would have a radius of $\frac{10^{-7}}{3{\cdot}6}$. It would be premature to go more deeply into this subject in this paper, as experiments are in the process of being carried out which may throw more light upon it. In conclusion I desire to state that my best thanks are due to Professor Thomson for the advantages that have been derived from his very valuable suggestions.

Monday, 22 *February*, 1897.

Mr F. Darwin, President, in the Chair.

The following were elected Fellows of the Society:—

A. Liversidge, M.A., Christ's College.
H. C. Robson, M.A., Fellow of Sidney Sussex College.

The following, having been nominated by the Council, were elected Associates:

S. W. Richardson, Trinity College.
H. F. Hyndman, Trinity College.

The following Communications were made to the Society:

(1) *On the Diffraction Pattern near the Focus of a Telescope.* By R. H. D. MAYALL, M.A., Sidney Sussex College.

The problem here considered is the diffraction pattern formed by the light from a star on a screen placed near the focus of a telescope. The exact theory of the diffraction of light through a circular opening has been developed by Lommel, in his well-known memoir on the subject*. It will be convenient to give here the general conclusions at which he arrived, as applied to the case under consideration.

If δ be the distance of the screen from the focus measured towards the object-glass, f and r the focal length and radius respectively of the object-glass, ζ the distance of any point on the screen from the centre of the image, and if we use the abbreviations

$$y = \frac{2\pi}{\lambda} \cdot \frac{\delta r^2}{f(f-\delta)}, \quad z = \frac{2\pi}{\lambda} \cdot \frac{\zeta r}{f-\delta},$$

the intensity of light at the point ζ is

$$M^2 = \frac{\pi^2 r^4}{f^2 (f-\delta)^2 \lambda^2} \left(\frac{2}{y}\right)^2 (U_1^2 + U_2^2)$$

$$= \frac{1}{\delta^2} (U_1^2 + U_2^2),$$

where

$$U_1 = \left(\frac{y}{z}\right) J_1(z) - \left(\frac{y}{z}\right)^3 J_3(z) + \dots,$$

and

$$U_2 = \left(\frac{y}{z}\right)^2 J_2(z) - \left(\frac{y}{z}\right)^4 J_4(z) + \dots,$$

in which the J's are Bessel's functions with the usual notation, and both series are continued to infinity.

Now $\frac{z}{y}\left(= \frac{f\zeta}{\delta r}\right)$ expresses the ratio of the distance of the point ζ from the centre of the image to the radius of the *geometrical* image, and is therefore < 1 for points inside the geometrical boundary and > 1 for points outside. Hence for points inside the geometrical image the series U_1 and U_2, although absolutely convergent, are not adapted to numerical calculation. They are however related to two other series

$$V_1 = \left(\frac{z}{y}\right) J_1(z) - \left(\frac{z}{y}\right)^3 J_3(z) + \dots,$$

$$V_0 = J_0(z) - \left(\frac{z}{y}\right)^2 J_2(z) + \dots,$$

* *Abh. der Kön. Bayer. Akad.* vol. xv.

by the equations

$$U_1 + V_1 = \sin \frac{1}{2}\left(y + \frac{z^2}{y}\right),$$

$$- U_2 + V_0 = \cos \frac{1}{2}\left(y + \frac{z^2}{y}\right).$$

If $\frac{z}{y}$ is not nearly equal to unity, i.e. for points which are not far from the centre of the image compared with its radius, the series V_1 and V_0 are rapidly convergent and may be calculated with ease from the tabulated values of the Bessel functions. They were in fact calculated by Lommel for values of y equal to π, 2π, ...10π, and for values of z from 1 to 12. But if $\frac{z}{y}$ is nearly equal to unity and at the same time y and z are fairly large, the calculation would become extremely laborious, for the function J_n does not become negligible in comparison with J_1, J_2 ... when z is large until n is also large, so that a great number of terms of the series would have to be taken in order to get even a moderate approximation to their true value. Now in the present case y may be much greater than 10π, even when the screen is very close to the focus, thus if $\frac{r}{f} = \frac{1}{20}$ the screen would have to be placed at a distance of only about a millimetre from the focus in order that the value of y should be so small as 10π. And at the same time, experiment shows that it is just when $\frac{z}{y}$ is nearly equal to unity that it becomes most important to determine theoretically what the intensity of illumination ought to be.

A very interesting series of star-photographs has been obtained by Mr Newall on plates exposed to the light of a star near the focus of the large refractor at the Observatory. It appears from these that when the light has been rendered as nearly as possible monochromatic the image takes the shape of a bright ring surrounding a darker space more or less uniformly illuminated. The position of this ring is just about the boundary of the geometrical image, and within it can be seen traces of one or two other rings but much inferior to the first in brightness. A great portion of the light in the image is thus confined to the immediate neighbourhood of the geometrical boundary, and as there is no reason to suspect any excessive concentration of light there due to spherical aberration, it is important to know whether the phenomenon is a direct result of the undulatory theory of light or not.

An examination of the curves traced by Lommel, to represent the variation in intensity of the light at different points of the

image, gives no indication of any well-defined ring at the outer boundary. It shows also no reason for thinking that, when the values of y and z are larger than for those curves, the intensity curve takes a kind of limiting form at its boundary which is independent of the value of y, and yet this is certainly indicated by the photographs.

In the following I have attempted to find an approximation, in the general case, to the values of the series V_0 and V_1 when y and z are both great and their ratio nearly equal to unity.

Expressing $J_n(z)$ as an integral by means of the formula

$$(i)^n J_n(z) = \int_0^\pi e^{iz\cos\phi} \cos n\phi \, d\phi,$$

and writing $\frac{z}{y} = \kappa$, where $\kappa < 1$, we have

$$\begin{aligned} V_0 + iV_1 &= J_0(z) + i\kappa J_1(z) + i^2\kappa^2 J_2(z) + \ldots \\ &= \frac{1}{\pi}\int_0^\pi e^{iz\cos\phi}\,(1 + \kappa\cos\phi + \kappa^2\cos 2\phi + \ldots)\,d\phi \\ &= \frac{1}{\pi}\int_0^\pi e^{iz\cos\phi}\,\frac{1-\kappa\cos\phi}{1-2\kappa\cos\phi+\kappa^2}\,d\phi \\ &= \frac{1}{2\pi}\int_0^\pi e^{iz\cos\phi}\left\{1 + \frac{1-\kappa^2}{1-2\kappa\cos\phi+\kappa^2}\right\}d\phi \\ &= \frac{1}{2}J_0(z) + \frac{1-\kappa^2}{2\pi}\int_0^\pi e^{iz\cos\phi}\,\frac{d\phi}{1-2\kappa\cos\phi+\kappa^2} \quad \ldots\ldots\ldots(1). \end{aligned}$$

Now consider the integral

$$\int_0^\pi e^{iz\cos\phi}\,\frac{d\phi}{\mu - \cos\phi}.$$

If $\mu > 1$ it may be expanded in the form

$$\begin{aligned} &\frac{1}{\mu}\int_0^\pi e^{iz\cos\phi}\left(1 + \frac{\cos\phi}{\mu} + \frac{\cos^2\phi}{\mu^2} + \ldots\right)d\phi \\ &= \frac{1}{\mu}\left(1 + \frac{1}{i\mu}\cdot\frac{d}{dz} + \frac{1}{i^2\mu^2}\cdot\frac{d^2}{dz^2} + \ldots\right)\int_0^\pi e^{iz\cos\phi}\,d\phi \\ &= \frac{\pi}{\mu}\left(1 + \frac{1}{i\mu}\cdot\frac{d}{dz} + \frac{1}{i^2\mu^2}\cdot\frac{d^2}{dz^2} + \ldots\right)J_0(z). \end{aligned}$$

Since $\mu > 1$ and $\frac{d^n J_0(z)}{dz^n}$ can never exceed a certain finite

quantity, the series above are absolutely convergent. The last line is what we should obtain by integrating by parts the integral

$$i\pi e^{i\mu z}\int_z^\infty e^{-i\mu z}\,J_0(z)\,dz.$$

Hence we have

$$\int_0^\pi e^{iz\cos\phi}\,\frac{d\phi}{\mu-\cos\phi}=i\pi e^{i\mu z}\int_z^\infty e^{-i\mu z}\,J_0(z)\,dz,$$

and putting $\dfrac{1+\kappa^2}{2\kappa}=\mu$ in (1) it becomes

$$V_0+iV_1=\frac{1}{2}J_0(z)+i\,\frac{1-\kappa^2}{4\kappa}\,e^{i\mu z}\int_z^\infty e^{-i\mu z}\,J_0(z)\,dz\ldots..(2),$$

in which κ, and therefore μ, are nearly equal to unity, while z is large.

To get an approximation to the value of the integral on the right-hand side of (2) we may use the approximate expression for J_0

$$J_0(z)=\sqrt{\frac{2}{\pi z}}\left[\cos\left(\frac{\pi}{4}-z\right)-\frac{1}{8z}\sin\left(\frac{\pi}{4}-z\right)\right],$$

with which value the integral becomes

$$\sqrt{\frac{2}{\pi}}\int_z^\infty e^{-i\mu z}\left[\cos\left(\frac{\pi}{4}-z\right)-\frac{1}{8z}\sin\left(\frac{\pi}{4}-z\right)\right]\frac{dz}{\sqrt{z}}\ldots..(3).$$

This may be expressed in terms of Fresnel's integrals

$$P=\int_0^\nu\cos\frac{1}{2}\pi y^2dy,\quad Q=\int_0^\nu\sin\frac{1}{2}\pi y^2dy,$$

the values of which have been tabulated by Gilbert* from $\nu=\cdot01$ to $\nu=5\cdot5$. For we have

$$\int_z^\infty e^{-i\mu z}\cos\left(\frac{\pi}{4}-z\right)\frac{dz}{\sqrt{z}}=\frac{1}{2}\int_z^\infty e^{-i\mu z}\left\{e^{i\left(\frac{\pi}{4}-z\right)}+e^{-i\left(\frac{\pi}{4}-z\right)}\right\}\frac{dz}{\sqrt{z}}$$

$$=\frac{1}{2}e^{i\frac{\pi}{4}}\int_z^\infty e^{-i(\mu+1)z}\frac{dz}{\sqrt{z}}+\frac{1}{2}e^{-i\frac{\pi}{4}}\int_z^\infty e^{-i(\mu-1)z}\frac{dz}{\sqrt{z}}\ldots(4).$$

If we neglect terms in $z^{-\frac{3}{2}}$ the first of the integrals in (4) is

$$-\frac{1}{2}ie^{\frac{i\pi}{4}}e^{-i(\mu+1)z}/(\mu+1)\sqrt{z},$$

* *Mem. couronnés de l'Acad. de Bruxelles* vol. xxxi.

while the second is

$$\frac{1}{2}e^{-\frac{i\pi}{4}}\int_z^\infty \left\{\cos(\mu-1)z - i\sin(\mu-1)z\right\}\frac{dz}{\sqrt{z}}$$

$$= \frac{1}{2}e^{-\frac{i\pi}{4}}\sqrt{\frac{2\pi}{\mu-1}}\int_\nu^\infty \left(\cos\frac{1}{2}\pi y^2 - i\sin\frac{1}{2}\pi y^2\right)dy$$

$$\left(\text{where } \nu = \sqrt{\frac{2(\mu-1)z}{\pi}}\right) \ldots\ldots\ldots (5)$$

$$= \frac{1}{2}e^{-\frac{i\pi}{4}}\sqrt{\frac{2\pi}{\mu-1}}\left\{\frac{1}{2} - P - i\left(\frac{1}{2} - Q\right)\right\}.$$

The ratio of the first integral in (4) to the second is therefore of the order $\sqrt{\frac{\mu-1}{z}}$, i.e. of the order $\frac{1-\kappa}{\sqrt{z}}$, so that the first integral may be neglected. We have then approximately

$$\int_z^\infty e^{-i\mu z}\cos\left(\frac{\pi}{4} - z\right)\frac{dz}{\sqrt{z}} = \frac{1}{2}e^{-\frac{i\pi}{4}}\sqrt{\frac{2\pi}{\mu-1}}\left\{\frac{1}{2} - P - i\left(\frac{1}{2} - Q\right)\right\}$$

$$= -\frac{1}{1-\kappa}\sqrt{\frac{\kappa\pi}{2}}\left\{i(1-P-Q) + P - Q\right\}.$$

The rest of the integral (3) is of an order which has already been neglected. Hence, by substituting the value of (3) in (2), we have approximately

$$V_0 + iV_1 = \frac{1}{2}J_0(z) + \frac{1+\kappa}{4\sqrt{\kappa}}e^{i\mu z}\left\{1 - P - Q - i(P-Q)\right\},$$

giving

$$V_0 = \frac{1}{2}J_0(z) + \frac{1+\kappa}{4\sqrt{\kappa}}\left\{(1-P-Q)\cos\mu z + (P-Q)\sin\mu z\right\},$$

$$V_1 = \frac{1+\kappa}{4\sqrt{\kappa}}\left\{(1-P-Q)\sin\mu z - (P-Q)\cos\mu z\right\},$$

or, since $U_1 = \sin\mu z - V_1$, $U_2 = V_0 - \cos\mu z$,

$$U_1 = \left\{1 - \frac{1+\kappa}{4\sqrt{\kappa}}(1-P-Q)\right\}\sin\mu z + \frac{1+\kappa}{4\sqrt{\kappa}}(P-Q)\cos\mu z,$$

$$U_2 = \frac{1}{2}J_0(z) - \left\{1 - \frac{1+\kappa}{4\sqrt{\kappa}}(1-P-Q)\right\}\cos\mu z + \frac{1+\kappa}{4\sqrt{\kappa}}(P-Q)\sin\mu z,$$

and the intensity of light at points of the image close to the geometrical boundary is given by

$$\delta^2 M^2 = U_1^2 + U_2^2$$

$$= \left\{1 - \frac{1+\kappa}{4\sqrt{\kappa}}(1-P-Q)\right\}^2 + \left\{\frac{1+\kappa}{4\sqrt{\kappa}}(P-Q)\right\}^2$$

$$- J_0(z)\left[\left\{1 - \frac{1+\kappa}{4\sqrt{\kappa}}(1-P-Q)\right\}\cos\mu z - \frac{1+\kappa}{4\sqrt{\kappa}}(P-Q)\sin\mu z\right].$$

This may be still further simplified, for $(1+\kappa)/4\sqrt{\kappa}$ differs from $\frac{1}{2}$ by $\frac{1}{16}(1-\kappa)^2$ when κ is nearly equal to unity, and the error in $\delta^2 M^2$ produced by putting $\kappa = 1$ in the squared terms is

$$\tfrac{1}{16}(1-\kappa)^2(2P^2 + 2Q^2 - 1)$$

or about $\frac{1}{20}(1-\kappa)^2$ at the greatest, while the error in the $J_0(z)$ term would be of order $\dfrac{1}{\sqrt{z}} \cdot \dfrac{(1-\kappa)^2}{16}$.

We have then $M^2 = M_1^2 + \epsilon$,

where $\delta^2 M_1^2 = \frac{1}{4}[(1+P+Q)^2 + (P-Q)^2]$

and $\delta^2 \epsilon = -\frac{1}{2} J_0(z)[(1+P+Q)\cos\mu z - (P-Q)\sin\mu z]$.

It appears from this that the variation in the intensity of illumination consists of two parts, one of which M_1^2 represents all the main features in the variation, the other being a small correction of irregular form to be applied to it.

For points outside the geometrical boundary, M^2 takes a different form. Here $\kappa > 1$ and

$$iU_1 - U_2 = i\frac{y}{z}J_1(z) + i^2\left(\frac{y}{z}\right)^2 J_2(z) + \ldots$$

$$= \frac{1}{\pi}\int_0^\pi e^{iz\cos\omega}\left(\frac{\cos\omega}{\kappa} + \frac{\cos 2\omega}{\kappa^2} + \ldots\right)d\omega$$

$$= \frac{1}{\pi}\int_0^\pi e^{iz\cos\omega}\,\frac{\kappa\cos\omega - 1}{1 - 2\kappa\cos\omega + \kappa^2}\,d\omega$$

$$= \frac{1}{2\pi}\int_0^\pi e^{iz\cos\omega}\left(-1 + \frac{\kappa^2 - 1}{1 - 2\kappa\cos\omega + \kappa^2}\right)d\omega$$

$$= -\frac{1}{2}J_0(z) + \frac{\kappa^2 - 1}{2\pi}\int_0^\pi e^{iz\cos\omega}\,\frac{d\omega}{1 - 2\kappa\cos\omega + \kappa^2},$$

which as in the preceding case may be reduced to

$$iU_1 - U_2 = -\frac{1}{2}J_0(z) + \frac{1+\kappa}{4\sqrt{\kappa}}\left\{1 - P - Q - i(P-Q)\right\}e^{i\mu z},$$

giving

$$U_1 = \frac{1+\kappa}{4\sqrt{\kappa}}\left\{(1-P-Q)\sin\mu z - (P-Q)\cos\mu z\right\},$$

$$U_2 = \frac{1}{2}J_0(z) - \frac{1+\kappa}{4\sqrt{\kappa}}\left\{(1-P-Q)\cos\mu z + (P-Q)\sin\mu z\right\},$$

and
$$\delta^2 M^2 = U_1^2 + U_2^2$$
$$= \tfrac{1}{4}\left[(1-P-Q)^2 + (P-Q)^2\right]$$
$$-\tfrac{1}{2}J_0(z)\left\{(1-P-Q)\cos\mu z + (P-Q)\sin\mu z\right\}$$

or
$$M^2 = M_2^2 + \epsilon',$$

where
$$\delta^2 M_2^2 = \tfrac{1}{4}\left[(1-P-Q)^2 + (P-Q)^2\right]$$
$$\delta^2\epsilon' = -\tfrac{1}{2}J_0(z)\left\{(1-P-Q)\cos\mu z + (P-Q)\sin\mu z\right\}.$$

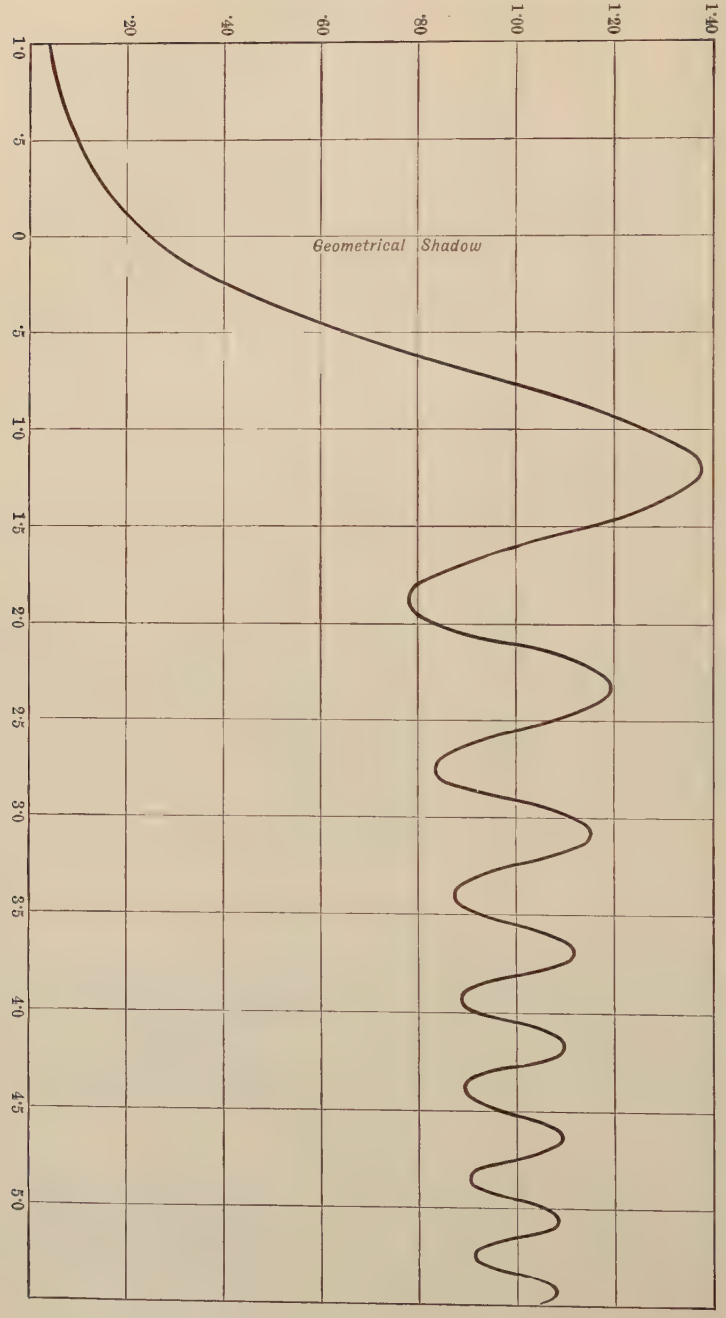
The values of M_1^2 and M_2^2 may be calculated from Gilbert's tables for P and Q: it will be remembered that the argument in this case is

$$\nu = \sqrt{\frac{2(\mu-1)z}{\pi}}$$
$$= \sqrt{\frac{(1-\kappa)^2 z}{\pi\kappa}}$$
$$= \pm\left(1 - \frac{z}{y}\right)\sqrt{\frac{y}{\pi}},$$

the upper sign being taken in M_1^2 and the lower in M_2^2, so that ν is proportional to the distance of the point of the image considered from the geometrical boundary. The curve in the figure represents $\delta^2 M_1^2$ and $\delta^2 M_2^2$ for different values of the abscissa ν, the origin corresponding to a point on the geometrical boundary and

$$\nu = \sqrt{\frac{y}{\pi}}$$

to the centre of the image. The numerical values of $\delta^2 M_1^2$ and $\delta^2 M_2^2$ are given in Tables I. and II. The figure shows that the intensity increases from zero outside the boundary until it reaches


·20
·40
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·80
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2·0
2·5
3·0
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4·0
4·5
5·0
Geometrical Shadow

its greatest maximum between $\nu = 1{\cdot}2$ and $\nu = 1{\cdot}3$ which will be within a small fraction of the radius, it then diminishes to its least minimum between $\nu = 1{\cdot}8$ and $1{\cdot}9$. Afterwards it passes through a series of maxima and minima until at length it becomes practically constant and equal to unity. The appearance on a screen would be that of a bright ring surrounding a series of fainter rings, within a very small distance from it, these latter rings gradually disappearing into a uniformly illuminated space. The outer ring would be most conspicuous, not only by reason of its greater intensity but also because of its greater width. On a photographic plate exposed with a view to showing the existence of this outer ring the probability would be that the light from the inner rings would hardly affect the plate at all; in any case the maxima and minima would be so close together and differ so little from each other that they would be indistinguishable, except perhaps the first two or three. Moreover, unless the light used were of a single definite wave-length, the maxima and minima for different wave-lengths would overlap. If the light were confined to a small portion of the spectrum, this might not affect the outer ring very much, since it is separated from the next inner ring by a wider and darker interval than exists between any of the others, but a very small range of wave-length would be sufficient to fill up nearly all the other minima.

The curve M^2 only represents the variation of the intensity for points which are near the outer ring in comparison with its radius, for points nearer the centre the terms which have been neglected would become more and more appreciable until the regularity of the curve was entirely destroyed. These terms were of order $\frac{1}{16}(1-\kappa)^2(2P^2+2Q^2-1)$, and $1/\sqrt{z}$, of which however the first is always small, since P and Q become ultimately equal to $\frac{1}{2}$; even the second would not make much alteration in the shape of the curve unless z were as small as 100, so that the curve will represent the intensity for a considerable distance towards the centre when y is large. At the centre itself, as is well known,

$$\delta^2 M^2 = 4 \sin^2 \frac{y}{4},$$

or the intensity varies with the position of the plate from zero to 4. From the centre outwards the intensity varies irregularly until $1-\kappa$ and $1/\sqrt{z}$ become small enough to allow the application of the preceding results.

The figure (Fig. 2) gives some idea of the shape of the intensity curve from the edge of the image up to the centre, the variation near the centre being quite irregular in the sense of varying rapidly with the position of the screen, while the constant part and the variation at the edge are the same for all positions of the screen so

long as it is far enough away from the focus of the telescope to make y large.

TABLE I.

v	P	Q	$\delta^2 M_1^2$	v	P	Q	$\delta^2 M_1^2$
·01	·01000	·00000	·2551	2·1	·5814	·3743	·9669
·02	·02000	·00000	·2602	2·2	·6362	·4556	1·1021
·03	·03000	·00001	·2655	2·3	·6268	·5525	1·1887
·04	·03999	·00003	·2708	2·4	·5550	·6197	1·1834
·05	·05000	·00007	·2763	2·5	·4574	·6192	1·0846
·06	·06000	·00011	·2819	2·6	·3889	·5500	·9463
·07	·07999	·00018	·2933	2·7	·3926	·4529	·8524
·08	·08999	·00027	·2992	2·8	·4675	·3915	·8654
·09	·09999	·00038	·3052	2·9	·5624	·4102	·9786
·10	·10999	·00052	·3113	3·0	·6057	·4963	1·1076
·15	·1500	·0018	·3372	3·1	·5616	·5818	1·1487
·20	·1999	·0042	·3720	3·2	·4663	·5933	1·0645
·25	·2498	·0082	·4102	3·3	·4057	·5193	·9296
·30	·2994	·0141	·4517	3·4	·4385	·4297	·8726
·35	·3487	·0224	·4966	3·5	·5326	·4153	·9520
·40	·3975	·0334	·5450	3·6	·5880	·4923	1·0842
·45	·4455	·0474	·5968	3·7	·5419	·5750	1·1206
·50	·4923	·0647	·6518	3·8	·4481	·5656	1·0172
·55	·5377	·0857	·7099	3·9	·4223	·4752	·9008
·60	·5811	·1105	·7707	4·0	·4984	·4205	·9220
·65	·6183	·1441	·8327	4·1	·5737	·4758	1·0524
·70	·6597	·1721	·8983	4·2	·5417	·5632	1·1077
·75	·6935	·2089	·9635	4·3	·4494	·5540	1·0061
·80	·7230	·2493	1·0286	4·4	·4383	·4623	·9032
·85	·7469	·2932	1·0920	4·5	·5258	·4342	·9624
·90	·7648	·3398	1·1525	4·6	·5672	·5162	1·0858
·95	·7760	·3885	1·2088	4·7	·4914	·5669	1·0606
1·00	·7799	·4383	1·2593	4·8	·4338	·4968	·9328
1·10	·7638	·5365	1·3358	4·9	·5002	·4351	·9374
·20	·7154	·6234	1·3696	5·0	·5636	·4992	1·0648
·30	·6386	·6863	1·3519	5·1	·4998	·5624	1·0631
·40	·5431	·7135	1·2803	5·2	·4389	·4969	·9377
·50	·4453	·6975	1·1638	5·3	·5078	·4406	·9502
·60	·3655	·6386	1·0274	5·4	·5572	·5140	1·0729
·70	·3238	·5492	·8897	5·5	·4786	·5538	1·0341
·80	·3363	·4509	·8018				
·90	·3945	·3734	·7815				
2·00	·4883	·3434	·8440				

TABLE II.

ν	$\delta^2 M_2^2$	ν	$\delta^2 M_2^2$	ν	$\delta^2 M_2^2$	ν	$\delta^2 M_2^2$
·05	·2262	·30	·1382	·55	·0865	·80	·0563
·10	·2008	·35	·1255	·60	·0791	·85	·0519
·15	·1854	·40	·1141	·65	·0703	·90	·0479
·20	·1679	·45	·1039	·70	·0665	·95	·0443
·25	·1522	·50	·0948	·75	·0611	1·00	·0411

(2) *On the marks made by stars on photographic plates exposed near the focus of a telescope.* By H. F. NEWALL, M.A., Trinity College.

The appearances presented by some photographs of stars taken with the 25-inch visual refractor of the Cambridge Observatory in the years 1893–4 were described and illustrated in a paper "On the formation of photographic star-discs" by the present writer, in the *Monthly Notices of the Royal Astronomical Society*, Vol. LIV. p. 515. In that paper attention was called to the existence of a considerable concentration of light in the boundary of the circle of diffusion for light of any wave-length; and I was led by a series of observations and experiments, which gave much information as to the excellent performance of the object-glass, to express the view that "it may be a straightforward result in the theory of diffraction through a circular aperture," that this annular concentration must exist.

The extension of Lommel's work (*Bavar. Acad. Mem.* 1886, Vol. XV., p. 235), which Mr Mayall was good enough to undertake at my suggestion and has accomplished and described in the preceding paper (page 259), shows that near the focus of a *perfect* object-glass with circular aperture the circles of diffusion must exhibit an intensification of light of any given wave-length near the boundary. Mr Mayall has in fact found that the undulatory theory accounts for just such an effect as my experimental results led me to anticipate.

A few more observations, and also some photographs taken since 1894, have only confirmed my view that whilst there are signs of some *slight* outstanding spherical aberration in the 25-inch refractor, the main factor in the production of the rings which I have measured in the star-marks obtained with special exposures of photographic plates has been the concentration of light near the boundaries of the circles of diffusion for different colours.

Of these photographs I propose only to speak of two. In the taking of one of them, the aperture of the object-glass was reduced to 24 inches, and an opaque circular disc, 12 inches in diameter, was suspended in front of the middle of the object-glass, so that light was prevented from falling on the central part. The telescope was then pointed to a bright star and a photographic plate was exposed near the focus, at a distance $1\frac{1}{2}$ inches within the visual focus (*i.e.* nearer to the object-glass). The appearance on the plate, which was sensitive to the yellow and green especially between λ 5900 and λ 5200, is mainly due to the light focussed at the visual focus; and the mark produced consists of a circular disc, whose outer edge is brighter than the average and whose central part—that which corresponds to the obstructed part of the object-glass—is blank; but round the central blank the intensity of photographic action is greater than the average. To put it in other words, the mark is a broad ring, whose external radius is twice the internal radius, and the intensity at the edges of the ring, both external and internal, is greater than the average within the broad ring; moreover the brightness of the internal edge is greater than when no obstructing disc is used. The excess of brightness of the internal edge is the point to which I wish to call attention.

The second photograph was taken under similar circumstances except that the obstructing circular disc was displaced from its central position, and set so that its centre fell halfway between the centre and edge of the full aperture. In this case also the marks on the developed plate exhibit an intensification of the light round the edge of what may be called the shadow of the displaced disc.

In both of the photographs described there were several exposures of various durations on each plate. The effect of long exposures is to obliterate the differences of intensity in different parts of the marks. If the exposure is so short that the plate is barely affected by the average intensity, then the parts where the intensity is greater are seen with exaggerated distinctness. It is easy to pick out amongst the various marks made with different exposures, one or more that show the ring very plainly.

The features seen in the photographs suggest the idea that the intensification of the light near the boundaries of these star-marks is a result which is to be expected with an object-glass of any shape of aperture, whether circular, square or triangular.

[Some photographs lately taken with a triangular aperture show that this expectation is realized in fact. The triangular marks are bounded by edges of which the intensity is greater

than the average intensity within the boundary. Note added 1897, Mar. 15.]

The effects of spherical aberration would be distinguishable from the diffractional intensification near the boundary by the fact that the aberration effects would be different inside the focus from those outside the focus, but the diffractional effects would be the same on opposite sides of the focus.

(3) *Theorems on the contacts of spheres.* By Mr W. M^{c}F. Orr, M.A.

Few, if any, of the following theorems are new. Theorem (1) is equivalent to the well-known theorem that all the generating spheres of one system of a cyclide touch all the generating spheres of the other system. Theorem (5) is proved analytically by Casey (*Proceedings Royal Irish Academy*, Vol. IX.). I believe they are not generally known, however, and it may be worth while to bring them together.

1. The problem to describe a sphere touching four spheres may admit of an infinite number of solutions. For any four tangent planes from a point to a sphere are touched by an infinite number of spheres; by inversion from any point the four planes become four spheres touched by an infinite number of spheres.

2. Any two spheres and their inverses with respect to a third are touched by two infinite families of spheres. For if A, B be two spheres and A', B' their inverses with respect to another sphere O, the spheres which touch A and A' are divided into two families each doubly infinite, all the members of one of which cut O orthogonally. Any sphere of this family which touches B must therefore touch B' and belongs to one or other of two singly infinite families according as the natures of its contacts with A and B are the same or different.

3. Any three spheres and their inverses with respect to a fourth are touched in common by eight spheres and in general by no more. For if A, B, C be three spheres and A', B', C' their inverses with respect to another sphere O, of the sixteen spheres which can be described touching A, B, C, A' (in case the solutions of that problem are determinate), eight cut O orthogonally and therefore touch B' and C'.

4. The sixteen spheres which can be described to touch four given spheres (in case the solutions of that problem be determinate), consisting of eight pairs of conjugates, the members of each pair being inverse to each other with respect to the sphere orthogonal to the original four, any two pairs of conjugates are touched by two infinite families of spheres. This follows from theorem (2).

5. In the same case any three pairs of conjugates are touched by four spheres besides the original four. This follows from theorem (3).

6. In the same case if the original four spheres be A, B, C, D, the four pairs of conjugates which belong to the same doubly infinite family touching any two of the given four as A, B are touched by two other spheres besides the original four, namely, the inverses of C, D with respect to one of the spheres of similitude of A and B. This is obvious as the eight spheres mentioned, touching C and D and cutting the sphere of similitude orthogonally, must touch the inverses of C, D with respect to that sphere of similitude. In this theorem as in the last we have a group of eight spheres touched by a group of six.

(4) *Change of the independent variable in a differential coefficient.* By E. G. Gallop, M.A., Gonville and Caius College.

[This paper will appear in the *Transactions*, Vol. xvi. Part II.]

Monday, 8 *March*, 1897.

The President, Mr F. Darwin, in the Chair.

The following Communications were made:

(1) *On the injection of the intercellular spaces occurring in the leaves of* Elodea *during recovery from plasmolysis.* By the President, Mr F. Darwin, and Miss D. F. M. Pertz.

Elodea continues to assimilate in salt solutions strong enough to plasmolyse the cells. On replacing the plant in water assimilation ceases, the gas disappears from the intercellular spaces, and the leaf is injected with water. The disappearance takes place partly by the escape of bubbles at the open ends of the intercellular spaces, but chiefly by solution. The first of these phenomena depends on the surface tension of salt solutions being greater than that of water. The authors are not able to explain the solution of the air in the intercellular spaces.

(2) *The phenomena of Carbon Dioxide production associated with reduced vitality in plants.* By Mr F. F. Blackman.

By the aid of an apparatus (which was exhibited), specially adapted for physiological research on very small outputs of carbon dioxide, several new phenomena of this nature have been brought to light in plants.

These comprise the liberation of carbon dioxide produced in the following four cases. Firstly, that resulting from the action of temperatures between 40° C. and 50° C. on dry resting seeds: at temperatures below 40° C. no appreciable formation of carbon dioxide takes place, and at continued higher temperatures the amount, which is at first large, does not remain so but steadily falls off, indicating the decomposition of a definite limited quantity of some substance. Secondly, the large amount of carbon dioxide produced in the first few hours after wetting coarsely ground dry seeds. This cannot be attributed to the action of micro-organisms and is hindered by the action of chloroform and other poisons. Thirdly, the varying production of carbon dioxide by the action of volatile poisons and of fatal temperatures on living leaves. Finally, the post-mortem production of carbon dioxide brought about by subjecting recently killed leaves to the action of a temperature of 100° C. This amount was shewn to vary with the method of killing adopted, and evidence was forthcoming to shew that in this, as in the other cases, these substances which easily oxidise with liberation of carbon dioxide are in some way to be associated with normal respiratory processes.

(3) *On the leaves of* Bennettites. By A. C. Seward, M.A., St John's College.

In the second volume of the British Museum Catalogue of Wealden plants[1], I brought forward evidence in favour of including *Williamsonia gigas* Carr., *Williamsonia Carruthersi* Sew., and other species in the same family or even genus with the Lower Cretaceous *Bennettites*. Although we have not as yet discovered any example of *Williamsonia gigas* showing internal structure, there are many reasons for believing that this Lower Oolite fossil represents an inflorescence very closely allied to that of *Bennettites Gibsonianus* Carr. The immediate purpose of this article is, however, to bring forward evidence in support of the somewhat discarded view that *Williamsonia gigas* is the inflorescence of the plant which bore the pinnate fronds known as *Zamites gigas* L. and H. It is by the gradual accumulation of facts and the correlation of various observations, that we are occasionally able to

[1] "Catalogue of the Mesozoic plants in the Department of Geology" (Brit. Mus.) *The Wealden Flora*, Part ii. 1895, p. 134.

surmount the difficulty presented by the almost constant occurrence of plant fragments as isolated and imperfectly preserved fossils. We are now in possession of most of the data as to the essential features of the Bennettitean inflorescence, but we have hitherto had no satisfactory proof as to the nature of the vegetative organs which were borne by *Bennettites*.

The juxtaposition of plant fragments in the same rock cannot as a rule be accepted as evidence of much value. There is no more serious difficulty in palæobotanical work than that of piecing together detached portions of the same species, and the isolation of members of one individual has given rise to a multiplication of generic terms which, though a matter of necessity, has been a source of no little confusion. Anyone at all familiar with the method of occurrence of *Williamsonia gigas* in the Jurassic beds of the Yorkshire coast must have recognised its almost constant association with *Zamites gigas*. The association is so frequent as to naturally suggest an original union of the two fossils in one plant. In the classic work by Young and Bird—*A Geological Survey of the Yorkshire Coast*—published in 1822[1], a portion of a *Zamites gigas* frond is represented in Plate II. fig. 2. and in fig. 6 of the same plate we have a typical example of *Williamsonia*. The authors do not give a name to either specimen, but express the opinion that "figs. 2 and 6 appear to belong to one plant; the former being a leaf, somewhat imperfect, and the latter the head or fruit of the plant[2]." Bird's drawing of the frond does not do justice to the specimen which is fairly well preserved, and represents a perfectly characteristic example of *Zamites gigas*. The original fossils figured in Young and Bird's work are preserved in the Whitby Museum. The late Prof. Williamson devoted a considerable time during the earliest years of his scientific life to the investigation of the fossils to which Carruthers gave the name *Williamsonia*. No one has had opportunities of studying this genus as it occurs in the rocks near Scarborough equal to those enjoyed by Williamson; and it is interesting to find that the most recent work has tended to support many of the conclusions arrived at by this observer. As early as 1834 Williamson[3] expressed the opinion that *Zamites gigas* was connected with *Williamsonia*, and in his very able paper, published in 1870[4], this author is confirmed in the view of the organic connection of these two sets of fossils. This opinion was also shared by Brongniart[5], who received an

[1] Young, E. and Bird, J., *A Geological Survey of the Yorkshire Coast*, Whitby, 1822.
[2] *Ibid.* p. 183.
[3] *Geol. Trans.* [2] Vol. v. p. 230, 1834.
[4] *Linn. Trans.* Vol. xxvi. p. 663, 1870.
[5] *Tableau Vég. foss.* p. 62, 1849.

unusually fine collection of English Williamsonias from the late Mr Yates. These specimens are now in the Natural History Museum, Paris, and many of them were drawn for Brongniart with a view to publication, but the work was unfortunately never completed. The drawings were afterwards made use of by Saporta in his comprehensive work on Jurassic plants. In describing the Yates specimens Saporta expresses himself strongly against the generally accepted view as to the union of *Williamsonia* and *Zamites*. He does not hesitate to separate the Zamites fronds from any connection with the Williamsonias. There is he admits "une certaine conformité apparente entre les appareils floraux auxquels on peut laisser le nom de *Williamsonia* et le *Zamites gigas*, tel que le fait voir le remarquable empreinte de la collection du Muséum de Paris (voy. Pl. 81, fig. 1). Nous avons tout bien de considérer les *Williamsonia* comme représentant l'inflorescence d'une monocotylédone primitive, révélant un type de Pandanées plus ou moins analogue aux *Yuccites*, aux *Podocarya*, aux *Eolirion* de Andrae, etc.[1]"

It is unnecessary to quote the various views advanced by different writers as the botanical affinity of *Williamsonia*, especially as these were in many cases largely influenced by Saporta's decision as to the complete independence of this genus and *Zamites gigas*. Nathorst proposed a comparison of *Williamsonia* with the *Balanophoreae*[2], but afterwards[3] reverted to the opinion of the earlier writers as to its connection with *Zamites*.

A recent examination of the Yates collection in Paris, and a comparison of the numerous specimens in the museums of London, Cambridge, Whitby and Scarborough has led me without hesitation to regard the pinnate Cycadean fronds of *Zamites gigas* as the leaves of the plant which bore a Williamsonian inflorescence.

One not infrequently finds a small bud or young *Williamsonia* borne on the end of a peduncle about 20 or 30 cm. long and 3 to 5 cm. broad. Such an axis is covered with linear lanceolate scale leaves spirally disposed and often clothed with delicate hair-like ramenta, such as occur on the scale leaves of *Dioon* and other recent Cycads. Such a peduncle is figured by Saporta on Pl. XV. of Vol. IV.[4] The original is in the Paris Museum; the scale leaves are less prominent and not so thick as those shown in the drawing, and in this and other specimens one sees traces of the ramental appendages. The best example of a peduncle is included in the series of specimens of *Williamsonia* now in the possession of Mrs Crawford Williamson, to whom my thanks are due for an

[1] *Pal. Franç. Plantes Jurassiques*, Vol. II. 1875, p. 55.
[2] *Öfr. k. Vet.-Akad. Förh.* 1880, No. 9, p. 33.
[3] *Ibid.* 1888, No. 6, p. 359.
[4] *Pal. Franç. Plant. Jurass.* Vol. IV. 1891.

opportunity afforded me of examining the fossils figured in Prof. Williamson's valuable memoir.

Saporta alludes to the resemblance of the peduncle which he figures to the stem of *Zamites gigas*[1] represented in his volume on Cycads, Pl. XI. fig. 1, but does not regard the similarity as evidence of relationship or identity. This specimen of *Zamites* referred to in the above quotation from the second volume of the *Plantes Jurassiques*, is of exceptional interest and furnishes the most important link in the argument in favour of the connection between *Williamsonia* and *Zamites gigas*. The figure in Saporta's work is very imperfect and conveys but a poor and erroneous idea of the actual specimen. At the base we have a stem about 5 cm. broad with the surface features indistinctly preserved, but showing a number of imperfect scale leaves. To one side of the stem, 5 cm. from the bottom of the specimen, are attached the petioles of two clearly preserved fronds of *Zamites gigas*, and above these occurs part of a third frond apparently in its natural position but without the petiolar attachment. The stem is prolonged obliquely upwards to the left in the form of a branch about 3 cm. broad and 14 cm. long. This branch is thickly clothed with hairy leaf scales and terminates in numerous spreading leaf scales of a narrow linear lanceolate form. The position and surface features of this branch are very inadequately and incorrectly reproduced in Saporta's figure. If we now turn to the specimen figured by the same author as a peduncle of *Williamsonia*[2], and which terminates in what appears to be a closed Williamsonian inflorescence, we find the characters are identical with those of the branch of the stem bearing Zamites fronds. Specimens of peduncles in the British Museum, and others in the collections of Whitby and Scarborough, afford similar proof of the identity of the detached peduncles and the obliquely placed branch of the leaf-bearing stem. There can be little doubt that the terminal bud-like structure on these peduncles is a young and unexpanded *Williamsonia*, but even if this be disputed, there can be no question as to the identity of the typical Williamsonia scale leaves and those of the terminal bud on the peduncles.

A specimen in the Whitby Museum shows a stem bearing two diverging peduncles, and evidence of the same habit of growth is afforded by an example in the British Museum. In all probability the stem figured by Saporta (*Plantes Jurassiques*, vol. II. Pl. XI. fig. 1) bore another peduncle in addition to that shown in the figure; this is suggested not only by the examination of other specimens but also by the oblique position of the peduncle which

[1] *Pal. Franç. Plant. Jurass.* Vol. II. p. 55.
[2] *loc. cit.* Vol. IV. Pl. XV.

is not brought out in the figure. The restoration of *Zamites* given by Williamson in his well-known paper (Pl. 53)[1] accurately represents what I believe to have been the manner of attachment of the inflorescence and foliage leaves to the main stem.

The above description must necessarily lack completeness in the absence of figures, but it is intended to publish drawings in a monograph on British Cycadean plants now in preparation for the Palæontographical Society.

The precise relation between *Williamsonia* and *Bennettites* must be left for future discussion, but evidence is not lacking in support of the view that the two genera are both members of the same family, and if not included in one genus they must at least be placed in the same family of *Bennettiteae*. In the title to this paper I have used the term *Bennettites* in a wide sense as including plants of the Bennettitean type, though possibly generically distinct.

The generic name *Williamsonia*, as expressing a more complete knowledge of the botanical affinity of the fossil than the provisional and comprehensive term *Zamites*, should be substituted for the latter in the case of Lindley and Hutton's species, *Zamites gigas*.

I am indebted to M. Renault for the opportunity of examining the specimens in the Paris Museum and for permission to have some of them photographed. My thanks are due also to Mr Newbitt of Whitby and Mr James Rowntree of Scarborough, for enabling me to examine the interesting examples of *Williamsonia* and *Zamites* in the museums of those towns.

[1] *Linn. Trans.* Vol. xxvi. 1870.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

On Lie's Solution of a Partial Differential Equation of the First Order.

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§ 1. The present paper gives an account and a proof of Lie's method* of solving a differential equation in one dependent and any number (including unity) of independent variables. The arrangement of the proof is such as to facilitate the examination of certain cases of exception. Such are afforded by the tac-locus and cusp-locus of the ordinary theory with one independent variable, and their analogues, and also by an extensive class of equations in which the linear partial form is included, and the integration of which has been discussed by Mayer†. Notes are added on the nature of a complete primitive, the complete solution of the auxiliary linear equation, and the satisfaction of limiting conditions.

Method of Solution.

§ 2. Let z be the dependent variable, $x_1, x_2 \ldots x_n$ the independent variables and $p_1, p_2 \ldots p_n$ the partial differential coefficients of z. If u, v are any two functions of these $2n+1$ quantities, denote the expression

$$\sum_{r=1}^{r=n} \frac{\partial(u, v)}{\partial(x_r, p_r)} + \sum_{r=1}^{r=n} p_r \frac{\partial(u, v)}{\partial(z, p_r)}$$

by the symbol (u, v).

* See Forsyth, *Theory of Differential Equations*, Part I. pp. 238—9; or Lie, *Math. Annalen*, Vol. VIII. p. 242.

† *Math. Ann.* Vol. VIII. pp. 313—8.

Then we know that the integration of any partial differential equation

$$f(x_1, x_2, \dots x_n, z, p_1, p_2, \dots p_n) = 0 \quad \dots\dots\dots\dots\dots (1)$$

depends on the solution of the linear equation

$$(f, \phi) = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

for ϕ in terms of $x_1, \dots x_n, z, p_1 \dots p_n$. The general solution of this linear equation is derivable when that of the equation $f = 0$ is known.

§ 3. With regard to the expression (f, ϕ) two things are noticeable. In the first place it is indifferent whether we do, or do not, suppose that wherever p_n, say, occurs in ϕ it is to be regarded as a known function of the other $2n$ variables, defined by the relation $f = 0$. This is true for each term in the expression and therefore for the whole.

Secondly, the following relation holds

$$(u, (v, w)) + (v, (w, u)) + (w, (u, v))$$
$$= -(v, w)\frac{\partial u}{\partial z} - (w, u)\frac{\partial v}{\partial z} - (u, v)\frac{\partial w}{\partial z} \quad \dots\dots\dots (3).$$

Hence if u, v, w are all solutions of the equation $(f, \phi) = 0$ it follows that the ratios

$$(v, w) : (w, u) : (u, v)$$

are also solutions, unless they are constants.

§ 4. If now we suppose p_n to be defined as a function of

$$x_1 \dots x_n, z, p_1 \dots p_{n-1}$$

by the equation $f = 0$, and therefore not to occur in ϕ except apparently, the equation $(f, \phi) = 0$, which is linear and homogeneous in the $2n$ derivatives of ϕ, is satisfied by $2n - 1$ independent functions, say $u_1, u_2 \dots u_{2n-1}$, and the most general form of ϕ is a function of these.

We take any such form, say u_1, and seek next a common solution of the equations

$$(f, \phi) = 0, \ (u_1, \phi) = 0.$$

This must be a function of $u_1, u_2 \dots u_{2n-1}$, such that

$$\sum_{r=1}^{r=2n-1} (u_1, u_r)\frac{\partial \phi}{\partial u_r} = 0.$$

The coefficients in this equation, or at least their ratios, are functions of $u_1, u_2 \dots u_{2n-1}$ since they satisfy the equation

$$(f, \phi) = 0.$$

Hence the condition $(u_1, \phi) = 0$ gives a linear homogeneous equation connecting

$$\frac{\partial \phi}{\partial u_2}, \frac{\partial \phi}{\partial u_3} \dots \frac{\partial \phi}{\partial u_{2n-1}},$$

the coefficients being functions of $u_1, u_2 \dots u_{2n-1}$. The full number of independent integrals of such an equation is $2n-2$ and u_1 is one of them; we may suppose without loss of generality that the rest are $u_2, u_3 \dots u_{2n-2}$.

The condition $(u_2, \phi) = 0$, when ϕ is a function of

$$u_2, u_3 \dots u_{2n-2},$$

is similarly satisfied by $2n-4$ forms of ϕ of which u_2 is one and the rest may be taken to be $u_3, u_4 \dots u_{2n-3}$.

By carrying on this process we get a series of n expressions $u_1, u_2 \dots u_n$, functionally independent of each other and such that

$$(f, u_r) = 0, \ (u_r, u_s) = 0 \ (r = 1, 2 \dots n; \ s = 1, 2 \dots n).$$

We are now to shew how the integrals of the equation $f = 0$ depend on the functions $u_1, u_2 \dots u_n$.

§ 5. Let us write $\frac{du}{\partial x_i}$ for the differential coefficient of u with respect to x_i, account being taken of the dependence of $z, p_1, p_2 \dots p_n$ upon x_i.

Thus

$$\frac{du_r}{\partial x_i} = \frac{\partial u_r}{\partial x_i} + p_i \frac{\partial u_r}{\partial z} + \sum_{j=1}^{j=n} \frac{dp_j}{\partial x_i} \cdot \frac{\partial u_r}{\partial p_j},$$

and

$$\sum_{i=1}^{i=n} \left(\frac{du_r}{\partial x_i} \frac{\partial u_s}{\partial p_i} - \frac{du_s}{\partial x_i} \frac{\partial u_r}{\partial p_i} \right) = (u_r, u_s) + \Sigma\Sigma \frac{\partial (u_r, u_s)}{\partial (p_i, p_j)} \left(\frac{dp_i}{\partial x_j} - \frac{dp_j}{\partial x_i} \right)$$
$$= 0 \ (r = 1, 2 \dots n, s = 1, 2 \dots n) \dots\dots\dots (4).$$

Putting f in the place of u_s, we have

$$\sum_{i=1}^{i=n} \frac{du_r}{\partial x_i} \frac{\partial f}{\partial p_i} = 0 \ (r = 1, 2 \dots n) \dots\dots\dots\dots\dots (5),$$

since $f = 0$ by supposition.

The system of $\frac{1}{2}n(n+1)$ equations thus found is, in virtue of the relation $f = 0$, an algebraical consequence of the system

$$\frac{dz}{\partial x_r} = p_r, \ \frac{dp_r}{\partial x_s} = \frac{dp_s}{\partial x_r},$$

which contains the same number of equations.

§ 6. The n equations (5) offer three alternatives:

(α) $u_1, u_2 \dots u_n$ are all constants;

(β) $u_1, u_2 \dots u_n$ are variable, but

$$\frac{d(u_1, u_2 \dots u_n)}{\partial(x_1, x_2 \dots x_n)} = 0,$$

so that they are connected by one or more relations;

(γ) $$\frac{\partial f}{\partial p_r} = 0 \qquad (r = 1, 2 \dots n).$$

If we take (α) we may have an integral, the complete primitive, which is the result of eliminating $p_1, p_2 \dots p_n$ from

$$f = 0, \quad u_1 = a_1, \quad u_2 = a_2 \dots u_n = a_n.$$

If we take (γ) we may have an integral, the singular solution, which is the result of eliminating $p_1, p_2 \dots p_n$ from

$$f = 0, \frac{\partial f}{\partial p_1} = 0, \frac{\partial f}{\partial p_2} = 0 \dots \frac{\partial f}{\partial p_n} = 0.$$

For (β), suppose the relations to be m in number, namely,

$$\phi_i(u_1, u_2 \dots u_n) = 0 \quad (i = 1, 2 \dots m) \dots\dots\dots\dots (6).$$

Then only $n - m$ of the equations (5) are independent—from (4) we may deduce that

$$\sum_{s=1}^{n} \frac{du_r}{\partial x_s} \cdot \frac{\partial \phi_i}{\partial p_s} = \sum_{s=1}^{n} \frac{d\phi_i}{\partial x_s} \cdot \frac{\partial u_r}{\partial p_s} = 0 \quad \begin{pmatrix} r = 1, 2 \dots n \\ i = 1, 2 \dots m \end{pmatrix}.$$

Each value of i gives a set of n equations of which only $n - m$ are independent and by comparison with (5) we deduce the $n - m$ relations

$$\left\| \begin{matrix} \frac{\partial f}{\partial p_1}, \dots & \frac{\partial f}{\partial p_s}, \dots & \frac{\partial f}{\partial p_n} \\ \frac{\partial \phi_1}{\partial p_1}, \dots & \frac{\partial \phi_1}{\partial p_s}, \dots & \frac{\partial \phi_1}{\partial p_n} \\ \frac{\partial \phi_2}{\partial p_1}, \dots & & \\ \dots\dots\dots & & \\ \frac{\partial \phi_m}{\partial p_1}, \dots & & \frac{\partial \phi_m}{\partial p_n} \end{matrix} \right\| = 0 \dots\dots\dots\dots (7).$$

If (β) leads to an integral it will be the result of eliminating $p_1, p_2 \dots p_n$ from $f = 0$ by means of the n equations (6) and (7). The equation has thus $n - 1$ distinct types of general integral

since m may be any of the integers 1, 2, ... $n-1$, and the complete and singular integrals are extreme types for which m has the values n and 0 respectively.

All solutions of the equation $f=0$ are included among those thus derived from any one system of functions

$$u_1, u_2 \dots u_n,$$

but the system of functions is by no means unique, as is clear from their mode of formation.

§ 7. We may verify these solutions in the following way.

Considering $z, p_1, p_2 \dots p_n$ as functions of $x_1, x_2 \dots x_n$ we have at once, since $f=0$,

$$\frac{\partial f}{\partial x_r} + \frac{dz}{\partial x_r} \cdot \frac{\partial f}{\partial z} + \sum_{s=1}^{s=n} \frac{dp_s}{\partial x_r} \cdot \frac{\partial f}{\partial p_s} = 0 \quad (r=1, 2 \dots n),$$

and also

$$\frac{\partial u_i}{\partial x_r} + \frac{dz}{\partial x_r} \cdot \frac{\partial u_i}{\partial z} + \sum_{s=1}^{s=n} \frac{dp_s}{\partial x_r} \frac{\partial u_i}{\partial p_s} = \frac{du_i}{\partial x_r} \quad \begin{pmatrix} r=1, 2 \dots n \\ i=1, 2 \dots n \end{pmatrix}.$$

Thus

$$\sum_{r=1}^{r=n} \frac{du_i}{\partial x_r} \cdot \frac{\partial f}{\partial p_r} = (u_i, f) + \sum_{r=1}^{r=n} \left(\frac{dz}{\partial x_r} - p_r \right) \frac{\partial (u_i, f)}{\partial (z, p_r)} + \sum_{s=1}^{s=n} \sum_{r=1}^{r=n} \frac{dp_s}{\partial x_r} \cdot \frac{\partial (u_i, f)}{\partial (p_s, p_r)} \quad \dots\dots(5\,a),$$

and

$$\sum_{r=1}^{r=n} \left(\frac{du_i}{\partial x_r} \frac{\partial u_j}{\partial p_r} - \frac{du_j}{\partial x_r} \cdot \frac{\partial u_i}{\partial p_r} \right) = (u_i, u_j) + \sum_{r=1}^{r=n} \left(\frac{dz}{\partial x_r} - p_r \right) \frac{\partial (u_i, u_j)}{\partial (z, p_r)} + \sum_{s=1}^{s=n} \sum_{r=1}^{r=n} \frac{dp_s}{\partial x_r} \cdot \frac{\partial (u_i, u_j)}{\partial (p_s, p_r)} \quad \dots\dots(4\,a).$$

(In the double summations r, s must be unequal.)

The first term on the right-hand side in each of the equations (5 a) and (4 a) vanishes, since $f=0$.

Now let these equations be multiplied respectively by the determinants of the matrix

$$\begin{vmatrix} \frac{\partial f}{\partial p_2}, & \frac{\partial u_1}{\partial p_2}, & \frac{\partial u_2}{\partial p_2} \dots & \frac{\partial u_n}{\partial p_2} \\ \frac{\partial f}{\partial p_3}, & \frac{\partial u_1}{\partial p_3}, \dots\dots\dots & \frac{\partial u_n}{\partial p_3} \\ \vdots & & & \\ \frac{\partial f}{\partial p_n}, \dots\dots\dots & & & \end{vmatrix}$$

and the products added. We thus have on the right

$$\left(\frac{dz}{\partial x_1} - p_1\right)\frac{\partial(f, u_1, u_2 \dots u_n)}{\partial(z, p_1, p_2 \dots p_n)},$$

and on the left

$$\begin{vmatrix} 0, & \frac{du_1}{\partial x_1}, & \frac{du_2}{\partial x_1}, & \frac{du_3}{\partial x_1} \cdots & \frac{du_n}{\partial x_1} \\ \frac{\partial f}{\partial p_1}, & \frac{\partial u_1}{\partial p_1}, & \frac{\partial u_2}{\partial p_1} \cdots & & \frac{\partial u_n}{\partial p_1} \\ \frac{\partial f}{\partial p_2}, & \frac{\partial u_1}{\partial p_2} \cdots & & & \frac{\partial u_n}{\partial p_2} \\ \vdots & & & & \\ \frac{\partial f}{\partial p_n} \cdots & & & & \end{vmatrix}.$$

In like manner

$$\left(\frac{dz}{\partial x_r} - p_r\right)\frac{\partial(f, u_1 \dots u_n)}{\partial(z, p_1 \dots p_n)}$$

= the same determinant with x_r written for x_1 in the first row.

In these equations we may substitute for $u_1, u_2 \dots u_n$ any n independent functions of them, say $\phi_1, \phi_2 \dots \phi_n$, the effect being to multiply each equation by the Jacobian

$$\frac{\partial(\phi_1, \phi_2 \dots \phi_n)}{\partial(u_1, u_2 \dots u_n)}.*$$

We thus have

$$\left(\frac{dz}{\partial x_r} - p_r\right)\frac{\partial(f, \phi_1, \phi_2 \dots \phi_n)}{\partial(z, p_1, \dots p_n)} = \begin{vmatrix} 0, & \frac{d\phi_1}{\partial x_r}, & \frac{d\phi_2}{\partial x_r} \cdots & \frac{d\phi_n}{\partial x_r} \\ \frac{\partial f}{\partial p_1}, & \frac{\partial \phi_1}{\partial p_1} \cdots & & \frac{\partial \phi_n}{\partial p_1} \\ \frac{\partial f}{\partial p_2}, & \frac{\partial \phi_1}{\partial p_2} \cdots & & \\ \vdots & & & \\ \frac{\partial f}{\partial p_n}, & \frac{\partial \phi_1}{\partial p_n} \cdots & & \end{vmatrix}.$$

* If the relations among $u_1, u_2 \dots u_n$ expressed by (6) are such that they cause this Jacobian to vanish for arbitrary forms of $\phi_{m+1}, \dots \phi_n$, then the equations (6) must be satisfied doubly by these relations, and this state of things can always be avoided.

The right-hand side of this equation vanishes if the equations (6) and (7) are satisfied, m being zero or any whole number up to n inclusive. Thus in general the equations (6) and (7) give p_r as the value of $\frac{dz}{\partial x_r}$ and therefore afford a solution of the differential equation (1), which was to be proved.

The relations (6) with $f=0$ form a 'complete system,' the singular solution of which is given by (7). Thus we may say that all the solutions of $f=0$ are singular solutions of complete systems to which $f=0$ belongs. It is not necessary to form all such complete systems in order to have all solutions.

Cases of failure.

§ 8. The proof will fail if

$$\frac{\partial(f, u_1, \dots u_n)}{\partial(z, p_1, \dots p_n)} = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots(8).$$

If we multiply the equations

$$(f, u_1)=0, \text{ etc.}, \quad (u_1, u_2)=0, \text{ etc.}$$

by the second minors of this Jacobian, formed by leaving out the derivatives with respect to z and p_r, and add, we have, as other forms of the relation

$$\frac{\partial(f, u_1, \dots u_n)}{\partial(z, p_1, \dots p_n)} = 0,$$

that

$$\frac{\partial(f, u_1, \dots u_n)}{\partial(x_r, p_1, \dots p_n)} = 0, \quad (r=1, 2 \dots n) \quad \dots\dots\dots\dots(9).$$

Hence it follows that all the determinants of the matrix

$$\left\| \begin{matrix} \frac{\partial f}{\partial x_1}, & \frac{\partial f}{\partial x_2} \cdots \frac{\partial f}{\partial x_n}, & \frac{\partial f}{\partial z}, & \frac{\partial f}{\partial p_1} \cdots \frac{\partial f}{\partial p_n} \\ \frac{\partial u_1}{\partial x_1}, & \frac{\partial u_1}{\partial x_2} \cdots \frac{\partial u_1}{\partial x_n}, & \frac{\partial u_1}{\partial z}, & \frac{\partial u_1}{\partial p_1} \cdots \frac{\partial u_1}{\partial p_n} \\ \frac{\partial u_2}{\partial x_1} & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \vdots & \\ \frac{\partial u_n}{\partial x_1} & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\

vanish unless those of the matrix

$$\left\| \begin{matrix} \dfrac{\partial f}{\partial p_1}, & \dfrac{\partial u_1}{\partial p_1} \cdots & \dfrac{\partial u_n}{\partial p_1} \\ \dfrac{\partial f}{\partial p_2} \cdots\cdots & & \dfrac{\partial u_n}{\partial p_2} \\ \vdots & & \\ \dfrac{\partial f}{\partial p_n} \cdots\cdots & & \end{matrix} \right\| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

are all zero.

There are therefore two kinds of exceptional cases, which we shall now examine*.

§ 9. The equations (7) may be written

$$\lambda \frac{\partial f}{\partial p_s} + \sum_{i=1}^{i=m} \mu_i \frac{\partial \phi_i}{\partial p_s} = 0, \quad (s = 1,\ 2 \ldots n).$$

Thus if (7) and (8) are both true it follows that

$$\lambda \frac{\partial f}{\partial z} + \Sigma \mu_i \frac{\partial \phi_i}{\partial z} = 0,$$

unless the determinants (11) vanish. In the same way from (9), unless the determinants (11) vanish,

$$\lambda \frac{\partial f}{\partial x_r} + \Sigma \mu_i \frac{\partial \phi_i}{\partial x_r} = 0, \quad (r = 1,\ 2 \ldots n).$$

* The proof also fails if

$$\frac{\partial (\phi_1,\ \phi_2 \ldots \phi_n)}{\partial (u_1,\ u_2 \ldots u_n)} = 0,$$

that is to say, since $\phi_{m+1}, \ldots \phi_n$ may be chosen arbitrarily, if

$$\left\| \begin{matrix} \dfrac{\partial \phi_1}{\partial u_1}, & \dfrac{\partial \phi_1}{\partial u_2} \cdots & \dfrac{\partial \phi_1}{\partial u_n} \\ \dfrac{\partial \phi_2}{\partial u_1}, \cdots\cdots & & \\ \cdots\cdots & & \\ \dfrac{\partial \phi_m}{\partial u_1} \cdots\cdots & & \dfrac{\partial \phi_m}{\partial u_n} \end{matrix} \right\| = 0.$$

These equations are $n-m+1$ in number, and as $u_1 \ldots u_n$ already satisfy the m equations (6) they will generally be inconsistent. If however any values satisfy them they will also satisfy (7). If the values are isolated then these relations afford a solution which is included in the complete primitive, since the values of $u_1 \ldots u_n$ are given. If the values are not isolated then these relations are not numerous enough to enable us to eliminate $p_1 \ldots p_n$.

The values satisfying these conditions will occur twice in the ∞^{2n-m} determined by the $m+1$ equations (1) and (6) in the $2n+1$ quantities $x_1, \dots x_n, z, p_1 \dots p_n$.

Thus if (8) is not a new relation this ∞^{2n-m} must contain a double ∞^n*, which will satisfy (7) but not be a solution of the *differential* equations (1) and (6). Any complete primitive of the system will define an ∞^n on the ∞^{2n-m} and it will not generally happen that a complete primitive will, in geometrical language, meet the double ∞^n in the same point on both sheets.

The system of values of $x_1, \dots x_n, z, p_1 \dots p_n$ will therefore be common to two complete primitives, which thus have contact with each other, so that the complete primitives of the system (1), (6) have a tac-locus. This is one of the cases of failure.

The second is that in which the determinants (11) all vanish in virtue of a single relation. This relation will clearly be included in (7), at least as an alternative, and the other alternative, if any, must be taken in order to find the singular solution of the system. In this case the complete primitive ∞^n has an ∞^{n-1} of cusps, that is of points where $x_1, \dots x_n, z$ are all the same for two consecutive points. The cusps of the system (1), (6) will form an ∞^{2n-m-1} within which the equations (7) define an ∞^n. This will generally not satisfy the differential equations.

The occurrence of a cuspidal ∞^{n-1} on the complete primitive ought perhaps to be considered as normal, since generally the vanishing of the determinants (11) will be secured by the vanishing of a factor of the left-hand side of (8)†.

If we consider a singly infinite series of complete primitives of (1), regarding only the $\infty^{n+1}(z, x_1, \dots x_n)$, each primitive has the cuspidal ∞^{n-1} as part of its intersection with the consecutive. If we consider a doubly infinite series then each primitive has an ∞^{n-2} of points on the cuspidal ∞^{n-1} as part of its intersection with two consecutives, and so on. Thus, whatever the value of $m (< n)$ in (6) there will be an ∞^n of cuspidal values satisfying (1), (6) and (7) algebraically but not generally satisfying (1) and (6) as differential equations.

§ 10. The condition (8) is satisfied also by the vanishing of any factor that occurs to a higher power than the first in any

* Possibly the double systems may form a manifoldness of more than n dimensions. In such a case the equations (1), (6), (7) will be satisfied by an arbitrary ∞^n contained therein, but in general this will not be a solution.

† For instance, take the equation $z = p_1x_1 + p_2x_2 + p_1p_2$. It has any number of complete primitive forms, representing different doubly infinite series of developable surfaces, and it is only in exceptional simple cases that a developable surface has not a cuspidal edge.

particular complete primitive. In such a case however the solution does not fail as may be seen. The failure in general is due to the fact that the first derivatives of the proposed solution do not give determinate values of the derivatives of z, but must be taken in combination with the second derivatives of the proposed solution. From these, one or more equations have to be formed involving only the first derivatives of z, and these equations are not generally satisfied by the values $p_1, p_2 \dots p_n$.

If however for particular values of the constants an equation of the form $\lambda^2\mu = 0$ can be formed from the equations

$$f = 0, \quad u_1 = a_1, \quad u_2 = a_2 \dots u_n = a_n \dots\dots\dots\dots (12),$$

then, on the supposition that $\lambda = 0$, the relations found from the second derivatives of $\lambda^2\mu = 0$ will be simply repetitions of those found by removing the factor λ from the first derivatives. The effect will be the same as that of removing the factor λ from the final equations of § 7, and thus the results

$$\frac{dz}{\partial x_r} = p_r \ (r = 1, \ 2 \dots n),$$

hold under these conditions, and there is no failure.

§ 11. Another possibility is that the condition (8) may be satisfied identically or in virtue of the equation $f = 0$ only. Since $u_1 \dots u_n$ are supposed independent, it is not possible for the conditions (10) to be satisfied identically, and hence (11) must hold good.

In this case the equations (12) give more relations than one connecting $z, x_1, x_2 \dots x_n$ and therefore the supposition (α), (§ 6), cannot be entertained. Let k be the number of independent relations not involving $p_1, p_2 \dots p_n$ that can be deduced from (12); then in (6) we must suppose m not to be greater than $n - k + 1$ or it will be possible to deduce more relations than one among $z, x_1, x_2 \dots x_n$. Also the relations (7) will reduce to $n - m - k + 1$ in number, $k - 1$ being consequences of the rest, since the determinants of order $n - k + 2$ formed from the matrix (11) vanish.

§ 12. The equations ($4\,a$) and ($5\,a$) now give, in virtue of (6) and (7),

$$\sum_{r=1}^{r=k} \left(\frac{dz}{\partial x_r} - p_r \right) \frac{\partial (f, u_1, u_2 \dots u_{n-k+1})}{\partial (z, p_r, p_{k+1} \dots p_n)} = 0 \dots\dots\dots\dots (13)$$

and other such equations, each being linear and homogeneous in k of the quantities

$$\frac{dz}{\partial x_r} - p_r \ (r = 1, 2 \dots n).$$

But the relations (6) and (7) are now only $n-k+1$ in number and therefore fail to determine $z, p_1, \dots p_n$ in terms of $x_1, \dots x_n$. They may be taken as determining $z, p_k, p_{k+1} \dots p_n$ in terms of $x_1, \dots x_n, p_1, \dots p_{k-1}$ and it is then easily found that

$$\frac{dz}{\partial p_r} = 0 \quad (r = 1, 2 \dots k-1),$$

$$\frac{dz}{\partial x_r} = -\frac{\partial (f, \phi_1, \phi_2 \dots \phi_m)}{\partial (x_r, p_1, \dots p_m)} \div \frac{\partial (f, \phi_1, \phi_2 \dots \phi_m)}{\partial (z, p_1, \dots p_m)},$$

$$(r = 1, 2 \dots n).$$

Thus the relations (6) and (7) yield an equation connecting $z, x_1, \dots x_n$ and leave $k-1$ of the quantities $p_1, p_2 \dots p_n$ undetermined.

If then we suppose $p_1, p_2 \dots p_{k-1}$ to have the values just found for $\frac{dz}{\partial x_r}$ $(r = 1, 2 \dots k-1)$ it will follow from the equation (13) that

$$\frac{dz}{\partial x_k} = p_k,$$

and in the same way

$$\frac{dz}{\partial x_r} = p_r \quad (r = 1, 2 \dots n).$$

Thus we may say generally that the equations (6) and (7) yield at least one relation among $z, x_1 \dots x_n$, and when they yield only one, that is when $m \geqslant n-k+1$, that one relation satisfies the differential equation $f = 0$.

Exceptional cases may arise here, again corresponding to the tac-locus and cusp-locus. In the one case all the minor determinants of order $n-k+2$ formed from the matrix (10) vanish, in the other those of order $n-k+1$ formed from (11).

§ 13. We are thus introduced to classes of differential equations in which certain of the forms of solution are wanting. Lagrange's linear equation is an extreme case, in which $k = n$. It is of course the only case that can occur when $n = 2$. In general the class includes equations formed by eliminating the arbitrary constants and function from an equation of the form

$$\psi (v_1, v_2 \dots v_k) = 0$$

where $v_1, v_2 \dots$ are known functions of $z, x_1 \dots x_n$ involving $n-k$ arbitrary constants, and ψ is an arbitrary function. The differential equation is of such a form that it is satisfied by equating $n-k+1$ of the quantities $p_1, p_2 \dots p_n$ to linear functions of the rest, the coefficients involving $z, x_1 \dots x_n$ and $n-k$ parameters. For instance

when $n=3$, $k=2$ the equation $f=0$ must so far as p_1, p_2, p_3 are concerned satisfy the condition for a ruled surface. This is necessary but not sufficient unless $k=n$.

§ 14. The number of complete primitives is unlimited, but it should be noticed that not every solution involving n arbitrary constants is a complete primitive in the sense that all other solutions can be deduced from it.

It may be that an equation

$$F(z, x_1, x_2 \dots x_n, a_1 \dots a_n) = 0$$

is such that from it and the n derived equations

$$p_r \frac{\partial F}{\partial z} + \frac{\partial F}{\partial x_r} = 0 \quad (r = 1, 2 \dots n)$$

two or more equations can be deduced in which the arbitrary constants do not occur. Let these equations be

$$f_1 = 0, \quad f_2 = 0 \dots f_m = 0.$$

Then all the solutions derived from $F=0$ by the ordinary method of variation of parameters will satisfy this system of m equations, and therefore will not include all the solutions of any one of the system, say $f_1=0$.

For instance the equation

$$x_1 z = a_1 x_1^2 + a_2 x_1 x_2 + a_3 x_2^2$$

is a solution of

$$z = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_1 p_2 p_3$$

containing three arbitrary constants, but since it satisfies the equation $p_3=0$ also it is not a complete primitive in the true sense.

§ 15. If any complete primitive of the equation $f=0$ is known, the general solution of the auxiliary equation $(f, \phi)=0$ can be deduced* as follows.

Let $$F(z, x_1, x_2 \dots x_n, a_1, a_2 \dots a_n) = 0$$

be the complete primitive. Then from this equation and its n derivatives expressions can be found for $a_1, a_2 \dots a_n$ in terms of $z, x_1 \dots x_n, p_1 \dots p_n$; otherwise it is not properly a complete primitive. Let these expressions be $u_1, u_2 \dots u_n$ respectively. Then $u_1, u_2 \dots u_n$ are n of the $2n-1$ solutions of $(f, \phi)=0$.

Another complete primitive can be found by taking

$$a_n = a_1 b_1 + a_2 b_2 + \dots + a_{n-1} b_{n-1} + b_n,$$

* See Mayer, *Math. Ann.*, Vol. VIII. p. 311.

and eliminating $a_1 \dots a_n$ by means of the equations

$$\frac{\partial F}{\partial a_r} + b_r \frac{\partial F}{\partial a_n} = 0 \ (r = 1, 2 \dots n-1).$$

We still have $a_1 = u_1$, $a_2 = u_2 \dots a_n = u_n$ and the values of $b_1, b_2 \dots b_n$ are solutions of the auxiliary equation. Thus the $2n-1$ necessary solutions of the auxiliary equation are $u_1, u_2 \dots u_n$ and the ratios of

$$\frac{\partial U}{\partial u_1}, \ \frac{\partial U}{\partial u_2} \dots \frac{\partial U}{\partial u_n},$$

where U denotes

$$F(z, x_1, x_2 \dots x_n, u_1, u_2 \dots u_n).$$

It is easy to shew that these are independent, and thus we are in a position to write down the general solution of the auxiliary equation.

§ 16. It would appear at first sight that the determination of the forms of the arbitrary functions to satisfy limiting conditions would involve the solution of differential equations, since the derivatives of the arbitrary functions occur in the solution; this is however not the case.

Suppose for instance that the value of z is given when $x_1 = 0$, say

$$z = \zeta(x_2, x_3 \dots x_n).$$

The values of $p_2, p_3 \dots p_n$ are therefore given when $x_1 = 0$, for then

$$p_r = \frac{\partial \zeta}{\partial x_r} \ (r = 2, 3 \dots n).$$

These values may be substituted for $x_1, z, p_2, p_3 \dots p_n$ in the equations

$$f = 0, \ \ u_1 = a_1, \ \ u_2 = a_2 \dots u_n = a_n,$$

and the n quantities $p_1, x_2 \dots x_n$ may be eliminated.

Thus one or more relations will be found among $a_1, a_2 \dots a_n$ and these are what were required.

More generally if the solution is to include all systems of values that satisfy the equations

$$\chi_1(z, x_1, x_2 \dots x_n) = 0, \ \ \chi_2(z, x_1, x_2 \dots x_n) = 0,$$

the values of $p_1, p_2 \dots p_n$ corresponding must satisfy the $n-1$ relations

$$\left\| \begin{matrix} -1, & p_1, & p_2 \dots & p_n \\ \frac{\partial \chi_1}{\partial z}, & \frac{\partial \chi_1}{\partial x_1}, & \dots\dots & \frac{\partial \chi_1}{\partial x_n} \\ \frac{\partial \chi_2}{\partial z}, & \frac{\partial \chi_2}{\partial x_1}, & \dots\dots & \frac{\partial \chi_2}{\partial x_n} \end{matrix} \right\| = 0.$$

The elimination of $z, x_1 \ldots x_n, p_1 \ldots p_n$ from these $n-1$ relations, and

$$\chi_1 = 0,\ \chi_2 = 0,\ f = 0,\ u_1 = a_1 \ldots u_n = a_n,$$

will give one or more equations connecting $a_1, a_2 \ldots a_n$, and these are what must be used to derive the particular solution sought.

Monday, 26 April, 1897.

Mr F. Darwin, President, in the Chair.

The following were elected Fellows of the Society:

R. Lachlan, Sc.D., Trinity College.
W. M. Coates, M.A., Queens' College.
S. S. Hough, M.A., St John's College.
Rev. P. E. Bateman, M.A., Jesus College.

The following Communications were made to the Society:

(1) *On the Apparent Electrification in an Electric Field at the Bounding Surface of Two Dielectrics.* By Prof. A. Anderson, M.A., Queen's College, Galway.

In Maxwell's *Treatise on Electricity and Magnetism*, Vol. I. 3rd edition, p. 100, the apparent surface charge σ' on the boundary of two media of specific inductive capacities K_1 and K_2 is defined by the equation $\frac{dV}{dn_1} + \frac{dV}{dn_2} + 4\pi\sigma' = 0$.

Maxwell then proceeds to state, giving a reference to Faraday, that if, during the action of the inducing force, the apparent electrification of the surface be discharged by passing a flame over the surface, then, when the inducing force is taken away, there will appear a true electrification opposite to σ'. In an Appendix to the chapter in which this occurs the apparent electrification is described as acting on the conducting flame which is in connexion with the earth, attracting electricity of opposite sign, and repelling electricity of the same sign, the consequence being the production of a real electrification which masks the effect of the apparent one.

Prof. Gray, in his *Absolute Measurements in Electricity and Magnetism*, Vol. I. p. 113, says that the action of the flame is to reduce the surface to zero potential, and that the real electrification is equal and opposite to the apparent electrification.

In *Mascart and Joubert*, Vol. I. p. 101 (English edition), it is stated that, though the layer of density σ is a fictive layer, if its surface is brought to the neutral state by means of a flame, and if the sources of induction are removed, a real layer of density σ will be found on the surface.

Again, Wiedemann, in *Die Lehre von der Elektricität*, Band II., asserts that the flame removes the apparent electrification σ', and produces a real electrification of opposite sign depending on the action of the apparent electrification on the interior of the body.

There seems to be some obscurity as regards what is meant by removing the apparent charge. Prof. Gray, however, clearly states, and I think the same thing is implied in the Appendix to the chapter in Maxwell above referred to, that it means reducing the surface to zero potential. It would, I think, be impossible to do this unless for the portion of the surface affected by the flame at one instant: if the flame be taken to another portion of the surface the real electrification produced there will alter the potential of all parts of the field and, therefore, of that part which has already been reduced to zero potential. To reduce the whole surface to zero potential, a flame large enough to affect all parts of it at once would be necessary. But, if this is what is meant by removing the apparent electrification, the real electrification, as will be shewn presently, is not necessarily either equal in amount or opposite in sign to the apparent electrification. Another way of putting it, used by Mascart and Joubert, is 'reducing the surface to the neutral state.' It is not clear what this means, unless it be making the electric stress zero at every point of the surface, on both sides of it. But this by Gauss's theorem is impossible if a real electrification is produced by the flame.

What seems to be properly implied by removing the apparent charge is making the surface density of this charge, as defined by Maxwell, zero; that is to say, a real charge must be given to the surface which will make the normal force at any point of it continuous. But, generally, this could not be done by means of a flame.

Let σ' denote the apparent electrification, and σ_1 the real electrification necessary to remove the apparent charge. Also let V denote the potential at any point before and V' after the apparent charge is removed, and V_1 the potential due to the real electrification of the surface. Then, $V' = V + V_1$, and we have

$$\frac{dV}{dn_1} + \frac{dV}{dn_2} + 4\pi\sigma' = 0,$$

$$K_1 \frac{dV}{dn_1} + K_2 \frac{dV}{dn_2} = 0;$$

also

$$K_1 \frac{d}{dn_1}(V + V_1) + K_2 \frac{d}{dn_2}(V + V_1) + 4\pi\sigma_1 = 0,$$

$$\frac{d}{dn_1}(V + V_1) + \frac{d}{dn_2}(V + V_1) = 0.$$

Hence, $$K_1 \frac{dV_1}{dn_1} + K_2 \frac{dV_1}{dn_2} + 4\pi\sigma_1 = 0,$$

and $$\frac{dV_1}{dn_1} + \frac{dV_1}{dn_2} - 4\pi\sigma' = 0.$$

To remove the apparent electrification is, then, equivalent to finding the actual surface density σ_1 of a distribution of electricity on the bounding surface of two media of specific inductive capacities K_1 and K_2 which would produce an apparent surface density $-\sigma'$, there being no other electricity in the field. This problem admits, in many cases, of an easy solution.

As an example, consider the case of a charge e at the centre of a sphere whose radius is a and specific inductive capacity K_1, the medium outside the sphere being of specific inductive capacity K_2.

Inside the sphere the potential is $\frac{e}{K_1 r} + \frac{e}{a}\left(\frac{1}{K_2} - \frac{1}{K_1}\right)$, and outside it is $\frac{e}{K_2 r}$. The surface density, σ', of the apparent electrification is, consequently, $\frac{e}{4\pi a^2}\left(\frac{1}{K_2} - \frac{1}{K_1}\right)$.

To find the real electrification which will remove all apparent electrification, we must find the distribution σ_1 on the sphere which, of itself, would produce an apparent distribution of surface density $-\frac{e}{4\pi a^2}\left(\frac{1}{K_2} - \frac{1}{K_1}\right)$. This is easily shewn to be $-\frac{e}{4\pi a^2}\left(1 - \frac{K_2}{K_1}\right)$, and is, therefore, not equal to $-\sigma'$ unless when $K_2 = 1$. When this surface distribution is placed on the sphere, the potential both inside and outside is $\frac{e}{K_1 r}$, and the force is consequently the same on both sides of the surface, as it ought to be when there is no apparent electrification.

Suppose, on the other hand, we bring the surface to zero potential by means of a flame. The potential at all points outside will be zero, and the potential inside $\frac{e}{K_1}\left(\frac{1}{r} - \frac{1}{a}\right)$. The surface density of the actual electrification of the surface is, therefore, $-\frac{e}{4\pi a^2}$, which is not $-\sigma'$ and not opposite to σ' unless when $K_1 > K_2$. Besides, there is still an apparent electrification of surface density $-\frac{e}{4\pi a^2 K_1}$.

(2) *On Luminosity attending the compression of certain rarified gases.* By H. F. Newall, M.A., Trinity College.

1. Morrem (*Pogg. Ann.* 1862, Vol. 115, pp. 350—2, and 1865, Vol. 126, pp. 643—654) and Sarasin (*Pogg. Ann.* 1870, Vol. 140, p. 425) have given accounts of phosphorescence produced in rarified gases by the passage of electrical discharges. The phenomenon itself was recognized in the earliest days of vacuum tubes.

Morrem and Sarasin agreed in thinking that no pure single gas (as distinct from a mixture of gases) exhibits phosphorescence. Morrem found the phosphorescence very strong in a mixture of gases containing 200 parts of Oxygen, 100 parts of Nitrogen, and 150 parts of SO_2. Sarasin regards the presence of oxygen as essential to the production of phosphorescence. J. J. Thomson (*Recent Researches*, 1893, p. 184) has described observations of such phosphorescence and has explained it in accordance with his view that the conduction of electricity by a gas is associated with decomposition. He states, "I have never detected any glow in a single gas (as distinct from a mixture) unless that gas was one which formed polymeric modifications, but all the gases I have examined which do polymerize have shown the afterglow."

2. In the course of an investigation of the spectra of rarified gases rendered luminous by electrodeless discharges by Prof. J. J. Thomson's method, I was dealing with the case of mixtures of Oxygen with other gases, chiefly Nitrogen and Carbonic Oxide in different proportions, and frequently obtained mixtures which at certain exhaustions phosphoresced so brilliantly that I was induced to devote some time to the study of the phenomena, in particular by means of the spectroscope.

3. A description of the apparatus used will make the conditions clearer.

A glass tube, in which electrodeless discharges were to be passed, was connected with a Hagen-Töpler mercury-pump, fused glass joints being used throughout. There was also attached to the pump a graduated tube in which mixtures of gases could be made in any desired proportions, and which served as a reservoir of such mixtures. Samples of the mixtures could be let into the pump by an arrangement very similar to that figured by Salet in his *Traité de Spectroscopie* (Masson, Paris, 1883), p. 209. In most of the experiments the electrodeless discharge was produced near the middle of a large tube, about 2 inches in diameter and 6 feet long. In a tube like this the phosphorescence begins near the bright discharge and spreads along

the tube, in some cases quite slowly (about 1 foot per second), in other cases too quickly for any estimate to be made without special apparatus. The rate varies with the pressure and with the mixture used.

4. The colour of the phosphorescence is very varied; I find among my notes that at various times the following colours have been observed:—white, silvery-grey, yellow, yellow-green, old gold, ruddy yellow, brilliant blue. The phosphorescence is undoubtedly diffused throughout the whole volume of the gas enclosed in the tube. There is an appearance of semi-opacity in the phosphorescent gas, except in the case of a certain brilliant blue phosphorescence. The contrast in this respect between the yellow and the blue afterglows is as marked as that between a clear solution of copper sulphate and a liquid rendered turbid by a slight yellow precipitate in suspension. The blue phosphorescence is marked, in my mind, as of an exceptional nature; I have only seen it four or five times, but it has generally been of so short duration, that I have not been able to study it fully, nor can I say what conditions are necessary for its appearance, except that I connect it with oxygen and sulphur.

Spectrum of the phosphorescent gas.

5. A careful and long continued spectroscopic investigation shows that the spectrum of the phosphorescent glows that I have examined is a continuous spectrum. Prof. Thomson records the observation of a few bright lines, if very high dispersion is used, but I have been unable to corroborate this observation. (In view of what follows in this paper, I should explicitly state that I refer here to the spectrum of phosphorescence which has been excited electrically in the rarified gas.)

6. An arrangement somewhat similar to a phosphoroscope was used.

Between the spectroscope and the tube in which the phosphorescence was produced, a sector or sectors of a circle were rotated by clockwork, which also drove a contact-break in the circuit that produced the bright discharge in the tube. It was arranged that the sector intercepted the light of the bright discharge, and allowed only the light from the phosphorescent gas to pass into the spectroscope. The spectra were observed visually and were also photographed. Both methods showed a continuous spectrum without any trace of bright lines, except on one occasion to which I refer later (§ 27).

7. The spectroscopic study of the phosphorescence itself has then given no evidence of its chemical origin. But the fact that a *continuous* spectrum is visible in a gaseous mixture of such tenuity as that dealt with (about $0^{mm}{\cdot}3$—$0^{mm}{\cdot}1$ pressure) is noteworthy. I was unable to detect diffuse bands or a maximum of brightness though the difference of the colours of the phosphorescent hazes in different mixtures of gas would lead one to expect the existence of such bands.

8. The spectrum of the bright discharge which precedes the phosphorescence has only given results difficult to interpret when taken in connection with one's knowledge of the care that has been taken in preparing the gases.

For instance, a bulb had been carefully filled with electrolytic oxygen, after 'rinsing' the pump and tubes out with the gas, and after heating the tubes several times at very low pressure, and after leaving the pump and tubes exhausted for two or three days. The bright discharge gave only the band spectra of nitrogen and cyanogen, and it was only after continued sparking that the oxygen spectrum asserted itself.

It seems to me that the spectrum of the bright discharge possibly affords no more certain evidence of the chemical origin of the phosphorescence than of the nature of the mixture of gases in which the discharge takes place.

9. The appearance of the phosphorescent gas led me to apply a test, which Sir G. G. Stokes has described as affording proof of the existence of solid particles in the flame of a candle. A beam of sunlight was concentrated by means of a lens upon the phosphorescent gas: and the track of the beam in the gas, as also the scattering of polarized light, were looked for. But no evidence could be obtained that supports the idea that we might have to deal with the phosphorescence of solid particles momentarily set free by decomposition accompanying the bright discharge.

10. In the course of my work with electrodeless discharges I have frequently found black or brown deposits on the inside of the glass tubes. In one or two cases the deposit has been thick enough to form a film which could be detached from the glass; the film was translucent and had a highly offensive smell. The formation of this film is a proof that permanent chemical changes are brought about by the discharge; and there can be little doubt but that the phosphorescence is a sign that temporary chemical changes are also produced.

11. Taking into account the difference in the colour of different phosphorescent hazes and the difference in the conditions of their appearance, but setting only small weight on the

indications of the spectrum of the *bright* discharge preceding phosphorescence, I am led to connect

white phosphorescence with mixtures containing oxygen and carbon compounds,

yellow phosphorescence with mixtures containing oxygen and nitrogen,

blue phosphorescence with mixtures containing oxygen and sulphur.

Pressure conditions.

12. So far as my experiments have gone, the phosphorescence appears in rarified gases between the pressures $0^{mm}{\cdot}6$ and $0^{mm}{\cdot}01$. It rises in brightness as the pressure is reduced and reaches a maximum at a pressure of about $0^{mm}{\cdot}4$, and fades away into invisibility as the pressure is further reduced. These pressures vary within small limits according to the nature of the gaseous mixture.

13. To measure the pressure, the pump has been calibrated in the well known method. The capacity of the pump-bulb is close upon 500 c.c.; and the bubble of air in the exit tube, measured at atmospheric pressure (or, at a pressure of one-tenth of the atmospheric pressure when very high exhaustion is to be dealt with), affords means of estimating the pressure of the permanent gases in the vacuum tube connected with the pump. A careful set of experiments on the indications of the gauge has shown within what limits they are trustworthy if certain precautions are attended to, the chief being that sufficient time should be given for the equalization of pressure throughout the tubes and bulb of the pump. Unless the connecting tubes are of large diameter the time required, when the pressure is of the order of 0·1—0·01, is very great, something like 20 to 30 minutes, and it is consequently probable that if the pump is worked at a rate greater than one stroke in 20—30 minutes, the relative proportions in a mixture of permanent gases will be altered considerably by the inevitable differences in the rates of transpiration of the components. The connecting tubes used in my experiments are about 2 yards long and have an internal diameter of about $\frac{1}{4}$ inch.

14. Striking effects may be observed if the pressure of the phosphorescent gas be altered whilst the gas is actually phosphorescing after a bright discharge has been passed. Thus suppose the pressure of the gas is higher than that at which the maximum brightness for the given mixture occurs; then if during phosphorescence the pressure is reduced, a wave of increased brightness passes from the end of the tube near the pump to that remote from it.

Pressure-glow, or luminosity accompanying compression.

Up to this point the present note deals with phosphorescence excited electrically and I now turn to the observations which suggested the title of the note.

15. The Hagen-Töpler pump and tubes were filled with a mixture of gases capable of exhibiting the yellow phosphorescence that I attribute to oxygen with a trace of nitrogen in it. The pressure was reduced below the value for which electrically excited phosphorescence was a maximum for that mixture, namely below $0^{mm}{\cdot}4$ or $0^{mm}{\cdot}3$. Then the gas in the bulb of the pump itself was compressed in the usual process of pumping, preparatory to its being driven out of the pump. As the compression proceeded, the gas was observed to become luminous throughout that part of the pump-bulb which was not occupied by the rising mercury. Its brightness increased considerably and then gradually faded away. The maximum brightness was enough to enable one to read ordinary small print with ease. The amount of compression necessary for maximum brightness was found to depend upon the initial pressure.

16. After the gas has been compressed and has given out the 'pressure-glow,' then if it be allowed to return to its original large volume and low pressure and be again compressed in the pump-bulb, the gas does not glow, unless a certain operation has taken place at low pressure; it consists in (i) lowering the mercury in the pump-bulb enough to open connection between the pump and the tube used for the electrodeless discharge, and (ii) producing an electrical discharge in this tube.

17. It would appear that before compression the gas can be put into a state of combination or grouping, which is stable at the lower pressure; and when compression has raised the pressure to a certain value, the original grouping of atoms or molecules is no longer stable and the gas passes into a new state of grouping with an evolution of energy, part of which would account for the 'pressure-glow.' In some way the electrodeless discharge supplies the energy necessary to the formation of the combination which is stable at low pressure. Inasmuch as the mixture of gases, in which pressure-glow occurs, consists of oxygen with only a trace of nitrogen (and possibly some other gases which would ordinarily be treated as impurities), it immediately occurs to one that we have to deal with the formation of ozone at the low pressure. It is however not clear what part is played in the phenomena by the impurities, though it seems certain that their presence is of importance, if not essential for the production of the phosphorescence.

18. With reference to the amount of compression necessary for maximum brightness in the pressure-glow, a number of experiments were made under varied conditions and the results were found consistent with the idea that the maximum brightness of the compressed gas occurred at a pressure between $0^{mm}{\cdot}5$ and $0^{mm}{\cdot}3$, fairly definite for a given mixture of gas. If the initial pressure of the rarified gas in the pump-bulb were small (e.g. $0^{mm}{\cdot}01$) the pressure-glow did not begin to take place with any brilliance until such a diminution of volume (e.g. about 20 or 30 fold) had been brought about as would increase the pressure to within the limits above stated. If, on the other hand, the initial pressure was comparatively high (e.g. $0^{mm}{\cdot}1$), then a much smaller diminution (two or three fold) of volume was required to produce the afterglow.

19. By regulating the rate at which compression was produced the brightness and duration of the glow could be controlled to a certain extent. The phenomena observed in this connection were all in conformity with the idea that in a given mixture and quantity of gas in the pump-bulb a definite amount of energy was to be got rid of by radiation for a given alteration of pressure, and that the time-integral of the intensity of the pressure-glow was approximately constant.

20. Attempts were made to obtain the pressure-glow without any electrical excitation like that described in § 16. The apparatus was left in complete working order one evening, and the next morning (i.e. after an interval of 15 or 16 hours), the rarified gas was compressed, but there was no glow:—that is to say, the energy requisite for the formation of the low-pressure-compound had not passed into the gas.

A strong beam of light was passed through the gas, in the hope that this would communicate the requisite energy. But there was no glow.

Electrical excitation was applied (as described in § 16), and the pressure-glow was as bright as before.

21. The actual distance between the tube for the electrodeless discharge and the pump-bulb was not more than 18 inches, but the connecting tubes were about 6—7 feet long. Very few electrodeless discharges in the tube were enough to put the gas in the pump-bulb into the state requisite for the production of the pressure-glow. The same gas was used again and again with apparently no uncertainty in the phenomena exhibited.

22. In order to make the conditions more simple, I surrounded the pump-bulb itself with a coil of thick wire and attempted to produce the electrodeless discharges in the pump-bulb: but though I have generally found it easier to produce the

discharges in large bulbs than in small tubes, I could not get the discharge to pass. The failure was attributed to the fact that the inner walls of the bulb were always wet with strong sulphuric acid, some of which floated on the surface of the mercury.

23. It may be noted here that the mercury in the pump was connected with earth through wires and the gaspipe system. This fact constitutes a difficulty in the way of explaining the pressure-glow as simply a phenomenon of electrical origin in which the compression plays a secondary part.

24. The explanation of the pressure-glow appears to me rather to be in general agreement with the explanation given by Sutherland (*Phil. Mag.* 1897, Mar.) of anomalies observed by Bohr and Crookes in the compression and rarification of oxygen. According to Sutherland, in the rarification of oxygen a point is reached (pressure $0^{mm}{\cdot}7$) when oxygen begins to be converted into ozone, and below the pressure $0^{mm}{\cdot}15$ the gas is entirely ozone. Between the pressures 15^{mm} and $0^{mm}{\cdot}7$ the gas obeys Boyle's Law as oxygen, and below $0^{mm}{\cdot}15$ it obeys Boyle's Law as ozone. Between the pressures $0^{mm}{\cdot}7$ and $0^{mm}{\cdot}15$ the gas is a mixture of oxygen and ozone. Sutherland is led to the view that in the compression of ozone from the lowest pressures, the ozone begins, when the pressure reaches the value $0^{mm}{\cdot}15$, to be knocked to pieces in virtue of the frequency of collision between ozone molecules being the same as that of some natural vibration in the molecule.

25. If Sutherland's views be correct in the case of ozone and oxygen, it is probable that there must follow a generalization of them to cases where mixtures of different gases are concerned.

[Since the subject of this note was communicated to the Society, an abstract has appeared of a paper by Prof. Threlfall and Miss Martin (*Nature*, Vol. 56, p. 288) in which an experiment was made with the object of investigating whether oxygen can form ozone simply by virtue of a reduction of pressure, and the authors come to the conclusion that either no ozone is formed, or if such formation does occur, it is to an extent less than 0·005 per cent. of the volume of the gas employed. The nature of the indicator is not described in the abstract referred to.]

Spectrum of the 'pressure-glow.'

26. The spectrum of the gas glowing during compression is very marked and consists of a faint continuous spectrum with four conspicuous bright bands coinciding with those which Schuster has described as belonging to the spectrum of the negative glow of oxygen.

27. The contrast of this spectrum with that of the electrically excited phosphorescence (§§ 6, 7) is very striking. In one case of the electrical phosphorescence the oxygen bands were seen, under circumstances however which led me to believe that they were really due to compression. The pressure in the tube for the electrodeless discharge was so low that the electrically excited phosphorescence was very faint, but whilst the phosphorescence was still persisting, more gas was let out of the reservoir into the pump and tube. Without any fresh electrical excitation, the phosphorescence increased in brilliance and the oxygen bands were seen in the spectrum. In no other cases of electrically excited phosphorescence have I seen any but a continuous spectrum.

It may be that the difference in the spectra in the two cases will suggest further investigation. I am not able to offer any explanation.

It is to be regretted that the investigation of the pressure-glow is made so difficult by the apparent importance of what may be described as impurities. I am not able to recover with any certainty the mixture of gases capable of exhibiting the pressure-glow, and can only say that oxygen with traces of nitrogen and SO_2 is the mixture I should begin upon, if I wished to recover the conditions that obtained when I made the experiments about two years ago.

Possible bearing of the phenomena of pressure-glow on astronomical matters.

28. The origin and mechanism of the luminosity of nebulæ and the persistence of their general outlines and details are subjects that have occupied the attention of astronomers at various times; and the idea of necessarily elevated temperature is losing ground.

The mere fact that a rarified gas can be got into a state such that compression causes it to glow and emit a band spectrum is suggestive.

If we admit a slow condensation going on in a gravitating mass of gas, whether it be gathered into globular form or irregularly extended like the nebula in Orion or surrounding stars like the Pleiades, then we should be led by the phenomena of the pressure-glow described in this note to conceive of the possible existence of a surface or layer of luminous matter, *outside* which luminosity would be inappreciable because the pressure was too low, and *inside which* the pressure was too high for luminosity of the kind considered. The gradual streaming in of rarified gas through the region when the conditions are favourable to the production of pressure-glow would account for the maintenance of the luminosity and the persistence of outline.

Monday, 10 *May*, 1897.

PROFESSOR LIVEING, VICE-PRESIDENT, IN THE CHAIR.

The following Communications were made to the Society:—

(1) *Observations on Stomata by a new method.* By FRANCIS DARWIN, M.A.

I.

Our knowledge of the behaviour of stomata is defective principally for want of good methods. The earlier workers simply stripped off the epidermis and examined the stomata microscopically. This plan is obviously untrustworthy and has been replaced by a better method, *i.e.* cutting surface sections. This is less inaccurate but is still open to the objection that it does not give evidence of the condition of the stomata in the uninjured leaf. Where the stomata are large it is possible with a microscope to see whether they are open or shut in the uninjured leaf; this is the method followed by Kohl*. The chief objection to this plan is that it can only be used in certain selected plants, *e.g.* Trianea, Caltha, and that it is laborious and requires considerable practice. Lastly, there is the method of N. Müller†, in which the condition of the stomata is estimated by the rate of flow through them of a stream of air.

Another set of methods are indirect in character and depend on the fact that in aerial leaves the loss of watery vapour takes place almost entirely by the stomata and not by the cuticle. In a leaf with stomata on the lower surface only, transpiration is all but confined to that surface. Such a leaf placed between two glass plates will bedew the lower but not the upper plate. Various methods have been founded on this basis.

Merget‡ used paper soaked in mixture of certain salts of palladium and mercury, the paper is applied to the surface of the leaf and becomes black or brown owing to the moisture coming off from the stomatal side.

* *Botanisches Beiblatt zur Leopoldina*, 1895.
† *Pringsheim's Jahrbücher*, Vol. VIII.
‡ *Comptes rendus*, 1878.

In 1894 Stahl* introduced his cobalt method, in which he took advantage of the fact that paper impregnated with cobalt chloride is blue when dry, and becomes pink in damp air. Thus if dry cobalt paper is placed on each surface of a hypostomatal leaf, the paper rapidly changes to pink on the lower side, but remains blue on the upper surface. Whereas when the stomata are shut no such difference is seen. With this method Stahl has made out a number of most interesting facts. It is moreover a method especially applicable to demonstration and lecture purposes, since the effect is rapidly obvious, and clearly visible from a little way off.

In the *Practical Physiology*, 1894, of Mr Acton and myself, I described a rough hygrometer in which the index was the awn of Stipa, and which I used to demonstrate that transpiration is ordinarily stomatal rather than cuticular.

The method which I have to describe is like Merget's and Stahl's papers, and like my Stipa hygrometer, an indirect one, depending on the yield of watery vapour from the stomata.

Two or three years ago I came across a toy consisting of the figure of a man cut out of a thin sheet of horn prepared in such a way as to be extremely hygroscopic. When placed on a damp surface it writhes and twists, curving away from the source of moisture. The toy had lain hidden in an old house for something like 100 years; it was made for fortune-telling purposes, the degree of movement on the hand indicating the disposition of the patient. The material was described as Chinese sensitive leaf; it is mentioned in Holtzappfel's *Turning and Mechanical Manipulation*, Vol. I., p. 123, 1846, as made from pressed horn. Similar toys in the form of fishes were described by Maria Edgworth, 1808, and appear to have come from Japan. Figures of women and also of fish, made apparently of gelatine or gold-beaters' skin, are still sold for fortune-telling. But these are not so good for my purpose as horn sheets. Being unable to obtain any more of my original material I made use of shavings of horn and for a long time despaired of making them sufficiently sensitive. At last a chance observation showed me that the horn shavings are rendered sensitive by heat.

The method of using the material is as follows. A strip of horn sheet 8 or 9 mm. long by 3 or 4 wide is fastened by one end to a small block of cork so that the horn lies flat on any surface on which the instrument is placed: the horn bears at its free end a bristle to serve as an index. When it is placed on a damp surface the index instantly rises to an angle of 20° to 50° or even more, whereas it remains flat on a dry surface.

* *Bot. Zeitung*, 1894.

A paper-scale being fixed to the cork block serves to read off the movement: when applied to leaves, for instance a hypostomatal leaf, the reading is zero on the upper side and varies on the other side according to the state of the stomata. It has been hitherto impossible to get any physical measure of what the readings indicate; from a large experience of various leaves it may be said that 50° means an extreme amount of transpiration, 30° fair degree, 10° a small degree.

It is sometimes difficult to know how far the readings of the hygroscope can be taken to indicate the state of the stomata. Thus for instance in variegated leaves I get low readings on the white parts, high readings on the green parts. Is this due to partial closure of the stomata or to there being fewer stomata on the white patches, or is it because the absence of chlorophyll diminishes the yield of vapour? If the method fails in cases like this it is nevertheless applicable in broader, less critical issues. For instance, all microscopic observations from the time of Mohl agree that certain leaves close their stomata when they wither. It cannot be doubted therefore that the drop in the hygroscope readings in withering leaves corresponds with closure: the same argument applies to the effect of darkness. By taking readings on the uninjured leaf of Caltha and by looking at the stomata I have convinced myself that there is a rough agreement between the hygroscopic and microscopic methods. But I generally get a low hygroscope reading even when stomata look quite shut.

If we may believe that broad differences in hygroscope readings represent differences in the openness of the stomata, we must not assume that where the hygroscope stands at zero there is absolutely no loss of vapour from the leaf. Thus there is no doubt that leaves lose some water by cuticular transpiration, yet the hygroscope always reads at zero on a surface free from stomata. Also a leaf whose stomata are apparently shut as indicated by the hygroscope, continues to lose weight by evaporation.

Since the movement of the index depends on the difference between the moisture of the air on the two sides of the horn plate it might be supposed that a given surface would give very different readings in damp and dry air. But practically this is not so. I used a porous brick standing in a shallow layer of water; on the brick I placed a sheet of perforated zinc and on this the hygroscope; readings were taken in the dry air of the laboratory, in the less dry succulent house and in the various plant-houses, and I got no very serious differences in the readings.

Another source of error is that the horn plate is affected by heat, *i.e.* it curves away from a hot surface. This makes it difficult to experiment on the effect of heat, but otherwise is unimportant.

It is impossible to make two instruments identical, so that the readings of one instrument are not strictly comparable with another. The chief use of the hygroscope, and for this it is admirably suited, is to compare the leaves of an experimental plant with those of a control, or to observe sudden or gradual changes of reading on a given leaf.

II.

In a forthcoming paper I shall give a full account of the results which I have obtained. In the present publication I give a brief statement of the facts which may be easily demonstrated with the hygroscope, together with some results which are believed to be new.

1. The hygroscope index remains at zero on the upper surface of a hypostomatal leaf, indicating that there is a low degree of transpiration from that surface.

2. There is active transpiration on the lower surface of a floating leaf (*e.g.* Nymphaea) when exposed to air.

3. In a terrestrial leaf with stomata on both surfaces, the behaviour of the stomata is different on the two surfaces. Thus the upper stomata are more sensitive to external conditions and are frequently shut when the lower ones are opened.

4. The typical stoma closes (either wholly or partially) in darkness. The process is not a sudden one, the readings of the hygroscope fall gradually to zero. It is difficult to generalise as to the time which closure in darkness or opening in light requires; half-an-hour would represent fairly rapid effect.

5. The natural diurnal course of the stomata can be easily followed by taking readings throughout the day and the behaviour of the stomata thus graphically represented. The rise in the early morning and the fall at night are both rapid, the intermediate region of the curve may be either more or less flat, or it may present a continuous rise and fall.

6. A certain degree of closure at night is almost universal in typical terrestrial plants and is generally absent in water-plants and marsh-plants *. Since the hygroscope does not distinguish between partial and complete closure, these results are not necessarily at variance with those of Leitgeb †.

7. The nocturnal closure is a periodic phenomenon: this is not so strongly marked as the nyctitropic movements of leaves, since the stomata of a leaf placed in a dark room at night are not found to be open in the morning.

* *Caltha* and *Trianea* however close at night.

† *Mittheilungen aus dem Bot. Institut zu Graz*, 1886.

The existence of periodicity may be best demonstrated by the method employed by Pfeffer in observing the "sleep" of flowers, namely, by seeing that they behave differently to a given change of temperature at different times of day. The case is the same for stomata. In the morning a much longer period of darkness is required to close the stomata than is necessary at night: on the other hand stomata which are closed, open in the light much more easily in the morning than in the late afternoon.

8. The well-known result that the stomata close when a leaf is cut from the plant and allowed to wither is strikingly demonstrable with the hygroscope, the readings sinking gradually to zero.

In this way the fact observed by Stahl can be confirmed, namely, that the leaves of marsh and water-plants do not, as a rule, close their stomata on withering.

The closure of the stomata produced by watering plants with dilute NaCl solution has also been confirmed *.

9. In some cases the process of withering is strikingly different. In *Pointsettia, Tropaeolum, Campanula vidalii* and some other plants the closure of the stomata is preceded by well-marked opening—as the following figures will show.

hr	min	hygroscope
4	10	= 10
	12	= 10
	12·2	leaf cut off
	13	= 18
	14	= 15
	15	= 22
	18	= 20
	23	= 15
	25	= 10
	27	= 12
	53	= 0

The result is of some importance as bearing on the question of mechanism.

The effect of cutting off the water supply must be to diminish turgescence—and diminished turgescence of the guard cells must mean closure. Therefore the *opening* of the stomata must be due to the loss of turgor in the other elements of the leaf, especially of the ordinary epidermic cells—the final closure being due to the withering of the guard cells †. This effect is especially well seen in plants with milky juices, *e.g. Campanula*

* See Schimper, *Indo-Malayische Strand Flora*, 1891.

† On this subject see Leitgeb, *loc. cit.*

vidalii, Pointsettia. I imagine that the laticiferous vessels supply, as it were, an efficient system of drainage by which a large quantity of fluid can at once escape, and therefore quickly alter the internal pressure in the tissues.

My work also bears on the question of mechanism in another way. It has been assumed, I believe, incorrectly, that light opens the stomata because the assimilation of the guard cells leads to osmotic material being built up; the closure in darkness being due to loss of such osmotic matter.

Schellenberg* has sought to uphold this view by placing leaves in air free from CO_2, where he finds that the stomata are shut. In none of my experiments did I see any signs of closure being due to absence of CO_2.

But in any case, another of my results is unfavourable to this view. Many plants reopen their stomata if they are kept continuously in the dark for several days. Whatever is the explanation of this fact it is clearly not due to returgescence of the guard cells owing to assimilation. This fact was first observed by my assistant, Mr Elborn, to whom I am indebted for much valuable help.

(2) *Notes on hybrid Cinerarias produced by Mr Lynch and Miss Pertz.* By W. Bateson, M.A., St John's College.

It is stated by many writers that the garden Cineraria arose as the hybrid offspring of several species of *Senecio* from the Canary Islands. This statement has been questioned by Mr Thiselton Dyer on various grounds. The author exhibited hybrids raised from *S. cruentus*, *S. multiflorus* and *S. Heritieri* (= *lanatus*) raised in the Cambridge Botanic Gardens by Mr Lynch and Miss Pertz which illustrated the great variability which appears in the offspring of the various crosses. In particular, specimens of *Heritieri* ♀ × *cruentus* ♂ and of the reciprocal cross were produced, showing excessive variability and proving how greatly the peculiar characters of *Heritieri* may be obscured in the offspring, even of the first cross. Five specimens of *multiflorus* ♀ × *Heritieri* ♂ were exhibited, each of which was quite distinct from the rest. Experiments had entirely confirmed Darwin's observation that Cinerarias are self-sterile in a high degree. They hybridize on the contrary with great readiness. An accidental hybrid between *Heritieri* ♀ × garden Cineraria ♂ and the reciprocal were also shown, the two plants being quite unlike each other, the former being an upright plant of a rather woody habit, resembling poor specimens of the garden Cineraria, while

* *Bot. Zeitung*, 1896.

the reciprocal (illustrated by two specimens of independent origin) was a struggling plant with large pale flowers having almost a procumbent habit, differing considerably from both its parents. One seedling *multiflorus* ♀ × garden Cineraria ♂ had been produced which was almost entirely female, a few anthers only appearing in later inflorescences. These experiments were to be continued, but so far as they had gone they were entirely consistent with the view that the Cineraria was a hybrid between several species, *cruentus*, *Heritieri* and probably *multiflorus* being among them. The two first are named by most authorities as probable parents.

It was proposed to publish details of the experiments when those of the next year should be completed.

Monday, 24 *May*, 1897.

MR F. DARWIN, PRESIDENT, IN THE CHAIR.

The following were elected Honorary Members of the Society:

Major Percy Alexander MacMahon, R.A.
Prof. Charles A. Young, Princeton.
Prof. Albert A. Michelson, Chicago.
Dr Ludwig Boltzmann, Vienna.
Prof. Augusto Righi, Bologna.
Prof. Dimitri Ivanowitch Mendeleeff, St Petersburg.
Sir Archibald Geikie.
Prof. Edward Salisbury Dana, New Haven, Conn.
Sir John Kirk.
H. H. Albert, Reigning Prince of Monaco.
Rev. Canon Alfred Merle Norman, Durham.
Prof. Wilhelm Pfeffer, Leipzig.

The following Communications were made to the Society:

(1) *The relationship of* Amphioxus *and* Balanoglossus. By E. W. MACBRIDE, M.A., St John's College.

In seeking for some light as to the origin of the great Vertebrate phylum, the only safe course is to examine with great care the anatomy and development of the least differentiated forms known to us which show traces of Vertebrate structure.

The simplest animal which is universally recognised as a Vertebrate is Amphioxus, and the study of its anatomy and development has thrown a flood of light on the meaning and origin of many of the organs in the higher Vertebrates. If however we desire to trace the Vertebrates further back than Amphioxus, we find no consensus of opinion as to the direction in which we should prosecute our search. Dohrn supposes that we ought to go in the direction of the marine Annelid worms. This Society has recently had presented to it by a distinguished fellow, an elaborate paper, pleading for the affinity of Vertebrates with the Arthropoda; whilst, as we all know, Bateson's view, that Balanoglossus is the lowest animal with traces of Vertebrate structure, has been widely accepted here, though it has not met with the same favour in Germany.

It must be admitted that Balanoglossus is about as unlike a Vertebrate externally as one can well conceive, and it is difficult to imagine that the dominant Vertebrates should have taken their origin from a mud-inhabiting worm. One of the most prominent features in the organization of the lower Vertebrates is the division of their muscles into segments, a feature which they share with the Annelids, and to which it is hardly possible to compare the division of the body of Balanoglossus in three regions —head, collar and trunk. The main object of this essay is to show that there exists in the Amphioxus larva an analogous division of the body which precedes in order of development the segmentation of the muscles.

In a former paper on this subject, I showed that the cœlom in Amphioxus originated at three distinct points from the alimentary tube.

(1) At the dorsal lateral corners a pair of hollow ridges become constricted off from the rest of the gut, and then by the obliteration of their lumen at various points, divided into a series of distinct sacs. These processes go on progressively as the animal grows in length—the ridges continuing to communicate with the gut behind, long after they have been completely separated from it and divided into segments in front.

(2) A pair of pouches grows out of the gut in front of the foregoing and nearer the mid-dorsal line, these give rise to the first pair of muscular segments and to other structures as well.

(3) The most anterior portion of the alimentary canal becomes separated off from the rest, and becomes immediately divided into a pair of cavities of which the right becomes large and thin-walled whereas the left becomes a small thick-walled sac which acquires an opening to the exterior.

Now these sacs practically give rise to all that is contained between the skin and gut of Amphioxus with the exception of the notochord and the nervous tube. In the paper referred to I compared (3) to the head cœlom of Balanoglossus, and (2) and (1) to the collar and trunk cœloms respectively of that animal.

It is *à priori* improbable that any form exactly like the ancestor of a living animal has survived to this day. For *ex hypothesi* such a type of structure has been found to be unsuited to the environment and has been modified by the struggle for existence. If, however, whilst some of the ancestors of Vertebrates were undergoing evolution into higher forms under the stress of competition, others betook themselves to situations, like the mud, where they could escape the brunt of the struggle, such animals would preserve the general level of ancestral structure in a more or less degenerate form with special adaptations to a burrowing life, and this we suppose Balanoglossus to have done.

In the higher Vertebrates the homologue of the head cavity has been detected in that bilobed head cavity which in the embryos of fish underlies the eyes, and from whose walls the majority of the eye muscles are formed, but the homologue of the collar cavity has not yet been sought for and naturally has not yet been discovered.

In my former communication on this subject I alluded to a paper by Lwoff on the development of the germinal layers in Amphioxus. In this paper Lwoff not only maintains that the cœlom or body cavity does not originate from the alimentary canal, but that the whole mesoderm and the notochord itself originate from an ingrowth of the ectoderm. If this were true any possible comparison of Balanoglossus and Amphioxus would be rendered impossible; it would be a result of the most far-reaching character. It may be remarked that it used formerly in many cases to be asserted that the ectoderm gave rise to the mesoderm where later renewed research has shown that the mesoderm arose from the alimentary tube.

I have now further evidence to offer in favour of this view. As I have said, the trunk cœlom becomes divided into separate muscular segments, these later become differentiated into a dorsal muscular and a ventral thin-walled portion; and then the dorsal portions become completely separated from the ventral parts, and simultaneously the latter become fused with one another to form two longitudinal cœlomic canals.

What I have designated as the collar coelom undergoes also differentiation into a dorsal muscular and a ventral thin-walled portion. The two do not however for a long time become constricted off from one another; and even when this occurs the

ventral part does not become confluent with the ventral part of the trunk cœlom. The ventral part of the collar cœlom grows back externally to the trunk cœlom, forming two ventral lateral ridges which reach some little distance behind the last gill-slit which is formed. These ridges, which are traceable as soon as the mouth and first gill-slit have been formed, are really the first rudiments of the atrial folds, which later hang down over and cover the gill-slits. The cœlom contained in these ridges gives rise to the ventral muscle of the atrial cavity. Now in Balanoglossus the hinder end of the collar extends over and covers the first gill-slit, and the suggestion of Bateson that this represents the first beginning of the formation of an atrium like that of Amphioxus is borne out by the facts I have just narrated.

I have said above that it was difficult to suppose that active Vertebrates were descended from a sluggish worm like Balanoglossus. It is however unnecessary to make such a supposition: when zoologists point to one existing animal as "representing" the ancestor of another, they often use loose and inaccurate language.

The evidence that Lwoff brings forward in support of his position is exceedingly feeble. The first shape which the developing egg of Amphioxus assumes is that of a hollow sphere—the so-called blastula: this then becomes converted into the well-known gastrula by the process of invagination, one half being pushed inside the other. Now Lwoff maintains that in the blastula certain cells are distinguished from the rest by their size, and that these alone represent the endoderm of other animals, they are first invaginated, and then the invagination of cells which are really ectoderm succeeds to this. Considering that the invagination is a continuous process which proceeds regularly to its completion, and that when it is completed no difference in the character of the invaginated cells can be detected, it will be seen that this is a most violent and arbitrary interpretation of the facts, which could only be entertained if it were shown that in the blastula the endoderm was clearly and sharply defined. I have cut about a hundred of invaginating blastulæ to examine this point and can meet it with a direct denial.

It is true there are certain cells of the blastula distinguished by their larger size from the rest, but these merge gradually into the rest at the sides, and where the process of invagination just shows itself the smallest cells are found. This shows that the size of a cell depends on the rapidity with which division is going on, and as the process of invagination is connected and in fact caused by division and multiplication, it is natural that smaller cells should be found at the edges of the area which is about to be invaginated.

As a matter of fact, although these small cells just appear at the spot which will become the dorsal lip of the blastopore, where Lwoff imagines the ectodermal invagination which is to give rise to the notochord to take place, it is the other lip which in the later stages exhibits it most clearly.

It is true that in those higher Vertebrates in which the process of invagination can be clearly traced, the cells composing the upper wall of the primitive alimentary tube are small compared to those forming the under part of the wall, these being choked with yolk; and great stress is laid by Lwoff on this fact. The obvious and simple explanation is, that the processes of folding, which the dorsal wall has to undergo in order to give rise to the notochord and the mesoderm, would be rendered impossible were the cells composing it to be overcharged with yolk, and hence the yolk is confined to the cells forming the ventral wall of the gut.

(2) *On the degree of the Eliminant of Two Algebraic Equations.* By R. LACHLAN, Sc.D., Trinity College.

The object of this paper is to show how the degree of the eliminant of two algebraical equations may be determined in a very simple manner by geometrical considerations. So far as I am aware the only method given in the ordinary text-books is that given by Serret,—*Cours d'Algèbre supérieure*, 4th edit. vol. 1, §§ 278—280, which method is due to Minding. Although not difficult to apply to particular cases, it does not seem to be so simple as that here given.

1. Consider two curves C_m, C_n whose equations expressed in homogeneous coordinates are

$$f_m(x, y, z) = 0,$$

$$f_n(x, y, z) = 0.$$

If z be eliminated from these two equations we obtain an equation of degree mn, say

$$F_{mn}(x, y) = 0,$$

which represents the lines connecting the point $x = 0$, $y = 0$ to the mn points of intersection of the curves C_m, C_n.

Now of these mn points some will usually be found on the lines x, y, z forming the triangle of reference, and some will

usually be coincident with the vertices of the triangle. Excluding all these points let us suppose that there are r points of intersection which do not lie on the lines x, y, z; and let $\phi_r(x, y) = 0$ be the equation which represents the line connecting these points to the point $x = 0, y = 0$.

Then $\phi_r(x, y) = 0$ is what is usually called the z-eliminant of the equations

$$f_m(x, y, z) = 0, \quad f_n(x, y, z) = 0.$$

2. It is obvious from this method of forming the eliminant of two equations, that, whichever one of the three variables be eliminated from two homogeneous equations, the result is of the same degree.

This fact may often be used with advantage.

As an example, suppose we require the degree of the eliminant when x is eliminated from two equations of the form

$$u_m + \lambda u_p = 0,$$

$$u_n + \lambda u_q = 0,$$

where u_m, u_p, u_n, u_q denote functions of x, y of degrees m, p, n, q.

Here if λ be eliminated the eliminant is

$$u_m u_q - u_n u_p = 0,$$

and is of degree equal to the greater of the two numbers $m + q$ or $n + p$.

Hence, if x be eliminated from the two equations the eliminant will contain λ in the degree $m + q$ or $n + p$, whichever is the greater.

Similarly, it follows that the discriminant of

$$u_m + \lambda u_p = 0,$$

will contain λ in the degree $m + p - 1$.

3. The degree of the eliminant of any two equations is to be found by arranging the equations in a homogeneous form, introducing if necessary other variables. The equations are then to be considered as representing curves, and the number of points of intersection which lie on the lines forming the triangle of reference has to be determined. The degree of the eliminant is equal to the product of the orders of the curves diminished by the number just determined.

To illustrate the method let us take the example given by Serret. He takes the equations

$$(x, 2)\, y^4 + (x, 2)\, y^3 + (x, 4)\, y^2 + (x, 5)\, y + (x, 5) = 0,$$

$$(x, 8)\, y^5 + (x, 6)\, y^4 + (x, 9)\, y^3 + (x, 4)\, y^2 + (x, 3)\, y + (x, 4) = 0,$$

where (x, r) denotes an integral expression of the rth degree in x.

Introducing z, so as to make these equations homogeneous, we have the equations of two curves, C_6, C_{13};

$$(x, z)_2\, y^4 + (x, z)_2\, y^3 z + (x, z)_4\, y^2 + (x, z)_5\, y + (x, z)_5\, z = 0,$$

$$(x, z)_8\, y^5 + (x, z)_6\, y^4 z^3 + (x, z)_9\, y^3 z + (x, z)_4\, y^2 z^7 + (x, z)_3\, y z^9 + (x, z)_4\, z^9 = 0,$$

where $(x, z)_r$ denotes a homogeneous expression of the rth degree.

The only common points of the curves C_6, C_{13} which lie on the lines x, y, z coincide with the points $x = z = 0$, and $y = z = 0$.

In fact, at the point $x = z = 0$,

C_6 has two branches, given by equating the coefficient of y^4 to zero,

C_{13} has eight branches, given by equating the coefficient of y^5 to zero.

Hence at this point there are 2×8 points of intersection of the curves.

Again, at the point $y = z = 0$,

C_6 has one branch, given by equating the coefficient of x^5 to zero,

C_{13} has two branches of the form

$$y^2 + axz = 0 \text{ and } y^3 x^5 + 6z^8 = 0.$$

Hence at this point there are 4 points of intersection of the curves.

Hence the order of the eliminant of the equations

$$= 6 \times 13 - 2 \times 8 - 4 = 58,$$

which agrees with Serret's result.

4. The following examples are of some importance.

Ex. 1. If θ be eliminated from the equations

$$\frac{x_1}{a_1 + \theta} + \frac{x_2}{a_2 + \theta} + \ldots + \frac{x_n}{a_n + \theta} = 1,$$

$$\frac{A_1 x_1}{a_1 + \theta} + \frac{A_2 x_2}{a_2 + \theta} + \ldots + \frac{A_n x_n}{a_n + \theta} = 1,$$

the degree of the eliminant is $n - 1$.

Introducing ξ, η to make the equations homogeneous, we may write them in the form

$$C \equiv \frac{\xi x_1}{a_1\eta + \theta} + \frac{\xi x_2}{a_2\eta + \theta} + \ldots + \frac{\xi x_n}{a_n\eta + \theta} - 1 = 0,$$

$$C' \equiv \frac{A_1\xi x_1}{a_1\eta + \theta} + \frac{A_2\xi x_2}{a_2\eta + \theta} + \ldots + \frac{A_n\xi x_n}{a_n\eta + \theta} - 1 = 0,$$

C and C' are curves of the nth degree, having n common points on the line $\xi = 0$, viz.

$$a_1\eta + \theta = 0,\ a_2\eta + \theta = 0,\ \&c.,$$

and $(n-1)$ branches passing through the point $\eta = \theta = 0$, and no branch of one curve touches a branch of the other.

Hence the order of the eliminant is

$$n^2 - n - (n-1)^2 = n - 1.$$

This result may also be inferred from § 2, for if ξ be eliminated, the result is obviously of degree $(n-1)$ with respect to η and θ.

Ex. 2. The discriminant with respect to θ of the equation

$$\frac{x_1}{a_1 + \theta} + \frac{x_2}{a_2 + \theta} + \ldots + \frac{x_n}{a_n + \theta} = 1,$$

is of the degree $2(n-1)$.

We have to find the degree of the eliminant of

$$\frac{\xi x_1}{a_1\eta + \theta} + \frac{\xi x_2}{a_2\eta + \theta} + \ldots + \frac{\xi x_n}{a_n\eta + \theta} = 1,$$

$$\frac{x_1}{(a_1\eta + \theta)^2} + \frac{x_2}{(a_2\eta + \theta)^2} + \ldots + \frac{x_n}{(a_n\eta + \theta)^2} = 0;$$

while the latter equation represents $2(n-1)$ lines through the point $\eta = \theta = 0$, the degree of the eliminant must be $2(n-1)$ (§ 2).

Ex. 3. If θ be eliminated from the equations

$$\frac{x_1}{(a_1 + \theta)^p} + \frac{x_2}{(a_2 + \theta)^p} + \ldots + \frac{x_n}{(a_n + \theta)^p} = 1,$$

$$\frac{A_1x_1}{(a_1 + \theta)^q} + \frac{A_2x_2}{(a_2 + \theta)^q} + \ldots + \frac{A_nx_n}{(a_n + \theta)^q} = 1,$$

the degree of the eliminant is $np - q\ (p > q)$.

This is obvious by eliminating ξ from

$$\frac{\xi x_1}{(a_1\eta+\theta)^p}+\ldots+\frac{\xi x_n}{(a_n\eta+\theta)^p}=1,$$
$$\frac{A_1x_1\xi}{(a_1\eta+\theta)^q}+\ldots+\frac{A_n\xi x_n}{(a_n\eta+\theta)^q}=1.$$

Ex. 4. If θ be eliminated from the equations

$$\frac{x_1}{(a_1+\theta)^p}+\frac{x_2}{(a_2+\theta)^p}+\ldots+\frac{x_n}{(a_n+\theta)^p}=0,$$
$$\frac{A_1x_1}{(a_1+\theta)^q}+\frac{A_2x_2}{(a_2+\theta)^q}+\ldots+\frac{x_n}{(a_n+\theta)^q}=0,$$

the eliminant is of degree $(n-1)p-q\,(p>q)$.

Introducing ξ and η to make the equations homogeneous, we may write them in the form

$$\frac{\xi x_1}{(a_1\eta+\theta)^p}+\frac{\eta x_2}{(a_2\eta+\theta)^p}+\ldots+\frac{\eta x_n}{(a_n\eta+\theta)^p}=0,$$
$$\frac{A_1\xi x_1}{(a_1\eta+\theta)^q}+\frac{A_2\eta x_2}{(a_2\eta+\theta)^q}+\ldots+\frac{A_n\eta x_n}{(a_n\eta+\theta)^q}=0.$$

If now we eliminate ξ we get

$$\frac{1}{(a_1\eta+\theta)^p}\left\{\frac{A_2x_2}{(a_2\eta+\theta)^q}+\ldots+\frac{A_nx_n}{(a_n\eta+\theta)^q}\right\}$$
$$-\frac{1}{(a_1\eta+\theta)^q}\left\{\frac{x_2}{(a_2\eta+\theta)^p}+\ldots+\frac{x_n}{(a_n\eta+\theta)^p}\right\}=0.$$

If $p>q$, this result is of the $(n-1)p-q$ degree with respect to η and θ.

Hence by § 2 the result of eliminating θ from the equations will be of degree $(n-1)p-q$ with respect to ξ and η.

Ex. 5. If x be eliminated from the equations

$$a_0x^{2m}+\ldots+a_rx^{2m-r}+\ldots+a_r'x^2+\ldots+a_0'=0,$$
$$b_0x^{2n}+\ldots+b_rx^{2n-r}+\ldots+b_r'x^2+\ldots+b_0'=0;$$

where a_r, a_r', b_r, b_r' contain y in the rth degree, the eliminant is of degree $2mn$.

Consider the curves

$$C_{2m}\equiv u_0x^{2m}+\ldots+u_rx^{2m-r}+\ldots+u_r'x^rz^{2m-2r}+\ldots+u_0'z^{2m}=0,$$
$$C_{2n}\equiv v_0x^{2n}+\ldots+v_rx^{2n-r}+\ldots+v_r'x^rz^{2n-2r}+\ldots+v_0'z^{2n}=0;$$

where u_r, u_r', v_r, v_r' are homogeneous with respect to z and y of degree r.

These two curves have the point $x = z = 0$ as a multiple point. In fact C_{2m} has m branches of the form $z^2 + kxy = 0$ at this point, and C_{2n} has n branches of a similar form.

Hence the point $x = 0$, $z = 0$ counts for $2mn$ points of intersection of the curves.

Hence the degree of the eliminant

$$= 2m \times 2n - 2mn = 2mn.$$

Ex. 6. The discriminant with respect to x of the equation

$$a_0 x^{2n} + \ldots + a_r x^{2n-r} + \ldots + a_r' x^r + \ldots + a_0' = 0,$$

where a_r, a_r' contain y in the rth degree, is of degree $2r^2$ with respect to y.

The discriminant is the same as the eliminant of the equations

$$2na_0 x^{2n-1} + \ldots + (2n-r)\, a_r x^{2n-r-1} + \ldots + r a_r' x^{r-1} + \ldots + a_1' = 0,$$

$$a_1 x^{2n-1} + \ldots + r a_r x^{2n-r} + \ldots + (2n-r)\, a_r' x^r + \ldots + 2na_0' = 0.$$

Consider then the curves

$$C_{2n-1} \equiv 2nu_0 x^{2n-1} + \ldots + (2n-r)\, u_r x^{2n-r-1} + \ldots + r u_r' x^{r-1} z^{2n-2r} + \ldots + u_1' z^{2n-1} = 0,$$

$$C_{2n} \equiv u_1 x^{2n-1} + \ldots + r u_r x^{2n-r} + \ldots + (2n-r)\, u_r' x^r z^{2n-2r} + \ldots + 2nu_0' z^{2n} = 0;$$

where u_r, u_r' are homogeneous with respect to y and z of degree r.

The only common points of C_{2n-1} and C_{2m} which lie on the lines x, y, z coincide with $x = 0$, $z = 0$.

C_{2n-1} has $n-1$ branches of the form $z^2 + kxy = 0$ passing through $x = 0$, $z = 0$; and C_{2n} has n similar branches passing through the point.

Hence the point $x = 0$, $z = 0$ counts for $2n(n-1)$ points of intersection of the curves.

Hence the degree of the eliminant

$$= 2n \times (2n-1) - 2n(n-1) = 2n^2.$$

(3) *Tides on the Equilibrium Theory.* By C. Chree, Sc.D., King's College. [Published in *Transactions*, Vol. XVI. Part II.]

(4) *Experiments on the Effect of Ultra-violet Light on the Conductivity of Iodine Vapour.* By J. Henry, Trinity College, Cambridge (1851 Exhibition Science Scholar, Queen's College, Galway).

It is well known that ultra-violet light causes negatively charged bodies on which it falls to lose their charges, but it has not been tried so far as I am aware whether ultra-violet light, like Röntgen rays, enables a gas through which it passes to conduct electricity; the following paper is an account of some experiments made to test this point, the gases used being the vapours of Iodine and Amyl Nitrite.

These vapours were used because it seemed probable that they would show this effect of light very readily if it existed, since they are sensitive to ultra-violet light in other ways*.

The apparatus used in the experiments on iodine vapour consisted of an insulated copper disc, connected to a pair of quadrants of an electrometer and supported on a stiff copper wire inside a glass tube, which was closed at one end by a quartz plate, the other end being open. This tube was fixed in a sand bath so that it could be heated to a high temperature, the closed end being at the temperature of the bath; the diameter of the tube was 3·8 cms., and the length about 20 cms. The light was given by a carbon arc and brought in a converging beam on the copper disc by a quartz lens (see fig. 1).

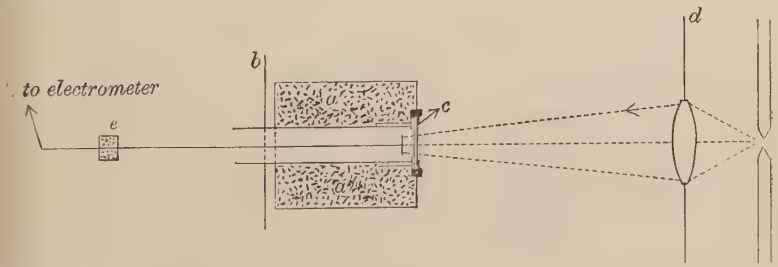


Fig. 1.

a is a sand bath.
b, a screen to protect insulated wire from hot gases.
c, a brass cap holding quartz plate.
d, a screen to cut off the scattered light.

The copper disc had to be supported on a wire insulated outside the tube at *e*, as the iodine always destroyed the insulation of any support inside, apparently by condensing on the insulating substance (discs of mica), even when its temperature was as high

* Contributions to Molecular Physics, Memoir X. Tyndall.

as 210° C., which was the temperature inside the tube during the experiments. The iodine was introduced at any time when required in the end of a small glass tube and soon surrounded the disc with vapour, which gradually escaped at the open end of the large tube, but allowed sufficient time,—three or four minutes—for the necessary observations.

Although a positively charged body is not discharged directly by ultra-violet light, yet I found that the disc, although positively charged, was discharged by the arc-light when the disc was in the tube as shown above, and also that the rate of discharge depended on the distance "x" between the disc and quartz plate.

Table I. gives the results of some experiments on this discharge, which show that it is due to the discharge of the negative electricity induced on the quartz plate by the positive charge on the disc, this induced charge becoming very small as "x" increases.

Column I. gives the distance "x" in cms.

„ II. gives the leak per minute in scale divisions for the positive charge.

„ III. gives the leak per minute in scale divisions for the negative charge.

„ IV. gives the total charge in scale divisions.

Table I.

I.	II.	III.	IV.
0·1 cm.	35	76	110
1 „	9·5	76	110
2 „	4	76	110
3 „	3	76	110

In trying the effect of the arc-light on the conducting properties of iodine vapour it was necessary to have the distance "x" small, otherwise all the effective light would have been absorbed by the vapour before reaching the disc, as was shown by experiments on the leak of the negative charge with iodine vapour present, so that it was not possible to get rid of the positive leak by increasing the distance of the disc from the end of the tube.

Table II. gives the rates of discharge of the disc for different values of the distance "x," the smallest being ·1 cm. in experiment 4; the potential of the charged disc was in each case 12 volts.

Column I. gives the sign of the charge on the disc.

„ II. gives the total charge on the disc in scale divisions.

„ III. gives the leak per minute due to light when there is no iodine present.

„ IV. gives the leak per minute due to light when there is iodine present.

By placing a plate of clear glass in the path of the light the leak whether of the positive or of the negative charge was at once destroyed.

We see by these figures that the leak when iodine vapour is present is always much less than when it is absent, and also, as was to be expected, that the negative leak diminished more rapidly with the increase of the distance "x" when the vapour was present than the positive leak did, since in the latter case the light fell on the quartz plate before being sifted of its ultra-violet rays by the vapour; the diminution of the positive leak when the iodine is present seems to show that the vapour not only does not conduct, but even tends to prevent the escape of the negative

Table II.

I.	II.	III.	IV.	
+	80	5	2	Exp. 1.
−	80	58	0	
+	85	5	2	Exp. 2.
−	85	61	2	
+	85	19	6	Exp. 3.
−	85	60	9	
+	85	27	15	Exp. 4.
−	85	63	40	

electricity from the quartz plate, this effect may however be due to the absorption in the iodine vapour of the light reflected from the copper disc, back again to the quartz plate, this light in the absence of iodine increasing the discharge from the quartz.

Owing to the leak of the positive charge, whether iodine

vapour is present or not, these experiments do not prove that there is absolutely no conduction of the charge through the vapour due to the light on it, but they do prove that if this conduction exists at all, it must be very small.

In testing amyl nitrite, the vapour was allowed to rise round an insulated copper disc connected to an electrometer as before,

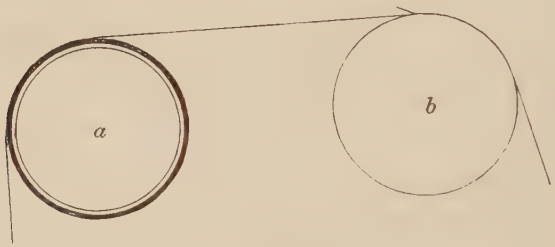


Fig. 2.

but when the disc was charged positively, the arc-light caused no discharge whatever, whether the vapour was present or not.

In another experiment on amyl nitrite, a beaker containing some of the liquid was placed in coil "*a*" (fig. 2), and a bulb for giving the electrodeless discharge in coil "*b*" which was in series with "*a*," and the number of turns in the coils adjusted, until a substance of very small conductivity placed in "*a*" destroyed the discharge in "*b*."

This arrangement showed the conductivity produced by the Röntgen rays in the air in coil "*a*," yet it gave no indication of conductivity in the vapour of amyl nitrite, when the light from an electric arc placed over the open beaker fell on the vapour.

In conclusion, I have to thank Professor J. J. Thomson for his many valuable suggestions in the course of these experiments.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

ANNUAL GENERAL MEETING.

Monday, 25 *October*, 1897.

Mr F. Darwin, President, in the Chair.

The following officers and new members of Council were elected:—

President:

Mr F. Darwin.

Vice-Presidents:

Prof. Newton, Prof. J. J. Thomson, Mr Larmor.

Treasurer:

Mr Glazebrook.

Secretaries:

Mr Newall, Mr Bateson, Mr Baker.

New Members of Council:

Mr Harker, Mr Hutchinson, Prof. Liveing, Mr Skinner.

The names of the Benefactors were read.

The following Communications were made to the Society:—

(1) *Electrical Oscillations in Wires.* By Mr H. C. Pocklington, St John's College.

1. In this paper are discussed some problems relating to the propagation of electrical oscillations along wires. The wire is always supposed to be a perfect conductor, and to have a circular cross-section, the diameter of which is small compared with the other dimensions of the system. We have therefore to solve the equations $\nabla^2(P, Q, R) = V^2 \frac{d^2}{dt^2}(P, Q, R)$, conv. $(P, Q, R) = 0$, with the further condition that at the surface of the wire the vector (P, Q, R) is perpendicular to the surface. The method of solution used is to start with the simplest solution of the general equations and by adding an infinite number of such solutions together to obtain one of sufficient generality. The arbitrary function which represents the infinite number of arbitrary constants introduced into this last solution is then found from an equation deduced from the surface condition. This last part of the work is conducted by means of approximations.

2. The simplest solution of the general equations, that corresponding to the solution $\phi = 1/r$ of the equation $\nabla^2\phi = 0$, is given by the formulae*

$$P = \frac{d^2\Pi}{dx\,dz}, \quad Q = \frac{d^2\Pi}{dy\,dz}, \quad R = \frac{d^2\Pi}{dz^2} + \alpha^2\Pi, \text{ where } \Pi = e^{i\alpha r} e^{ipt}/r,$$

in which $2\pi/p$ is the period of the disturbance, and $2\pi/\alpha\,(= 2\pi V/p)$ the wave-length corresponding in free ether to this period. This result can be expressed in words as follows. The electric force due to an elementary Hertzian oscillation with the element of length ds as axis, is compounded of two forces; the first of these is derived from a potential function $-\frac{d\Pi}{ds}$, and the second is a force $\alpha^2\Pi$ parallel to ds. This system of forces satisfies the equations of propagation of electric force everywhere excepting at the element ds. If we place an infinite number of such elements consecutively so as to form a curve, of which ds will then be an elementary arc, and attribute varying strengths λ to them, we shall obtain a system of forces which satisfies the equations of propagation everywhere except on the curve. The resulting system of forces is

$$(P, Q, R) = -\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) \int ds\,\lambda \frac{d\Pi}{ds} + \alpha^2 \int ds\,(l, m, n)\,\lambda\Pi.$$

* Hertz, *Wied. Ann.* 1889, vol. 36, p. 4; *Electrical Waves* (tr. Jones), p. 140.

If the curve is either closed or has its extremities at infinity, this is equivalent to

$$(P, Q, R) = \left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) \int ds \frac{d\lambda}{ds} \Pi + \alpha^2 \int ds\,(l, m, n)\,\lambda\Pi \ldots (1).$$

This is a general solution containing an arbitrary function λ.

3. It now remains to consider the equation derived from the surface conditions. At a point at a small distance ϵ from the curve we have, neglecting all terms that are not large,

$$\int ds\,\Pi \frac{d\lambda}{ds} = -2 \frac{d\lambda}{ds} \log \epsilon \,.\, e^{ipt},$$

and similarly for $\int ds\, l\lambda\Pi$, etc., so that, to this order of approximation,

$$P = -\left\{2 \frac{d}{dx} \frac{d\lambda}{ds} \log \epsilon - 2\alpha^2 l\lambda \log \epsilon\right\} e^{ipt},$$

and similarly for Q and R.

The component of force along the wire therefore is, to this order,

$$-2\left(\frac{d^2\lambda}{ds^2} + \alpha^2\lambda\right) \log \epsilon \,.\, e^{ipt}.$$

The force tangential to the cross-section of the wire $= 0$ to this order. Hence the system of forces given by (1) is a solution of the problem (to this order) provided that

$$\frac{d^2\lambda}{ds^2} + \alpha^2\lambda = 0 \text{ or } \lambda = e^{i\alpha s},$$

and the disturbance is propagated along the wire with velocity V and without diminution of amplitude. This is only what might have been expected from a knowledge of what happens in the case of a straight wire; for if in our case we take the electrical forces to be finite near the wire, at a finite distance they are zero.

4. It is clear that in order to obtain results of much interest we must approximate more closely. We will now consider the equations obtained by neglecting only small quantities of the first and higher orders.

As given by (1) the force at any point on the wire tangential to the axis is the same for all points on the same cross-section, and contains two terms, one containing $\log \epsilon$, the other finite. The force tangential to the cross-section is finite and varies for a given value of s as the cosine of some azimuth angle.

Suppose now that we shift the curve formed by the Hertzian elements through a small distance of the second order. The effect is to change the value of the component tangential to the cross-section by a finite amount which varies as the cosine of some azimuth angle, and that parallel to the axis of the wire by an amount of the first order of small quantities.

By making such a shift of appropriate magnitude and direction at every point, we can therefore eliminate the component tangential to the cross-section. At the same time, the component parallel to the wire is unaltered (to our order). Hence we may still derive the surface condition from (1) by taking the integration along the axis of the wire.

The condition thus obtained is

$$0 = \frac{d}{ds}\int ds\,\frac{d\lambda}{ds}\,\Pi + \alpha^2\{l\int ds\, l\lambda\Pi + m\int ds\, m\lambda\Pi + n\int ds\, n\lambda\Pi\}\ldots(2).$$

5. *Circular Ring.* The simplest case that we can consider is that of a circular ring. Let the radius of the ring be a, the radius of the wire ϵ as before, and let the axis of symmetry be chosen as the axis of z. We shall assume $\lambda = A\cos r\phi$, where ϕ defines a point on the axis of the wire. This assumption will be justified later.

At the point (ϖ, θ, z)

$$\int ds\,\frac{d\lambda}{ds}\,\Pi = -rA\int_0^{2\pi} d\phi\,\Pi\sin r\phi$$

$$= -rA\int_0^{2\pi} d\phi\,\Pi_0(\sin r\theta\cos r\phi + \cos r\theta\sin r\phi)\,e^{vpt},$$

where Π_0 is the value that Π takes when ϕ is put for $(\theta-\phi)$ and 0 for t, and is thus a function of ϕ, ϖ, z only,

$$= -2rAe^{vpt}\sin r\theta\int_0^{\pi} d\phi\,\Pi_0\cos r\phi;$$

$$\int ds\, l\lambda\Pi = -A\int_0^{2\pi} a\,d\phi\,\Pi\cos\phi\cos r\phi$$

$$= Aae^{vpt}\Big[\cos(r+1)\theta\int_0^{\pi} d\phi\,\Pi_0\cos(r+1)\phi$$

$$+\cos(r-1)\theta\int_0^{\pi} d\phi\,\Pi_0\cos(r-1)\phi\Big],$$

as above; and $\int ds\, n\lambda\Pi = 0.$

Hence (2) becomes

$$0 = \left[-2rA \frac{r\cos r\theta}{a} \int_0^\pi d\phi\, \Pi_0 \cos r\phi \right.$$
$$\left. + \alpha^2 A a \cos r\theta \int_0^\pi d\phi\, \Pi_0 \{\cos(r+1)\phi + \cos(r-1)\phi\} \right] e^{\iota pt},$$

or
$$\int_0^\pi d\phi\, \Pi_0 \cos r\phi\, (r^2 - \alpha^2 a^2 \cos\phi) = 0 \ldots\ldots\ldots\ldots (3),$$

the disappearance of θ justifying the assumption made as to the form of λ. In this equation for finding α we may give to ϖ and z in Π_0 any values which correspond to a point on the surface of the wire. The simplest values are $\varpi = a + \epsilon$, $z = 0$, and these should therefore be chosen.

6. The special case $r = 1$, which corresponds to the fundamental node of the wire, is that of the most interest and will be investigated in detail.

In this case, (3) becomes on substituting for Π_0 its value (when $z = 0$),

$$\int_0^\pi d\phi \frac{e^{\iota\alpha\sqrt{a^2 - 2a\varpi\cos\phi + \varpi^2}}}{\sqrt{a^2 - 2a\varpi\cos\phi + \varpi^2}} \{\cos\phi - \tfrac{1}{2}\alpha^2 a^2 (1 + \cos 2\phi)\} = 0,$$

or, putting $\varpi = a + \epsilon$,

$$\int_0^\pi d\phi \frac{\frac{1}{2}\alpha^2 a^2 - \cos\phi + \frac{1}{2}\alpha^2 a^2 \cos 2\phi}{\sqrt{2a(a+\epsilon)(1-\cos\phi) + \epsilon^2}}$$
$$+ \int_0^\pi d\phi \frac{e^{2\iota\alpha a \sin\frac{1}{2}\phi} - 1}{2a\sin\frac{1}{2}\phi} (\tfrac{1}{2}\alpha^2 a^2 - \cos\phi + \tfrac{1}{2}\alpha^2 a^2 \cos 2\phi) = 0 \ldots (4),$$

where in the second integral ϵ has been put $= 0$, since we are neglecting small quantities.

The first integral in (4) is, calling $2\alpha a = x$, $\log\frac{8a}{\epsilon} = L$, and neglecting small quantities,

$$\frac{1}{a}\left\{2 - \frac{x^2}{3} + \left(\frac{x^2}{4} - 1\right) L\right\}.$$

The second integral is, putting $\phi/2 = \psi$,

$$\int_0^{\frac{\pi}{2}} d\psi \frac{e^{\iota x\sin\psi} - 1}{\sin\psi} \left\{\frac{x^2}{8a}(1 + \cos 4\psi) - \frac{1}{a}\cos 2\psi\right\}$$
$$= \iota \int_0^{\frac{\pi}{2}} d\psi \frac{\sin(x\sin\psi)}{\sin\psi} \left\{\frac{x^2}{8a}(1+\cos 4\psi) - \frac{1}{a}\cos 2\psi\right\}$$
$$+ \int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x\sin\psi) - 1}{\sin\psi} \left\{\frac{x^2}{8a}(1 + \cos 4\psi) - \frac{1}{a}\cos 2\psi\right\} \ldots (5).$$

Now, n being an integer,

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} d\psi \cos 2n\psi \cos(x\sin\psi) = J_{2n}(x),$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} d\psi \cos 2n\psi \frac{\sin(x\sin\psi)}{\sin\psi} = \int_0^x dx\, J_{2n}(x)$$

$$= -2\left[J_{2n-1}(x) + J_{2n-3}(x) + \ldots + J_1(x)\right] + \int_0^x dx\, J_0(x).$$

Therefore the first integral in (5) is

$$\frac{\iota\pi}{2a}\left\{-\frac{x^2}{4}J_3(x) + \left(2 - \frac{x^2}{4}\right)J_1(x) + \left(\frac{x^2}{4} - 1\right)\int_0^x dx J_0(x)\right\},$$

and (4) becomes, on re-arrangement and multiplication by a,

$$\left(\frac{x^2}{4} - 1\right)L = \frac{x^2}{3} - 2 - \frac{\iota\pi}{2}\left\{-\frac{x^2}{4}J_3(x) + \left(2 - \frac{x^2}{4}\right)J_1(x) + \left(\frac{x^2}{4} - 1\right)\int_0^x dx\, J_0(x)\right\}$$

$$- \left\{\frac{x^2}{8}\int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x\sin\psi) - 1}{\sin\psi}(1 - 2\cos 2\psi + \cos 4\psi)\right.$$

$$\left. + \left(\frac{x^2}{4} - 1\right)\int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x\sin\psi) - 1}{\sin\psi}\cos 2\psi\right\} \quad \ldots(6).$$

This equation gives x and thence α with an error of the order of ϵ/a. It can only be solved by trial. If however ϵ is so small that errors of the order of $1/L^2$ can be neglected, we may use an approximate solution of the above. A first approximation is $x^2 = 4$ or $x = 2$. A second is obtained by putting $x = 2$ on the right-hand side of (6). This gives

$$\left(\frac{x^2}{4} - 1\right)L = \cdot 485 - \cdot 703\iota,$$

so that

$$\alpha = \frac{1}{a}\{1 + (\cdot 243 - \cdot 351\iota)/L\}.$$

Hence the period of the oscillation is equal to the time required for a free wave to traverse a distance equal to the circumference of the circle multiplied by $1 - \cdot 243/L$, and the ratio of the amplitudes of consecutive vibrations is $1 : e^{-2\cdot 21/L}$ or $1 : 1 - 2\cdot 21/L$. It is easy to verify from first principles that the decrease in amplitude of the vibrations is of this order.

7. *Induced Vibrations in a Ring.* We will now consider the case of a ring upon which plane waves are incident in a direction parallel to the axis of the ring.

Let the coordinate axes be chosen as before, and let the incident vibration be given by

$$P = R = 0, \quad Q = Ae^{\iota\alpha z}e^{\iota pt}.$$

The tangential force at the point θ due to this wave together with a disturbance induced in the wire of the fundamental mode and magnitude B, is to be equated to zero, giving

$$0 = Ae^{\iota pt}\cos\theta - 2Be^{\iota pt}\frac{\cos\theta}{a}\int_0^\pi d\phi\,\Pi_0\cos\phi\,(1-\alpha^2a^2\cos\phi).$$

If we neglect ϵ/a in comparison with unity, this becomes, as in § 6,

$$\begin{aligned}0 = A + \frac{2B}{a^2}\Bigg[&\left\{2-\frac{x^2}{3}+\left(\frac{x^2}{4}-1\right)L\right\}\\ &+\frac{\iota\pi}{2}\left\{-\frac{x^2}{4}J_3(x)+\left(2-\frac{x^2}{4}\right)J_1(x)+\left(\frac{x^2}{4}-1\right)\int_0^x dx\,J_0(x)\right\}\\ &+\frac{x^2}{8}\int_0^{\frac{\pi}{2}}d\psi\,\frac{\cos(x\sin\psi)-1}{\sin\psi}(1-2\cos2\psi+\cos4\psi)\\ &+\left(\frac{x^2}{4}-1\right)\int_0^{\frac{\pi}{2}}d\psi\,\frac{\cos(x\sin\psi)-1}{\sin\psi}\cos2\psi\Bigg].\end{aligned}$$

Unless $x = 2$, this gives

$$B = \frac{2Aa^2}{(4-x^2)L},$$

so that in general the induced vibration is small, and the thinner the wire the smaller the induced vibration. The phase is the same as or opposite to that of the electric displacement in the plane of the ring due to the incident wave. If, however, $x = 2 = 2 + \xi$, where ξ is small, we may put $x = 2$ in all the terms not involving L, and get

$$B = \frac{Aa^2}{-2L\xi + \cdot970 - 1\cdot055\iota}.$$

The maximum amplitude of the induced vibration is obtained when $2L\xi = \cdot970$, or $\alpha = x/2a = \{1 + \cdot243/L\}/a$, i.e. when the period of the incident wave is the same as the free period of the ring; the amplitude then is $\cdot948Aa^2$, and the phase is in quadrature

with that of the wave. It is noteworthy that the amplitude of the induced vibration is independent of the thickness of the wire. If the incident wave is not proceeding in a direction perpendicular to the ring, the problem can still be solved by a method similar to the above. The vibrations induced in the ring will however not be confined to the fundamental, but will include vibrations of all modes.

8. *Helix.* We will now consider the case of vibrations propagated along an infinite wire wound into a uniform helix. Let the equations of the axis of the wire be $x = a\cos\phi$, $y = a\sin\phi$, $z = a\phi\tan\omega$. We shall assume $\lambda = Ae^{\iota\beta\phi}$. This assumption is justified later. The value of the force tangential to the wire at the point (ϖ, θ, z) is $e^{\iota\beta\theta}$ times that at the point $(\varpi, \theta, z - a\theta\tan\omega)$. Hence (2) gives

$$0 = Ae^{\iota pt}\left[\left(\frac{d}{ds}\right)_{\theta=0} e^{\iota\beta\theta}\int_{-\infty}^{\infty} d\phi\,\iota\beta e^{\iota\beta\phi}\Pi_0 + \alpha^2\left\{\cos\omega\int_{-\infty}^{\infty} d\phi\, a\cos\phi e^{\iota\beta\phi}\Pi_0 + \sin\omega\int_{-\infty}^{\infty} d\phi\, a\tan\omega e^{\iota\beta\phi}\Pi_0\right\}\right],$$

where Π_0 is the value that Π takes when $t = 0$ and ϕ is put for $\phi - \theta$.

Hence $$0 = \int_{-\infty}^{\infty} d\phi e^{\iota\beta\phi}\,(\alpha^2a^2\tan^2\omega - \beta^2 + \alpha^2a^2\cos\phi)\,\Pi_0 \ldots (7).$$

In obtaining this equation small quantities only have been neglected. If however ϵ/a is very small, we may in this equation neglect all finite quantities in comparison with those of the order of $\log\epsilon/a$. In this case we may with advantage find an approximate value of the right-hand side of (7).

Assuming κ any finite quantity, and neglecting terms that are finite, the right-hand side of (7) is

$$\int_{-\infty}^{-\kappa} d\phi e^{\iota\beta\phi}\frac{e^{-\iota a\alpha\phi}\tan\omega}{-a\phi\tan\omega}\{\alpha^2a^2\tan^2\omega - \beta^2 + \alpha^2a^2\cos\phi\}$$

$$+\int_{-\kappa}^{\kappa} d\phi\,\frac{\alpha^2a^2\sec^2\omega - \beta^2}{\sqrt{2a\,(a+\epsilon)(1-\cos\phi) + a^2\phi^2\tan^2\omega + \epsilon^2}}$$

$$+\int_{\kappa}^{\infty} d\phi e^{\iota\beta\phi}\frac{e^{\iota a\alpha\phi\tan\omega}}{a\phi\tan\omega}\{\alpha^2a^2\tan^2\omega - \beta^2 + \alpha^2a^2\cos\phi\}.$$

The second integral is, neglecting finite quantities,

$$-2\,(\alpha^2a^2\sec^2\omega - \beta^2)\frac{\cos\omega}{a}\log\epsilon.$$

The first and last can be reduced to sums of integrals of the form $\int_{\kappa}^{\infty} d\phi \frac{e^{-\gamma\phi}}{\phi}$. This integral* is $-\log\gamma$ to our order, and thus the first and last integrals give

$$\frac{1}{a\tan\omega}[(\alpha^2a^2\tan^2\omega-\beta^2)\log(\alpha^2a^2\tan^2\omega-\beta^2)$$
$$+\tfrac{1}{2}\alpha^2a^2\log\{(1+\beta+\alpha a\tan\omega)(1-\beta+\alpha a\tan\omega)(1+\beta-\alpha a\tan\omega)$$
$$(1-\beta-\alpha a\tan\omega)\}],$$

and therefore the approximate form of (7) is

$$2(\alpha^2a^2\sec^2\omega-\beta^2)\sin\omega\log\epsilon=(\alpha^2a^2\tan^2\omega-\beta^2)\log(\alpha^2a^2\tan^2\omega-\beta^2)$$
$$+\tfrac{1}{2}\alpha^2a^2\log\{(1+\beta+\alpha a\tan\omega)(1-\beta+\alpha a\tan\omega)(1+\beta-\alpha a\tan\omega)$$
$$(1-\beta-\alpha a\tan\omega)\}\ldots(8).$$

Several cases may occur. (i) In general, if α is not small, the only term of importance is that on the left, so that

$$\alpha^2a^2\sec^2\omega-\beta^2=0, \text{ or } \beta=\alpha a\sec\omega,$$

and the velocity of propagation measured along the wire is

$$\frac{p}{\beta}a\sec\omega=\frac{p}{\alpha}=V,$$

the same result as that obtained in the case of a circle.

(ii) If however α and β are small, the first term on the right is also of importance. If α and β are so small that $\log\epsilon$ can be neglected in comparison with $\log(\alpha^2a^2\tan^2\omega-\beta^2)$, i.e. if the product of the wave-length of the disturbance into the radius of the wire is very large compared with the square of the radius of the helix, we have $\alpha^2a^2\tan^2\omega-\beta^2=0$, or $\beta=\alpha a\tan\omega$, and the velocity of propagation measured along the wire is

$$\frac{p}{\beta}a\sec\omega=\frac{p}{\alpha}\operatorname{cosec}\omega=V\operatorname{cosec}\omega,$$

so that the disturbance is propagated with a velocity V measured along the axis of the helix.

If α and β are small, but not so small that $\log\epsilon$ can be neglected, i.e. if the product of the wave-length into the radius of the wire is comparable with the radius of the helix, the velocity of propagation has an intermediate value†. It is easy to see that a like result does not follow if we try to make $(\alpha^2a^2\tan^2\omega-\beta^2)$ small without making α and β small.

* J. W. L. Glaisher, *Phil. Trans.*, 1870, p. 369.

† Hertz, *Wied. Ann.* 1889, vol. 36, p. 21; *Electrical Waves* (tr. Jones), p. 158, has proved experimentally that this is the case.

(iii) If one of the factors, e.g. the last, of the term under the second log sign be small we have, since α and β cannot then both be small,

$$2(\alpha^2 a^2 \sec^2\omega - \beta^2)\sin\omega \log\epsilon = \tfrac{1}{2}\alpha^2 a^2 \log(1-\beta-\alpha a\tan\omega).$$

In this case we must have $\beta < \alpha a \sec\omega$, so that the velocity of propagation measured along the wire must be greater than V. There is however no superior limit to the value that it may have. There is an inferior limit to α. Hence for periods which are not greater than a certain value, there are two velocities of propagation along the wire, one $= V$ given by (i), the other, $> V$, given by (iii).

No other cases arise by making α and β great, since we must then recur to (7), as (8) does not then hold; and here if α and β are great, we simply have $\alpha^2 a^2 \sec^2\omega - \beta^2 = 0$, the case considered in (i).

(2) *On Circles, Spheres and Linear Complexes.* By Mr J. H. Grace.

This paper is printed in the *Transactions*, Vol. xvi. Part iii.

(3) *Reduction of a certain Multiple Integral.* By Arthur Black. Communicated by Professor M. J. M. Hill, M.A., Sc.D., F.R.S.

This paper is printed in the *Transactions*, Vol. xvi. Part iii.

(4) *On the Gamma Function.* By Mr H. F. Baker.

The Gamma Function could be defined for real values of x by the conditions (i) that $\Gamma(1)=1$, (ii) that $\Gamma(x+1)=x\Gamma(x)$, (iii) that, for a fixed finite h, as x tends to $+\infty$, the difference

$$\frac{\Gamma'(x+h)}{\Gamma(x+h)} - \frac{\Gamma'(x)}{\Gamma(x)}$$

tends always to zero. The condition (iii) was well known, being deducible from the result

$$\frac{\Gamma'(x)}{\Gamma(x)} - \log x = \int_0^\infty [e^{-xt} - (1+t)^{-x}]\frac{dt}{t},$$

and was of suitable character for a definition; it was desirable however to deduce it immediately from the equation

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt;$$

this note dealt with such a deduction.

On the lines of striction of a hyperboloid. By Mr H. F. Baker.

It was a known fact that the lines of striction of a hyperboloid formed two unicursal quartic curves. The most commonly given equation shewed that the curve was an octavic curve with six double points; such a curve on a surface of the second order could not be a proper octavic curve; for a cubic surface drawn through the double points and thirteen arbitrary points of the curve would otherwise cut the curve in $12 + 13 = 25$ points. The question considered was what are the possible forms of such octavic curves.

(5) *On the Action of Uranium rays on the Condensation of Water Vapour.* By C. T. R. Wilson, M.A., of Sidney Sussex College, Clerk Maxwell Student.

I have already (*Proc. Roy. Soc.*, Vol. 59, p. 338, 1896; *Phil. Trans.* A. 189, p. 265, 1897) described experiments upon the effect of Röntgen rays on the condensation of water vapour. These experiments proved that the rays, in traversing moist air, introduce nuclei capable of acting as centres of condensation when supersaturation exceeding a definite limit is brought about by the sudden expansion of the gas. Nuclei, requiring exactly the same degree of supersaturation to enable condensation to take place upon them, are always present in very small numbers even without the action of the X-rays; but these rays enormously increase their number. To produce the degree of supersaturation necessary to bring these nuclei into play in air originally saturated, a sudden expansion is necessary such that v_2/v_1, the ratio of the final to the initial volume, exceeds 1·25; corresponding approximately to a fourfold supersaturation.

In the absence of X-rays and other disturbing influences, no condensation is observed (after the removal of all foreign nuclei) if v_2/v_1 is less than 1·252. If v_2/v_1 lies anywhere between this and 1·37 a rainlike condensation results, the drops being few and scattered. Beyond this second limit dense fogs appear.

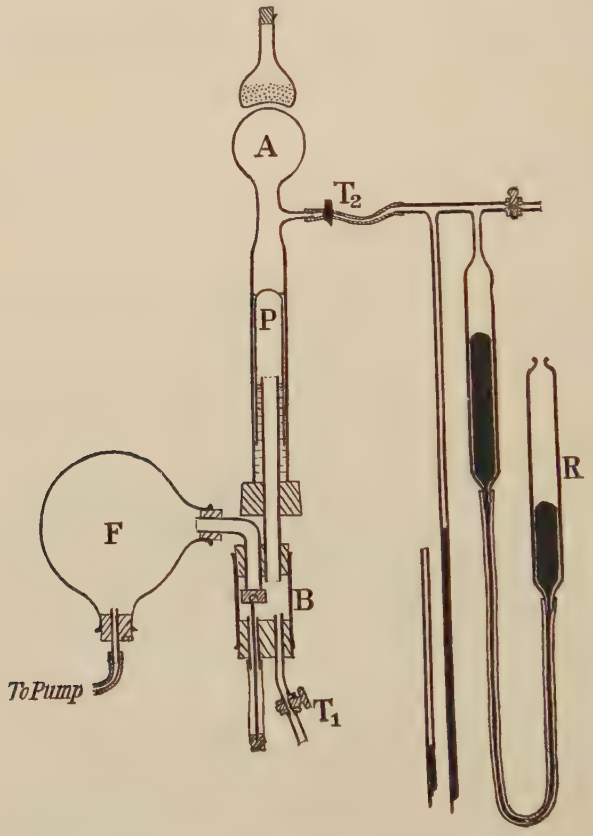
The action of the X-rays is simply to increase the number of the drops which are formed when v_2/v_1 lies between these limits; the minimum expansion required for condensation not being sensibly altered.

The experiments described in the present paper show that nuclei of exactly the same nature are produced in moist air under the action of the Uranium radiation discovered by Becquerel.

The form of apparatus used is represented in the accompanying figure.

The thin-walled glass bulb *A*, about 5 centims. in diameter, contains the air to be exposed to the action of the rays.

The tube, at the end of which the bulb is blown, is about 2·5 centims. in diameter. It has its open end ground perpendicular to the axis, and is wired tightly down in a vertical position against the flat surface of an indiarubber stopper.



Inside this tube there slides freely an inverted thin-walled test-tube *P*, with the lip removed and the open end ground smooth and perpendicular to the axis. The test-tube serves as a piston. Its open end is always immersed in the water which fills the lower part of the outer tube, in the way indicated in the figure.

By means of the arrangement *B* (described on p. 268 of the paper in the *Phil. Trans.* already referred to) the air-space in

the upper part of the test-tube can suddenly be put in communication with the exhausted vessel F. This causes P to fly till it strikes the indiarubber, thus causing a sudden expansion of the air in A. It remains tightly pressed against the surface of the indiarubber as long as the vacuum is maintained below it, thus preventing the lubricating water which surrounds it from escaping.

While the piston is in this position, the extent of the next expansion is arranged for, by opening the screw-clip T_2, and lowering the mercury reservoir R, till the pressure in A, as indicated by the gauge, is the desired amount below the atmospheric pressure. The clip is then closed and air readmitted to the interior of the piston P by opening the tap T_1. The piston then rises till the pressure in A only differs from the atmospheric pressure by that required to support the weight of the piston,—a small fraction of a millimetre of mercury and negligible in these experiments.

If B be the barometric pressure, then the pressure of the air before expansion is

$$P_1 = B - \pi,$$

where π is the maximum vapour pressure at the temperature of experiment. The pressure of the air after expansion, when the temperature has risen to its former value, is

$$P_2 = P_1 - p,$$

where p is the reading of the gauge, taken before the previous contraction, as described above.

Then the ratio of the final to the initial volume is

$$\frac{v_2}{v_1} = \frac{P_1}{P_2} = \frac{B - \pi}{B - \pi - p}.$$

Any condensation, in the form of drops, resulting from the expansion was made visible by the light of a luminous gas-flame, brought to a focus inside A by means of a condensing lens.

The Uranium salt (the double sulphate of Uranium and potassium) was contained in a thin-walled glass bulb, forming a layer about 1 centim. in depth.

The dust-particles originally present having been removed by repeated expansions, observations were made of the result of a given expansion, with the bulb containing the Uranium salt alternately 2 or 3 millimetres above A (as in the figure) and removed to a distance.

I. $v_2/v_1 = 1{\cdot}245.$

Result. No drops seen either with or without the rays.

II. $v_2/v_1 = 1{\cdot}260.$

Result. A dense shower was always seen when the expansion was made with the air in A exposed to the rays; only a few drops appearing when the Uranium salt was removed.

III. $v_2/v_1 = 1{\cdot}273.$

Result. Slight shower without the rays, a very dense shower under the action of the rays.

IV. $v_2/v_1 = 1{\cdot}286.$

Result. The same as in III. The showers were not appreciably denser than in III.

More exact measurements were now made of the minimum expansion at which the drops first begin to appear under the action of the rays. The experimental numbers are given in the following table.

Pressures are all given in millimetres of mercury.

Barometer = 769,

$t°$ C. = 15·5°,

π = vapour-pressure = 13,

P_1 = pressure of the air before expansion

$= B - \pi = 769 - 13 = 756.$

Gauge reading, p	Pressure after expansion, $P_2 = P_1 - p$	$P_1/P_2 = v_2/v_1$	Result
149	607	1·245	0
153	603	1·254	shower
149	607	1·245	0
151	605	1·249	shower

Thus the minimum value of v_2/v_1 required to bring about condensation under the influence of Uranium rays lies between 1·245 and 1·249. This agrees very closely with the value 1·252, found with very different apparatus for air under ordinary conditions (*Phil. Trans. loc. cit.*).

The Uranium salt shows a strong effect on the condensation, even when the thin glass vessel containing it is completely wrapped in tinfoil during the experiment and for many hours previously; showing both that this material is transparent to the agent which influences the condensation, and that the Uranium salt continues to be active when kept in the dark. There can be little doubt therefore that the effects on the condensation are really due to the radiation studied by Becquerel.

The nuclei introduced by the action of the Uranium rays, like those formed under the influence of the *X*-rays, persist for some seconds after the radiation has been cut off; for a comparatively dense shower can be obtained, by expansions of suitable amount, five or ten seconds after the removal of the tube containing the Uranium salt. If, however, an interval of one minute be allowed to elapse after cutting off the radiation, the drops produced on expansion are not appreciably more numerous than they would be, were the air not exposed to the radiation at all.

The nuclei produced in moist air by Uranium rays appear, from the experiments described above, to be identical with those formed under the action of Röntgen rays, as well as with those always present in very small numbers in the moist gas. They require the air, originally saturated, to suffer sudden expansion such that v_2/v_1 exceeds 1·25 in order that condensation may take place upon them; the density of the vapour at the moment when the expansion is completed being then approximately four times that of vapour in equilibrium over a flat surface of water at the same temperature (*v. Phil. Trans. loc. cit.*).

The electrical properties of gases under the action of Röntgen and Uranium rays point to the presence of free ions. It is natural to identify with these the nuclei made manifest in the gas under the same conditions by the condensation phenomena described in this and previous papers. As early as 1887 R. v. Helmholtz (*Wied. Ann.*, vol. 32, p. 1) and later the same author and Richarz (*Wied. Ann.*, vol. 40, p. 161, 1890) accounted for the effect of electrification and chemical action in bringing about dense condensation in a steam-jet by supposing free atoms or ions to be produced by these processes. Prof. J. J. Thomson (*Phil. Mag.*, vol. 36, p. 313, 1893) has explained how we should expect free ions, in virtue of the charge they carry, to have such an effect in helping condensation. Richarz has more recently (*Wied. Ann.*, vol. 59, p. 592, 1896) found that a steam-jet reacts also with Röntgen rays, and again explains the phenomenon by the hypothesis of free ions.

The great advantage of expansion experiments over those made upon steam-jets is that we are able in the former to distinguish between various kinds of nuclei, by means of the different

degrees of supersaturation necessary to make condensation take place upon them. Whatever view may be held as to the nature of the nuclei introduced by *X*-rays and Uranium rays, they are shown in this way to be identical with one another, as well as with those existing in very small numbers in ordinary moist, dust-free air.

Monday, 8 *November*, 1897.

Mr F. Darwin, President, in the Chair.

The following Communications were made to the Society:—

(1) *On Farmer's method of demonstrating assimilation.* By Francis Darwin, F.R.S., *President of the Society.*

The method here given is merely a simplification of the interesting experiment described by Farmer in the *Annals of Botany*[1]. He there pointed out that the protoplasmic circulation ceases in an *Elodea* leaf kept in the dark, in a current of hydrogen: and that the circulation recommences if the preparation is illuminated. The cessation of the movement is due to the absorption of the free oxygen, its reappearance to the oxygen produced by the assimilatory activity of the chloroplasts. Farmer's positive results should in any case I think outweigh Pringsheim's[2] failure to observe a revival of circulation in illuminated specimens of *Chara* &c., and the results which I have obtained fully confirm Farmer's conclusions.

My object in modifying Farmer's method was to place an experiment of great educational value within the reach of those who possess no apparatus beyond a microscope. This can be done by mounting an *Elodea* leaf[3] in water and sealing the preparation by careful ringing it with melted "wax-mixture."[4] The preparation is then placed in the dark, and the observer waits until the available oxygen has been absorbed and the circulation has come to rest, which takes place in from 3 to 6 hours.

[1] Vol. x. p. 284.

[2] *Sitzb. k. Preuss. Akad.* 1887. See also Vines' interesting discussion *Annals of Botany*, Vol. i. p. 371.

[3] For reasons given below it is better to mount two or even three leaves under a single cover-glass.

[4] Resin 15 parts, vaseline 50 parts, beeswax 35 parts.

It is easy to show that the cessation of circulation is due to want of oxygen by lifting the cover-glass with a needle and adding a drop of fresh water, when the protoplasm will begin to stream in a few minutes. The same result can—as in Farmer's experiment—be produced by illumination. The length of the period of illumination required to start the movement varies: in a preparation darkened for about 3 hours, in which the circulation has only recently come to a stand-still two or three minutes' exposure to incandescent gas light will revive the movement in two or three minutes. When the period of darkness has lasted over 6 hours it may require 20—30 minutes. This clearly depends on partial inhibition[1] of the assimilatory powers of the chloroplasts. If the period of darkness is still further prolonged it is easy to reach a condition in which it is apparently impossible to revive the circulation; but my experiments not being especially directed to this point I cannot say what might be the effect of very prolonged illumination. It is to avoid the inhibition of assimilation that I mount several leaves under one cover-glass, so that the dissolved oxygen may be exhausted before the chloroplasts fall into this condition.

The method here described seems to me likely to be of value to teachers in putting an interesting experiment within the power of all. And it is worth noting that Farmer's experiment has all the *educational* value of Engelmann's bacterial method. For it demonstrates: (1) The evolution of oxygen by green plants in the light. (2) The connection between movement and respiration and the fact that a chlorophyllous cell makes use for purposes of respiration of the oxygen evolved within itself. (3) It demonstrates in a rough way (what Boussingault showed to be true for aerial plants) that assimilation can begin in the absence of free oxygen. (4) It also demonstrates the inhibition of the assimilating function produced by prolonged darkness.

For this last-named purpose it must be shown by the addition of oxygenated water that the failure to circulate does not depend on any loss of power in the protoplasm.

The method also serves well to demonstrate the effect of light of different wave-lengths on assimilation. As a source of light I use incandescent gas, separated from the microscope by a glass trough through which cold water flows. For the different lights I use Landolt's screen solutions[2] in flat-sided glass vessels. In this way the maximum effect in the red 718—639 $\mu\mu$ can easily be shown. The circulation, as far as I have seen, does not begin in green[3] light unless the experiment is made after not more

[1] See Ewart in *Journal Linn. Soc.* Vol. xxxi.
[2] *Berichte D. Chem. Ges.* 1894, Vol. 27 (3), p. 2872.
[3] The light is 540—505 $\mu\mu$, *i.e.* it shows a narrow strip of blue.

than 2 or 3 hours of darkness, *i.e.* when the assimilative function is not seriously inhibited. It should be possible to demonstrate a second maximum in the blue[1], but I am here considering the question simply from the point of view of laboratory teaching, and in this connection I think it best to confine the demonstration to a comparison of red and green light.

(2) *Artificial Cultures of* Stereum, *a Timber-destroying Fungus.* By Prof. MARSHALL WARD.

(3) *On* Encephalartos Ghellinckii, *Lem., a rare Cycad.* By A. C. SEWARD, M.A., St John's College.

The subject of this note is one of the less known species of the African Cycadean genus *Encephalartos.* In 1895[2] I called attention to the unusual form of the fronds of *Encephalartos Ghellinckii,* Lem., and figured a small piece of a leaf from a specimen in the British Museum herbarium. A plant in the Royal Gardens, Kew, bearing a single frond, first attracted my attention to the species. The large and stout fronds of such species as *Encephalartos horridus,* Lehm., *E. Caffer,* Miq., *E. Lehmanni,* Lehm., *E. Altensteinii,* Lehm., and others with broad and occasionally spinous pinnae represent the type of plant which is usually regarded as characteristic of the genus. *Encephalartos cycadifolius*[3], Lehm. (= *E. Frederici-Guilielmi*) forms a connecting link between species with narrower pinnae and *E. Ghellinckii.*

The first account of *E. Ghellinckii* was published by Lemaire[4] in 1867, the description being founded on a plant in the nurseries of Verschaffelt, by whom the Cycad was obtained from the central region of South Africa. The following is Lemaire's diagnosis of the species:—

"Caudex ut in praecedente (*i.e. Encephalartos villosus,* Lem.). Frondes 5—12 erectae et illae ut praecedentis dispositae, totae gossypinae metrales et ultra (quas vidi solum in prima aetate frondationis sese evolventes; rhachi cylindrica serius siccata subplana) foliolis approximatissime oppositis v. rarius alternis omnino ob arctissimam marginum revolutionem lineari-filiformibus apice mucrone robusto aculeiformi pungentissimo terminatis, marginibus integris acus recte simulantibus.

Coni ♂ et ♀ nobis hucusque ignoti."

[1] Using a screen of ammoniacal copper sulphate which lets through a good deal of green as well as the blue end of the spectrum.

[2] "The Wealden Flora" (*Catalogue of the Mesozoic plants in the British Museum*), vol. II. 1895, p. 22, Pl. III. fig. 3.

[3] *ibid.* Pl. III. fig. 6.

[4] Lemaire, C., "L'illustration horticole," *Journal spécial des serres et des jardins,* vol. XIV. 1867, p. 80.

ENCEPHALARTOS GHELLINCKII, LEM. $\frac{1}{11}$ Nat. Size.

Lemaire describes both *E. villosus* and *E. Ghellinckii*, and speaks of the two species as quite distinct from the other members of the genus. The two woodcuts and the coloured plate which accompany the above diagnosis convey an imperfect idea of the appearance of the latter species[1].

The plant reproduced in the Plate[2] was obtained by Mr Lynch, of the Cambridge Botanic Gardens, from Messrs Sander and Co., of St Albans, who received it from the interior of the Province of Natal. When sent to Cambridge the stem shewed no sign of life and bore traces of having been considerably charred, probably in a forest fire. After lying for some months in an apparently dead condition a fine crown of leaves was produced, and the accompanying photograph was taken before all the fronds had attained their full development. The stem measures about 28 cm. in height and 70 cm. in circumference, and is covered with the characteristic armour-like investment of persistent and corky leaf-bases. The leaves are a little over 90 cm. in length, and in those which are not fully expanded the slender apices are bent over at the tip, and the linear-filiform pinnae are crowded together like the barbs of a goose-quill. The rachis of the mature frond bears two rows of opposite or alternate pinnae, about 11 cm. in length and 2 mm. broad, which are attached laterally to the leaf axis. Both the rachis and pinnae present a woolly or tomentose appearance, especially in the young condition. The pinnae terminate in an acute spinous apex, and the margins are strongly revolute; four parallel veins traverse each pinnae, but these are hardly visible in a surface-view. As shewn in the plate, the two rows of pinnae form a fairly wide angle with one another, and are not extended in one plane. In some of the fronds the rachis is slightly twisted in a spiral, as in *Macrozamia spiralis*, Miq. The general appearance of *Encephalartos Ghellinckii* suggests a distinctly xerophytic habit, and this is borne out by the thick cuticle and sunken stomata of the pinnae.

The rachis is traversed by numerous vascular bundles of the typical mesarch form, characteristic of the petioles and peduncles[3] of recent Cycads. Each bundle is surrounded by a more or less complete ring of large thick-walled fibres; the parenchymatous ground-tissue contains several large mucilage canals and the hypoderm consists of strands of fibrous elements alternating with parenchyma. The anatomical characters agree with those of other Cycads, and especially with other species of *Encephalartos*. A transverse section near the tip of a rachis bears a striking

[1] Lemaire, *loc. cit.*, vol. xv. 1868, Pl. 567.

[2] The photograph reproduced in the plate was taken for me by Mr Hayles of the Cavendish Laboratory.

[3] *vide* Scott, D. H., *Annals of Botany*, vol. xi. 1897, p. 399.

resemblance, in the scattered collateral bundles and large canals, to the fossil petioles known as *Myeloxylon*. The hypoderm in the younger portion of a rachis consists in part of collenchyma, which afterwards developes into uniformly thickened fibrous tissue.

The following points may be briefly noticed in the anatomy of the pinnae. In tranverse section a pinna has the form of a somewhat flattened crescent, the two edges being bent over towards the under surface. The upper surface is covered by a thick cuticle, and the epidermis is succeeded by two or three layers of thick-walled fibres, and internal to these there is exceedingly well-marked palisade tissue of one layer of long cells; internal to this the mesophyll consists of spongy parenchyma followed by one or two rows of fibres and the lower epidermis. There are numerous stomata on the lower surface, the two guard-cells being situated at the bottom of a fairly deep depression. The mesophyll is traversed by four collateral bundles surrounded by a ring of fibrous elements. The woolly appearance of the young fronds is due to the occurrence of numerous long unicellular hairs borne on a short and thick-walled stalk-cell as in *Divon edule*, Lind., and other Cycads[1].

The importance of such a species as *Encephalartos Ghellinckii* in connection with fossil Cycads has previously been discussed. The necessity of examining as many recent species as possible, especially the less known and rarer forms, cannot be too strongly urged. There are several fossil fronds in which the rachis bears numerous long and narrow pinnae, which may well be compared with *Encephalartos Ghellinckii*. Some of the Mesozoic fronds, which have been erroneously referred to the genus *Cycadites*[2], bear a very close resemblance to those African Cycads with the narrow linear or linear-filiform segments.

[1] My thanks are due to Mr A. W. Hill, of King's College, for preparing several sections of the frond of *Encephalartos Ghellinckii* and other species.

[2] For references to fossil fronds *vide Wealden Flora*, vol. II. pp. 36 *et seq.*

Monday, 22 *November*, 1897.

MR F. DARWIN, PRESIDENT, IN THE CHAIR.

The following communications were made to the Society:—

(1) *Partial Differential Equations of the Second Order.* By Professor FORSYTH.

This paper is printed in the *Transactions*, Vol. XVI. Part III.

(2) *Electrical Properties of Newly Prepared Gases.* By JOHN S. TOWNSEND, M.A., Cavendish Laboratory, Cambridge.

(1) The experiments which are described in this paper form a continuation to those which have already been published in the *Proceedings of the Cambridge Philosophical Society*, Vol. IX. Pt. V. It was there shown that the gases given off by the electrolysis of sulphuric acid or caustic potash carry with them an electric charge, a large percentage of which remains in the gas after it has been bubbled through a liquid, and passed through glass wool to remove the spray. Another property of these gases is their power of condensing moisture to form a cloud. No such cloud could be observed in newly prepared gases unless they were charged, and further the weight of the cloud was found to be proportional to the charge on the gas. These results go to show that the condensation of the moisture is connected with the charge, and the experiments described in section 16 of the above paper, and in section 19 of this paper prove that when the cloud is formed in a charged gas the electrification resides on the drops forming the cloud. So that we have definite proof of the fact that the drops are formed round the carriers of the electric charge.

(2) These results were used to find the charge on each carrier, and it was found to coincide with what we can calculate as being the atomic charge, on the supposition that at ordinary temperature and pressure there are 10^{20} molecules in each cubic centimetre of a gas. The experiments giving the weight of the cloud corresponding to the charge being of importance, were repeated by the following method which gave the same proportionalities in the different gases between the charge per c.c. and weight of cloud per c.c., as were previously obtained.

(3) The apparatus which was used is shown in figure 1. The positively charged oxygen and hydrogen given off from a dilute

sulphuric acid cell were first bubbled through a solution of potassium iodide in the small flask *A* and then through distilled water

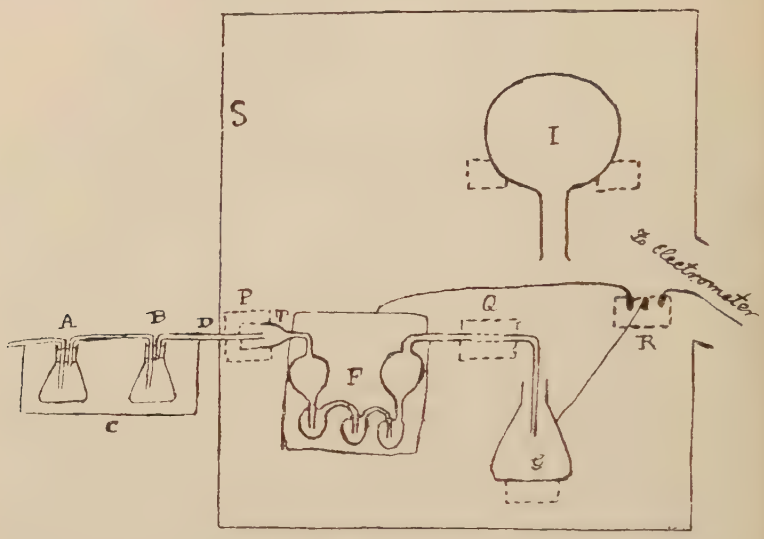


Fig. 1.

Insulators are drawn with dotted lines and conductors with continuous lines.

in *B*. Both *A* and *B* were immersed in a trough of water *C* so that they should remain at a fixed temperature while an experiment was being conducted. The charged gas thus formed a cloud and carried it along the delivery tube *B*, which led into the paraffin block *P* inside a large metallic screen *S*. The block *P* was arranged as shown in the figure in order that the moisture should not break down the insulation by settling on the paraffin. By this means the moist gas entered the larger tube *T* connected to three sulphuric acid bulbs *F* set up between the two paraffin tunnels *P* and *Q*. After the gas had bubbled through the acid it was not only cleared of the moisture, which under ordinary conditions it would have carried from *B*, but the cloud was also completely removed. The increase of weight in the bulbs thus arises from two causes, and when the weight of the moisture necessary to saturate the volume of the gas which passes through *F* is subtracted from the total, the weight of the cloud is obtained. The gas on leaving *F* entered one of the insulated inductors, the smaller one *G* being used for oxygen, and the larger one *I* for hydrogen. The inductor and the bulbs were both covered with tinfoil and connected to mercury cups in the paraffin block *R*, so

that either of them could be readily connected to the insulated quadrants of the eloctrometer.

The experiments were conducted in the following manner. The sulphuric acid bulbs were carefully weighed and then connected by indiarubber tubing to the glass tubes which were imbedded in the paraffin blocks as shown in the figure. The current through the cell was switched on for a few minutes before joining the bulbs into the series, so as to drive the air out of A and B. As soon as the gas began to bubble through the acid in F a stop watch was started. By connecting the conductors F and G alternately to the quadrants of the electrometer the rate at which each was acquiring a charge was found. The sum of the charges acquired by F and G denotes the total charge passing out of B and that acquired by F divided by the total is the discharging power of the bulbs. After the stream of gas from the cell had been running through the apparatus for a sufficient time the bulbs were removed, and dry air was drawn through them before weighing them a second time. In each experiment a current of 14 ampères was used which was kept constant by having an ammeter and an adjustable resistance in the circuit.

In order to reduce the readings on the electrometer scale to absolute units it was found that when F was connected to the quadrants and the other conductors to earth, each scale division represented ·0040 electrostatic units of quantity, and when G was connected to the quadrants the scale divisions each represented ·0036 electrostatic units. The inductor I which was used with hydrogen had a larger capacity than either of the other two conductors, and when it was connected to the quadrants it required ·0042 electrostatic units to make the spot of light on the electrometer scale move through a division.

The following tables give the results of the experiments which were made with the different gases, θ is the temperature in degrees centigrade of the water in B, W the increase in the weight of the sulphuric acid bulbs, and n_1 and n_2 the numbers of divisions that the spot of light on the electrometer scale moves per minute when F and G [or I] are connected to the insulated quadrants; w is the calculated weight of the moisture necessary to saturate the volume of gas v at temperature θ which passes through F in the course of the experiment, $\frac{W-w}{v}$ the weight of the cloud per c.c., and ρ the charge per c.c. on the gas. The volume v is easily calculated as the gas is evolved by a known current.

Table I. gives the results obtained with positive oxygen. In each experiment a current of 14 ampères was used, and the stream of the evolved gas passed through F for 20 minutes.

Table II. gives similar results for positively charged hydrogen.

Table III. refers to negatively charged oxygen from a caustic potash cell, the stream of gas being passed through F for 15 minutes in each experiment.

The experiments were performed with widely different densities of electrification ρ, in order to find how the weight of the cloud varied with the electrification. The ratio of the weight of the cloud to the electrification $\frac{W-w}{v\rho}$ is given in the last column for each experiment. The variations in ρ were obtained by varying the temperature of the cell from which the gas was given off. Both in the case of a sulphuric acid electrolyte and a caustic potash electrolyte the charge on the gas which is evolved by a given current increases as the temperature of the electrolyte is raised.

TABLE I. + *Oxygen from* H_2SO_4 *electrolyte.*

θ	n_1	n_2	W	w	$\frac{W-w}{v}$	ρ	$\frac{W-w}{\rho v}$
1	22	31	·0270	·0050	$22{\cdot}3 \times 10^{-6}$	$4{\cdot}04 \times 10^{-3}$	$5{\cdot}5 \times 10^{-3}$
1	13	20	·0187	·0050	$14{\cdot}1 \times 10^{-6}$	$2{\cdot}58 \times 10^{-3}$	$5{\cdot}4 \times 10^{-3}$
13	21	30	·0330	·0118	$20{\cdot}8 \times 10^{-6}$	$3{\cdot}8 \times 10^{-3}$	$5{\cdot}5 \times 10^{-3}$
14	9	19	·0240	·0125	$11{\cdot}5 \times 10^{-6}$	$2{\cdot}05 \times 10^{-3}$	$5{\cdot}6 \times 10^{-3}$

TABLE II. + *Hydrogen from* H_2SO_4 *electrolyte.*

θ	n_1	n_2	W	w	$\frac{W-w}{v}$	ρ	$\frac{W-w}{\rho v}$
0	21·5	28·5	·0264	·0092	$8{\cdot}8 \times 10^{-6}$	$2{\cdot}1 \times 10^{-3}$	$4{\cdot}2 \times 10^{-3}$
0	27	34	·0296	·0092	$10{\cdot}5 \times 10^{-6}$	$2{\cdot}57 \times 10^{-3}$	$4{\cdot}1 \times 10^{-3}$
15	32	37	·0486	·0264	$10{\cdot}8 \times 10^{-6}$	$2{\cdot}8 \times 10^{-3}$	$3{\cdot}8 \times 10^{-3}$
15·3	26	31	·0445	·0271	$8{\cdot}5 \times 10^{-6}$	$2{\cdot}2 \times 10^{-3}$	$3{\cdot}9 \times 10^{-3}$

TABLE III. – *Oxygen from* KOH *electrolyte.*

θ	n_1	n_2	W	w	$\frac{W-w}{v}$	ρ	$\frac{W-w}{\rho v}$
0	15	26	·0188	·0035	21×10^{-6}	$3{\cdot}1 \times 10^{-3}$	$6{\cdot}8 \times 10^{-3}$
0	22	32	·0217	·0035	25×10^{-6}	$4{\cdot}1 \times 10^{-3}$	$6{\cdot}1 \times 10^{-3}$
12	14·5	24·5	·0230	·0081	20×10^{-6}	$2{\cdot}86 \times 10^{-3}$	$7{\cdot}0 \times 10^{-3}$
11·5	12·5	22·5	·0206	·0078	17×10^{-6}	$2{\cdot}51 \times 10^{-3}$	$6{\cdot}8 \times 10^{-3}$

(4) The numbers given in the tables show that in each case the weight of the cloud is proportional to the charge, and that this proportionality does not vary with the temperature of the water from which the cloud is formed, at least within the limits of 0° and 14° C. The ratio is not so exact in the case of the negative oxygen as it is for the other gases. The only chemical difference that was detected was that the former were slightly acid and the latter alkaline in their reactions. Since the vapour rising from hot caustic potash does not change red litmus to blue, the alkalinity of the gas must be due to small particles of caustic potash which are carried with the gas from the electrolyte and give it an alkaline reaction even after bubbling through sulphuric acid, this could easily be detected at all temperatures of the cell by letting the oxygen or hydrogen pass along a tube with red litmus.

The oxygen given off by the electrolysis of caustic potash has no appreciable charge till the temperature of the cell is up to about 20° C. and the cloud begins to appear at the same time as the charge on the gas, so that the presence of the alkaline spray can have only a very small effect as no cloud is observed in the oxygen coming from the cell at a temperature 10° C. although the presence of the spray of caustic potash can be easily detected.

(5) The following simple experiments show how intimately connected the charge on the gas is with the formation of the cloud.

The apparatus used is shown in figure 2. The oxygen given off by a current of ten ampères from a caustic potash cell of specific gravity 1·3 was bubbled through a solution of potassium iodide in the flash A to remove any traces of ozone. Before entering the inductor I the gas bubbled through water in B. The inductor I was insulated and connected to the quadrants of the electrometer so that the charge on the gas that entered it could be read off on

the electrometer scale. The gas was not passed through any glass wool before entering *I* so that as much spray as possible should be carried from the cell.

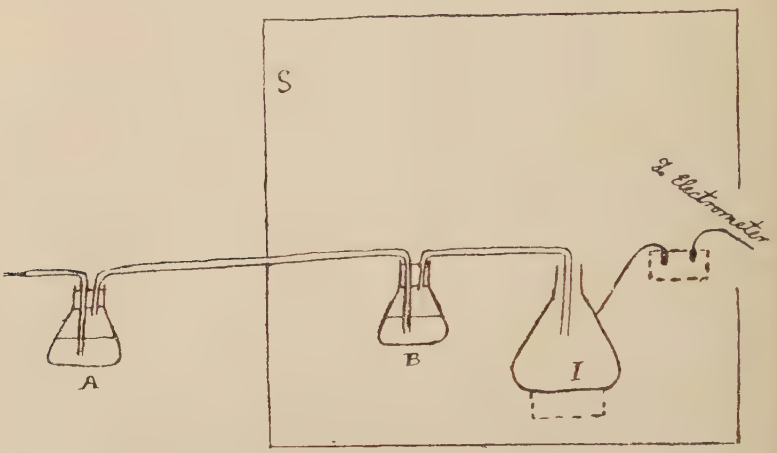


Fig. 2.

The cell was at a low temperature, 11° to start with, and a current of 10 ampères was switched on which gradually warmed the electrolyte. During the first nine minutes that the current was on and the gas entering the inductor, no electrification whatsoever was detected, and no cloud could be observed over the water in *B*. During these nine minutes the temperature of the cell rose from 11° to 18°.

During the next five minutes a deflection of 9 divisions negative was obtained and the temperature of the cell had risen to 21·5.

At a temperature between 22 and 23 a thin cloud began to appear over the water in *B* and the spot of light was then moving at the rate of 4 divisions per minute, so that before the cloud becomes distinctly visible a small electrification of about 2×10^{-4} Electrostatic units per c.c. is necessary. When the current is continued and the cell becomes hotter the electrification on the gas increases and the cloud becomes denser.

(6) In order to show that the cloud disappears when the charge on the gas is removed the temperature of the cell was raised to 48° and, with the same apparatus as was used for the last experiment, the three following results were obtained.

(*a*) When the gas passes from the cell to the inductor without filtering through glass wool a dense cloud was observed over

the water in A, and a deflection of 29 divisions negative per minute was obtained as the gas entered I.

(*b*) The tube connecting A and B was removed and a tube containing glass wool was substituted. With the same current through the cell a slight cloud was seen over the water in B and the deflection was reduced to 6 divisions per minute. The electrification of the gas was thus reduced to $\frac{1}{5}$ of its original value by passing through 15 centimetres of glass wool. The amount of spray carried through this length of wool must have been extremely small but the cloud was distinctly visible.

(*c*) The tube containing the glass wool was heated with a Bunsen burner, and its discharging power was considerably increased as the spot of light then gave only 2 divisions per minute and no cloud was observed over the water in B.

It is thus evident that the formation of the cloud and the presence of the charge are phenomena which accompany one another.

(7) The clouds which are formed are slightly different in appearance and for equal electrifications those formed in oxygen are whiter than those formed in hydrogen. A difference is also to be noticed in the positive and negative oxygen clouds, that formed in the latter being the whiter. This would point to the fact that the drops formed in the negative oxygen are larger than those in the positive, and that those formed in either positive or negative oxygen are larger than those in the hydrogen. A fairly approximate value for the radius of the drop may be obtained by observing the rate at which the cloud falls in a vessel. The velocity of the drop through the gas was obtained by taking two photographs of the cloud allowing some minutes to elapse between the two exposures. Figure 3 represents two such photographs taken of the cloud formed by bubbling the charged oxygen from a sulphuric acid electrolyte through water. Three minutes were allowed to elapse between the two exposures, and the scale shows that in that time the cloud had fallen between 9 and 10 millimetres. Similar experiments were made with the other gases, but in the case of the hydrogen the outline of the cloud never became so distinct as it did in the oxygen.

The sizes of the drops were obtained from the formula $\mu a V = \frac{4}{3}\pi a^3 g$ (Lamb, *Motion of Fluids*, p. 229).

This gives for the radius of the drop in the positive oxygen $6{\cdot}8 \times 10^{-5}$, the rate of fall being 10 millimetres in 3 minutes, and the radius of the drop in negative oxygen $7{\cdot}9 \times 10^{-5}$ the rate of fall being 18 millimetres in 4 minutes.

We thus know the weight of each drop and dividing this into the weight of the corresponding cloud, we obtain the number of drops per cubic centimetre.

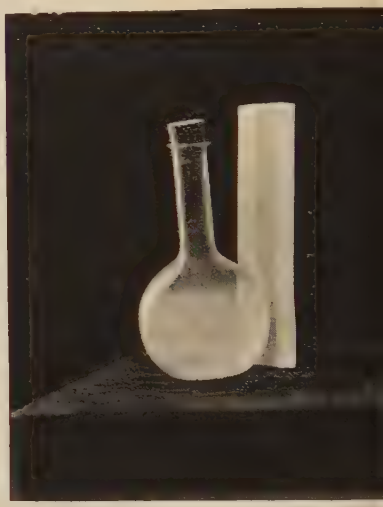


Fig. 3.

When the number of drops is divided into the charge per cubic centimetre the charge on each is obtained.

The charge on the positive carrier was thus found to be $2{\cdot}4 \times 10^{-10}$, and that on the negative carrier to be $2{\cdot}9 \times 10^{-10}$. When we take into account all the experimental errors these two charges may be considered equal and approximately 3×10^{-10}. The charge on the hydrogen was found to be between one-third and two-thirds of this value but could not be arrived at very accurately owing to the difficulty of finding the size of the drop. For present purposes this charge will be taken as $\frac{3}{2} \times 10^{-10}$.

Velocity of the carrier when acted on by an electric force.

(8) It has been shown that the radius of the drop forming the cloud is in the case of negatively charged oxygen 810^{-5}, so that its weight is 210^{-12} grammes and when acted on by gravity, it moves at the rate of 18 millimetres in four minutes. The force acting on the drop is practically 210^{-9}, so that it would require an electric force of $\frac{210^{-9}}{3 \times 10^{-10}}$ absolute units to produce an equal velocity, or 2000 volts per centimetre. As we are only dealing

with small electromotive forces it is evident that we may disregard the effect of mutual repulsion of the carriers when the gas is carrying a cloud. When the gas is bubbled through sulphuric acid the radius of the carrier is so much reduced, that the effect of the mutual repulsion of the particles carrying the charge is easily detected.

(9) The following is a general method of investigating the motion of a gas in a vessel of any shape, the initial distribution being uniform. Let ρ be the density of electrification in any part of the gas, u, v and w the velocities of the carriers along the axes of x, y and z.

The equation of continuity is $\frac{1}{\rho}\frac{\delta\rho}{\delta t}+\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0$, the notation being the same as that used in Lamb, *Motion of Fluids.*

Let ϕ be the electric potential; $-\frac{d\phi}{dx}$, $-\frac{d\phi}{dy}$ and $-\frac{d\phi}{dz}$, are the forces which act on the charged carriers, and their velocities u, v, and w, are given by the equations:—

$$(1)\quad \kappa u=-e\frac{d\phi}{dx};\quad \kappa v=-e\frac{d\phi}{dy};\quad \kappa w=-e\frac{d\phi}{dz},$$

where e is the charge on the carrier and κ is a constant to be determined experimentally.

Substituting these values for u, v, and w, in the equation of continuity and we obtain:—

$$\frac{\kappa}{\rho}\frac{\delta\rho}{\delta t}-e\nabla^2\phi=0,\ \text{but}\ \nabla^2\phi=-4\pi\rho,$$

therefore

$$\frac{\kappa}{\rho^2}\frac{\delta\rho}{\delta t}=-4\pi e.$$

Integrating and we obtain:—

$$(2)\quad \rho=\frac{\rho_0}{1+\frac{4\pi e\rho_0}{\kappa}t},$$

where ρ_0 is the initial density, which is uniform throughout the space considered.

Equation (2) shows that the motion takes place in such a way that the density ρ is a function of the time only and does not vary from point to point in the gas, on this account no variation in the pressure of the charged gas takes place and the terms $\frac{dp}{dx}$, $\frac{dp}{dy}$,

and $\frac{dp}{dz}$, which have in general to be taken account of when dealing with the motion of gases, can be omitted in equations (1) since p does not vary from point to point.

When the gas is in a closed vessel the mutual repulsion of the carriers of the electricity drives them to the sides where they either remain so close to the walls that they cannot be blown out, or else get discharged against the sides. The charged gas remaining in the vessel has a uniform density given by equation (2), and if the vessel which contains it is an insulated conductor connected to a pair of quadrants of an electrometer the deflection on the electrometer scale will be proportional to ρ when the charged gas is blown out of the vessel.

(10) In order to investigate experimentally the rate at which the gas loses its charge to the sides of an inductor, or in other words to find how the density ρ will vary with the time, a metal

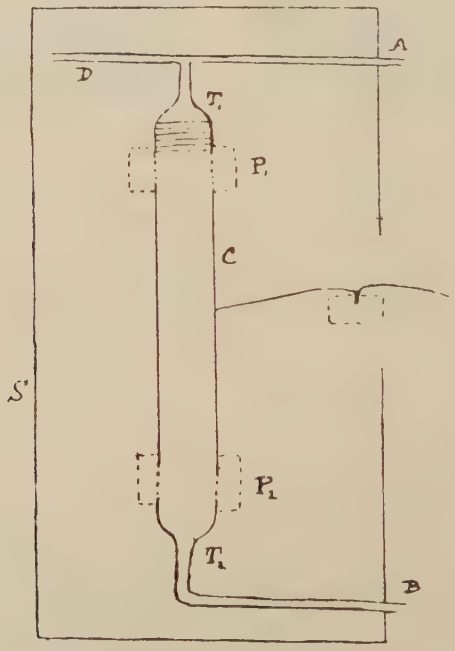


Fig. 4.

cylinder C, figure 4, 30·2 centimetres long and 1·6 centimetres in diameter, was used. The two ends of the cylinder fitted into paraffin blocks P_1 and P_2, into which were also fixed the glass

tubes T_1 and T_2. The tube T_1, through which the cylinder was filled, had several layers of fine copper gauze across the broad end, which tended to distribute the gas evenly across the section. The whole apparatus was contained inside a screen S, the ends A and B of the tubes leading to the cylinder projecting outside the screen; by this means the charged gas could be blown out of the cylinder, without stirring any apparatus or tubes inside S; so that the deflection on the electrometer scale, when C is connected to the insulated quadrants, was solely due to the removal of the charge inside C. Any effect of gravity, on the motion of the carriers towards the sides of the tube, was prevented by placing it in a vertical position.

(11) The gases that were examined were the oxygen and hydrogen given off by electrolysis of dilute sulphuric acid. The cell was raised to a temperature about 20° above the room, at which it could easily be kept by running the current. By means of an ammeter and variable resistance in the circuit the current could be kept constant. A method is thus provided of filling the cylinder several times with a gas having a constant electrification per c.c. When filling the cylinder with the hydrogen it is best to lead the gas in at the top, and when using oxygen at the bottom, in order that the air may be driven out in as short a time as possible, as it is not desirable to run the current for more than five minutes at a time, when a number of experiments have to be performed requiring the same electrification in each case.

The gases were first bubbled through potassium iodide; and then through strong sulphuric acid, so that they should enter the tube D perfectly dry. The cylinder and tubing connected with it were thoroughly dried by heating them with a Bunsen burner and then blowing dry air through. This precaution is absolutely necessary as it only requires a very slight amount of moisture to form a cloud, and this would impede the motion of the carriers.

(12) A series of experiments of the following type were then performed: The cylinder was connected to a pair of quadrants of the electrometer, which were kept to earth while the cylinder was being filled and insulated before blowing out the gas, so as to get the readings on the same part of the scale. The tube A was closed, and D connected to the delivery tube of the sulphuric acid wash-bottle, and a current of 14 ampères sent through the cell. It only required the current to run for about five minutes to completely fill C with the charged gas. The circuit was then broken and the tube B closed. At the same time a stop-watch was started. After the lapse of t minutes the ends A and B are opened, and dry unelectrified air blown through C to expel the

charged gas; this caused the spot of light on the electrometer scale to move n divisions, which is proportional to ρ the density of the electrification after the gas has remained in the cylinder for a time t.

The numbers thus obtained for n and t are given in Tables I. and II., the first being for charged hydrogen and the second for charged oxygen.

TABLE I.

n	t
32	0
25·5	1
20·5	2
17·5	3
15·5	4
14	5

TABLE II.

n	t
49	0
37·5	2
29·5	4
24·5	6
22	8

It will be seen that these numbers are nearly exactly the same as those given by the formula $n = \frac{n_0}{1+\theta t}$, if we put $n_0 = 32$ and $\theta = \cdot 0043$ in the case of the hydrogen, and $n_0 = 49$ and $\theta = \cdot 00255$ in the case of oxygen. The numbers calculated from this formula are given in the two following tables:

TABLE III.

n	t
32	0
25·4	1
21·1	2
18	3
15·7	4
14	5

TABLE IV.

n	t
49	0
37·5	2
30·3	4
25·5	6
22	8

This shows that $\rho = \frac{\rho_0}{1+\theta t}$. Hence we have from equation (2) § 9

$$\theta = \frac{4\pi\rho_0 e}{K}.$$

(13) The velocity of the charged carrier when acted on by an electric force can be found from the above numbers. The electrometer having been standardized, it was found that ·0037 electrostatic units of quantity would give a deflection of one division on the scale, so that we get

$$\rho_0 = \frac{32 \times \cdot0037}{\pi \times \cdot8^2 \times 30\cdot2} = 2\cdot10^{-3}$$

in the hydrogen experiments, and

$$\rho_0 = \frac{49 \times \cdot0037}{\pi \times \cdot8^2 \times 30\cdot2} = 3\cdot10^{-3}$$

in the oxygen experiments.

Let κ_H and e_H denote the values of κ and e for hydrogen and κ_O and e_O similar values for oxygen, and we get

$$\frac{4\pi\rho_0 e_H}{\kappa_H} = \cdot0043, \text{ where } \rho_0 = 2\cdot10^{-3},$$

and

$$\frac{4\pi\rho_0 e_O}{\kappa_O} = \cdot00255, \text{ where } \rho_0 = 3\cdot10^{-3}.$$

Hence

$$\frac{\kappa_H}{e_H} = 5\cdot6, \text{ and } \frac{\kappa_O}{e_O} = 15 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1).$$

From the equation $\kappa V = Fe$, which is similar to the equations (1) § (9), V being the velocity due to an electromotive force F, we obtain for the motion of the charged carrier in hydrogen

$$V = \frac{F}{5\cdot6},$$

and for the motion of the charged carrier in oxygen

$$V = \frac{F}{15}.$$

Hence under an electromotive force of 1 volt per centimetre, the hydrogen carrier travels at the rate of $\frac{1}{300 \times 5\cdot6}$ centimetres

per second, and the oxygen carrier at the rate of $\frac{1}{300} \times \frac{1}{15}$ centimetres per second.

The conclusions arrived at from the above investigations are based on the assumption that in each case we are dealing with a gas containing carriers all charged with the same sign, either positive or negative. Experiments on conductivity have been carried out in order to test this point, and it has been found that we are in reality dealing with mixtures. Thus in the case of oxygen, or hydrogen, from a sulphuric acid cell having a positive charge equal to $3e$, it is possible to obtain from the gas a positive charge of $4e$ and a negative charge of e. If we suppose the positive and negative to act independently the charge on the oxygen carrier would be 510^{-10} instead of 310^{-10}; also the velocities under a volt per centimetre would be smaller in the ratio of 3 : 4 than those given above.

We can arrive at an approximate value to the size of the carrier if we assume that the viscosity of a gas effects the motion of a small sphere and a large one according to the same law

$$6\pi\mu a V = P.$$

(Lamb, *loc. cit.*). Substituting

$$V = \frac{1}{300 \times 5{\cdot}6}, \quad \mu = 10^{-4}$$

for hydrogen, and

$$P = \frac{1}{300} \times \frac{3}{2} 10^{-10} = \frac{1}{2} 10^{-12},$$

we get the radius (a) for the hydrogen carrier $4{\cdot}510^{-7}$. Similar substitutions give the radius of the oxygen carrier 1210^{-7}. Hence the carriers are large compared with molecular dimensions.

The velocities obtained for the carriers in the conductivity due to Röntgen rays are far greater than the above. Thus for oxygen and hydrogen Mr Rutherford obtains velocities of 5·2, and 1·4 centims. per second under a volt per centimetre. [*Phil. Mag.*, Nov. 1897.]

If we assume that the charges on the carriers are of the same order as those obtained in § 7 we see that the dimensions of the carrier are smaller than those with which we are here dealing.

The discharging power of tubes.

(14) It has been shown that, if the gas with a uniform charge ρ_0 per c.c. be left in a tube for a time t, the density of the electrification falls to a uniform value ρ, given by the equation

$$\rho = \frac{\rho_0}{1 + \alpha\rho_0 t}$$

where α is a constant. Now if we suppose that the velocity of a gas as it passes along a tube be perfectly uniform and that no discharging takes place due to accidental circumstances, the values of t, ρ, and ρ_0, would be connected by the same equation, where now t denotes the time that any portion of the gas takes to traverse the tube, ρ_0 and ρ being the densities of electrification on entering and escaping from the tube.

When the gas is given off by a current through an electrolyte, the volume q which enters the tube per second is known when the current is known; so that $t = \frac{V}{q}$ where V is the volume of the tube, and the above equation becomes

$$\frac{\rho_0 - \rho}{\rho} = \frac{\alpha\rho_0 V}{q}.$$

The values of ρ_0 and ρ are easily found and satisfy this equation roughly, so that α can be determined; its value however is greater than $\frac{4\pi e}{K}$ found already. This is due to accidental discharging, as it can be shown that, even when the carrier is so large that the small E.M.F. driving it to the side of the tube could have no appreciable effect, still a measurable discharge takes place. (See § 21.)

(15) If the gas be run into an insulated inductor after traversing the tube, $\frac{\rho_0 - \rho}{\rho}$ is the ratio of the charge gained per minute by the tube to the charge gained per minute by the inductor. These charges can be easily found by connecting the tube and inductor alternately to the insulated quadrants of an electrometer. The tube C insulated in the manner described in § 10 was used in these experiments, the gas examined being the hydrogen given off by a current of 14 ampères from a dilute sulphuric acid electrolyte. The following deflections n_1 and n_2 were obtained, n_1 being the deflection per minute when the tube is connected to the

insulated quadrants, and n_2 the deflection per minute when the inductor is connected to the insulated quadrants.

n_1	n_2
16	47
9·5	36

The relative value of n_1 and n_2 in absolute units of quantity was 12 : 13·5, so that

$$\frac{\rho_0 - \rho}{\rho} = \frac{12 n_1}{13{\cdot}5 n_2}.$$

We thus obtain the following numbers showing the connection between $\frac{\rho_0 - \rho}{\rho}$ and ρ_0, so that $\frac{\rho_0 - \rho}{\rho}$ is (within 5 per cent.) proportional to the densities of the electrification.

$\frac{\rho_0 - \rho}{\rho}$	ρ_0	$\frac{\rho_0 - \rho}{\rho\rho_0}$
·30	2×10^{-3}	150
·23	$1{\cdot}46 \times 10^{-3}$	157

(16) Similar values were obtained for $\rho_0 - \rho$ and ρ for the oxygen given off from the same electrolyte by a current of 14 ampères. A different method was used in this case, which did not require an inductor. When the tube C (experiment § 10) was being filled with a charged gas, which enters with an electrification ρ_0 and escapes with an electrification ρ, the quantity $\rho_0 - \rho$ can be obtained by observing the rate (n_1 per minute) at which the spot of light moved on the electrometer scale, when the tube is full of the charged gas, and the stream kept running through it. The mean density of electrification in the tube is practically $\frac{\rho_0 + \rho}{2}$ and was obtained by blowing out the charged gas, and observing the deflection n_2 which is proportional to $\frac{\rho_0 + \rho}{2} \times V$, where V is

the volume of the tube. The following values of n_1 and n_2 were obtained:

n_1	n_2
14·5	62
9	49

The rate q at which the gas is supplied to the tube is ·85 c.c. per second: so that $(\rho_0 - \rho) \times 51$ is the charge given to the tube per minute as the gas passes through it, and is proportional to n_1, and $\frac{\rho_0 + \rho}{2} \times V$ is proportional to n_2, where V, the volume of the tube, is 60·4 cubic centimetres.

Thus we get the following values for $\frac{\rho_0 - \rho}{\rho}$ and ρ_0:

$\frac{\rho_0 - \rho}{\rho}$	ρ_0	$\frac{\rho_0 - \rho}{\rho\rho_0}$
·32	$4{\cdot}3 \times 10^{-3}$	74
·25	$3{\cdot}3 \times 10^{-3}$	76

(17) The ratio $\frac{\rho_0 - \rho}{\rho}$ was also found to increase when the current through the electrolyte diminished, thus showing that the ratio depended upon the rate at which the gas is supplied to the tube. Another tube of equal length and 1·12 centimetres in diameter was examined by similar methods, and it was found to have a far less discharging power than the larger tube; so that, when the supply q and density ρ_0 are given, the smaller the bore of the tubing that is used to convey the gas from one vessel to another, the less will be the loss of the charge. The above results showing the dependence of $\frac{\rho_0 - \rho}{\rho}$ on ρ_0 are important as we see from the formula

$$\frac{\rho_0 - \rho}{\rho} = \frac{4\pi e}{\kappa} \times \frac{\rho_0 V}{q}$$

that the increase in the charge due to heating the cell arises from an increase in the number of carriers with the same charge, and

not from an increase of the charge on the carriers already present; for, if the increase in density were due to an increase in ρ, we would have found $\frac{\rho_0 - \rho}{\rho}$ to be proportional to the square of ρ_0.

(18) It would be almost impossible to arrange to pass the gas along a tube in such a way that the discharging power would be as small as the theoretical value, calculated from the numbers given for κ, section 13. Small differences in temperature between the gas and the sides of the tube, and other effects which would give rise to currents in the gas, considerably increase the discharging power, but have a comparatively small effect on the loss of charge when the gas is allowed to stand in the tube. The velocity of the carrier towards the sides of the tube, due to an electromotive force, will on this account be too large when calculated from the formula $\frac{\rho_0 - \rho}{\rho} = \frac{4\pi e}{\kappa} \cdot \frac{\rho_0 V}{q}$.

Thus, when we substitute the values given in §§ 10, 11 for $\frac{\rho_0 - \rho}{\rho}$ and ρ_0, we obtain for K the values:

$$K_H = 3 \cdot e_H,$$
$$K_O = 12 \cdot e_O.$$

Causes which influence the discharging power.

(19) In the above experiments the gases were always treated in the same way before entering the tube C, being bubbled first through a solution of potassium iodide and then through strong sulphuric acid. When the gases are bubbled through water instead of sulphuric acid they will enter C carrying with them a cloud, the effect of gravity upon which makes itself very apparent by comparing the charges acquired per minute by the tube, when in the vertical and horizontal positions. Thus in the case of a stream of highly charged hydrogen, which bubbled through water before entering C, it was found that the charge acquired was 14 scale divisions per minute as the gas passed through C in the vertical position; and 47 divisions per minute with C in the horizontal position. The electric capacity of the conductor, consisting of C and the quadrants, was not altered by more than 3 per cent. by turning the tube round; so that the large increase in the discharging power must have been caused nearly entirely by the force of gravity acting on the drops forming the cloud. The experiment shows that the charge resides on the drops forming the cloud.

A similar result was obtained in the case of charged oxygen carrying a cloud; the corresponding numbers of divisions of the scale being 4 and 20, showed that five times the charge is removed by the tube when it is turned from the vertical to the horizontal position.

Rough values for the size of the drop might be deduced from these numbers; but the variations in density per c.c. (due to the cloud) are so great, that it would be impossible to arrive at satisfactory values, especially in the case of hydrogen, where the weight of the cloud can be as great as $\frac{1}{7}$th of the weight of the gas in the same volume.

(20) Even when the gas has been bubbled through sulphuric acid before entering C, a slight effect of this kind is still to be found. Thus, when the tube is turned from a vertical to a horizontal position, the charge acquired by it per minute increases from 16 to 19 divisions of the electrometer scale per minute. If the coefficient of viscosity can, even roughly, be applied as in section 13 to determine the size of the carrier, the effect of gravity could not exert on a carrier a force that would cause it to move through the gas surrounding it, with a velocity of the same order as $\frac{1}{300}$ centimetres per second. In fact, if we take the density of the carrier to be unity, its radius should be as big as 310^{-5}, in order that the force of gravity should make it move with a velocity $\frac{1}{300}$ centims. per second.

But if, instead of considering a single carrier, we consider the case of a finite quantity of gas (occupying unit volume say), as differing from the surrounding gas by ·05 per cent. in density, which could arise from various causes, a vertical motion far greater than the above would ensue, which would affect the discharging power of a horizontal tube.

The gases can be sent through a tube containing phosphorous pentoxide, after bubbling through sulphuric acid, so as to remove all traces of moisture; and if the powder is placed loosely in a horizontal tube, about 8 centims. long with glass wool at the end to prevent dust being carried along, not more than half the charge on the gas will be lost. The hydrogen when dried in this manner loses more of its charge, in passing along the vertical tube C, than if it had been dried by sulphuric acid alone; but the loss of charge was not found to be increased by turning the tube from the vertical to the horizontal position.

(21) The discharging power of a tube in a vertical position is, as we should expect, greatly diminished by having the gas moist, as the electric force is then too small to have any appreciable effect in causing the drops to move towards the sides.

Thus, in an experiment similar to that explained in section 15, the hydrogen, instead of being bubbled through sulphuric acid before entering *C*, was bubbled through water, and it was found that when *C* acquired a charge of 14 per minute the inductor acquired 75 per minute, whereas if a similarly charged dry gas were used, *C* would have acquired a charge at the rate of 28 per minute, and the inductor at the rate of 62.

The same property holds in the case of charged oxygen. Thus when the gas carries a cloud only 4 divisions per minute were acquired by *C*, whereas a similarly charged dry gas would have given 7 divisions per minute. The loss of charge of the clouded gas in passing through a vertical tube is due to the unevenness in the motion and the accidental contact of the cloud with the sides.

Diffusion.

(22) Experiments were performed on the diffusion of charged gases in order to find out whether any of the charge can pass

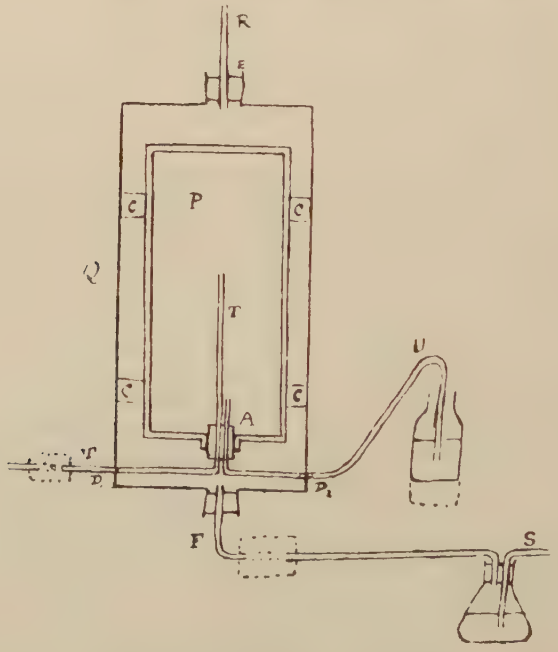


Fig. 5.

through porous earthenware. The apparatus which was used in these experiments is shown in figure 5. It consists of two

cylinders, the outer Q was of tin, and the inner P was a porous porcelain vessel fixed by means of corks (C) to the tin cylinder. The porous vessel had one aperture which was fitted with an indiarubber stopper carrying two tubes projecting through the sides of Q at D_1 and D_2. The tube T was broken, and fitted with a paraffin tunnel for purposes of insulation, and the tube U dipped into a sulphuric acid bottle and so acted as a manometer.

The outer cylinder had two apertures at E and F, the tube R from E led into a large insulated inductor, and the tube from F was connected through a paraffin tunnel to the delivery tube of a sulphuric acid bottle.

Before performing an experiment it is necessary to thoroughly dry the whole apparatus, this was done by blowing dry air through T and F. The charged hydrogen from a sulphuric acid electrolyte was dried and allowed to enter the cylinder P through T.

The bellows is connected to the tube S and a gentle current of air is blown into the cylinder through F. The cylinder Q is connected to earth, and the inductor into which R leads is connected to the insulated quadrants of the electrometer. By this means the hydrogen which diffuses through P is carried with the air current along the tube R into the inductor. The sulphuric acid in U during the course of the experiment rose in the tube showing that the hydrogen diffused as quickly as the tube T supplied it.

Only a very small fraction of the original charge was found to be on the gas after diffusion as the deflection per minute, as the diffused gas enters I, was found to be less than 2% of the deflection obtained by allowing the gas to enter I directly.

(23) Having seen that the hydrogen in diffusing carries with it no appreciable charge it is interesting to know whether the electricity is carried to the sides and gets caught in the pores of the porcelain or whether the carriers remain distributed throughout their original volume surrounded by the air which diffuses in to take the place of the hydrogen.

In order to test this the whole apparatus was insulated and experiments were conducted as follows:—

The outside cylinder Q was connected to the insulated quadrants of the electrometer, and the hydrogen from the cell after bubbling through strong sulphuric acid was admitted into P by means of the tube T. The draught of air between P and Q was maintained in order to make the diffusion as rapid as possible, no inductor was used so that the hydrogen after diffusion was carried

into the atmosphere of the room through F. As the charged gas entered P, the conducting cylinder Q being connected to the quadrant, the deflection on the electrometer scale registers the total charge that enters P.

When a sufficient charge had entered P, the tube T was closed and the sulphuric acid in the manometer U rose rapidly and showed a maximum difference of pressure between the atmosphere inside and outside P of about 3 centimetres of acid. As the blowing into E was continued the sulphuric acid in U began to fall gradually and in a few minutes the difference of pressure was reduced to one or two millimetres, which showed that practically all the hydrogen had diffused out.

The question was then decided by observing what charge could be blown out of P by blowing through T, and it was found that a large fraction of the original charge could be thus removed from inside P. The charge which is thus blown out does not represent quite all the electricity left in the atmosphere in P after the hydrogen had diffused out as about 10 per cent. got discharged in bubbling through the sulphuric acid before escaping through v.

The numbers obtained in three experiments which were performed in the above manner are given in the following table:— n_1 is the charge (in divisions of the scale) put into P, t_1 the time in minutes required to admit the charge, h_1 the maximum height in centimetres that the sulphuric acid rises in U after the tube T is closed, t_2 the time that elapsed while the manometer fell from h_1 to h_2, and n_2 the charge that was blown out of P.

n_1	t_1	h_1	h_2	t_2	n_2
210	3	3·0	·1	6	130
200	3	3·2	·2	5	125
132	5	3·0	·2	4	98

The current through the electrolyte was in each case 14 ampères.

In the last experiment the charge on the gas was reduced by passing it through a tube of phosphorus pentoxide before it enters T.

We thus see that the carriers of the electricity in the charged hydrogen can with ease be transferred to an atmosphere of any

other gas by diffusion. Also, by continuing the stream of hydrogen into P, the density of the charge in P increases, so that diffusion not only affords a method of removing the charge from a gas, but also gives us a means of increasing the electrification per unit volume.

Experiments with hydrochloric acid.

(24) When gases are given off by electrolysis both the quantity and sign of the electricity which they carry are affected by various causes. These changes which the charge undergoes are well illustrated by a series of experiments which were performed with a 20% solution of hydrochloric acid.

The form of the apparatus used to generate hydrogen and chlorine by electrolysis from hydrochloric acid is shown in figure 6. It consists of two glass bulbs, joined together, from

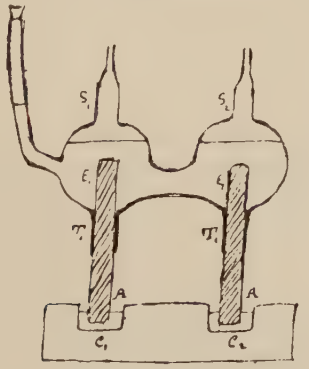


Fig. 6.

each of which project two tubes S_1, T_1 and S_2, T_2. The tubes S_1 and S_2 acted as delivery tubes for the gases, which collected in the spaces immediately underneath them. The electrodes, E_1 and E_2, consisted of carbon sticks which fitted into the tubes T_1 and T_2; indiarubber tubing projecting over the ends of T_1 and T_2 prevented the apparatus from leaking when the bulbs were filled with a liquid. The carbon sticks had a thin layer of copper deposited on the ends A, so as to make good contact with the mercury in the cups C_1, C_2. These cups acted as terminals to which the leads, from the cells supplying the current, could be joined. The current through the electrolyte was given by an ammeter in the circuit, and was easily adjusted to any required value by means of a variable resistance.

(25) The electric state of the gases was examined by leading them into an insulated inductor, connected to one pair of quadrants of an electrometer. The other pair of quadrants, the case of the electrometer, and the screen inside which the inductor was placed, were connected to earth.

The hydrogen was bubbled through a strong solution of caustic potash and the chlorine through water, before entering the inductor.

(26) The charge carried by the hydrogen evolved from a hydrochloric acid electrolyte varies in a very peculiar manner. When new carbon electrodes are used and the current turned on, the electrometer shows that the gas, at first, has a positive charge, but although the current is kept constant, this charge diminishes gradually and after some minutes becomes negative, it soon reaches its maximum negative value and the charge then remains constant, except for a small variation due to temperature. When the electrolyte is cooled, and the same current again sent through the acid, the hydrogen begins to come off with a negative charge.

(27) The chlorine, which has been evolved from the positive electrode, will be seen to have dissolved in the acid not only round the positive electrode, but also round the negative, before the change of sign is observed in the hydrogen. If now this acid be removed from G, and the electrodes well washed, and a fresh quantity of acid used, it will be found that the electrometer will indicate charges on the hydrogen, exactly similar to those at first observed except that the positive charges are not so large.

After washing the electrodes a second time, and experimenting again with a fresh quantity of acid, the same effect will be observed without any further diminution in the positive charges.

The number of divisions of the scale obtained in an experiment of this kind are :—

$$\begin{cases} 1\text{st} & \text{minute} & 14 & \text{divisions} & \text{positive,} \\ 2\text{nd} & \text{„} & 11 & \text{„} & \text{„} \\ 3\text{rd} & \text{„} & 5 & \text{„} & \text{„} \end{cases}$$

after a few minutes the hydrogen came off with a negative charge, corresponding to 5 divisions per minute. The current through the electrolyte being 10 ampères, and the value of divisions on the electrometer scale in absolute units of quantity, being obtained by multiplying them by ·003.

When the chlorine was similarly examined it was found to have a small negative charge, corresponding to 4 divisions of the

electrometer scale per minute. It did not vary in sign like the charge on the hydrogen.

(28) It is probable that the remarkable change of sign that occurs in the charge carried by the hydrogen is due more to the absorption of chlorine gas by the carbon electrode, than to the presence of chlorine gas in the electrolyte; for if the platinum electrodes were used the hydrogen was given off with a large positive charge and did not change when the chlorine got dissolved in the electrolyte.

It is improbable that the effect is due merely to the change from platinum to carbon, as there was no corresponding change when similar experiments were performed with sulphuric acid; the positive charge on the hydrogen evolved being large whether platinum or carbon terminals were used and no change occurred by running the current for half an hour, except a gradual increase in the charge which can be shown to be due to a rise in temperature. It was also found that the charge on the hydrogen was increased by using a new carbon electrode, instead of that used with the hydrochloric acid.

(29) In the case of direct electrolysis where no secondary action takes place, or other chemical effects at the electrode, considerations of polarization would lead us to expect that gases would come off carrying with them a charge of the same sign, as that which they carry in the electrolyte. Thus in the case of polarization due to a layer of hydrogen with a positive charge surrounding the electrode it is conceivable that, when the evolution of hydrogen takes place, some of the gas composing this layer would come off without losing its charge.

(30) In seeking to explain the changes of sign that occur, as in the case of the hydrogen evolved from hydrochloric acid, it must be remembered that, once the gas has acquired a charge, only a small quantity of electricity is lost by bubbling through liquids, so that causes which influence the sign of the charge should most naturally be sought for at the place where the gas is being evolved, and may be due to the state of the electrode or to impurities in the electrolyte in the immediate vicinity of the electrode.

(31) There are some well known phenomena which point to an explanation of the results obtained with hydrochloric acid, namely: the electrification of gases by glowing metals*. The typical effects of this kind are, the positive electrification of

* J. J. Thomson, *Phil. Mag.*, Dec. 1896.

oxygen when a metal is being oxidized, and the negative electrification of hydrogen when blown past a hot oxidized metal; the metal in the former case uniting more readily with the negative oxygen atoms, and the oxygen in the latter uniting more readily with the positive hydrogen atoms. The only necessity for the high temperature of the metal is to break up some of the molecules of the gas into atoms, but in the cases we are dealing with the gases, at the electrode, are already in the atomic state. We should therefore expect somewhat similar effects to take place when the gases which are evolved can form chemical compounds with the electrode, or with bodies either dissolved in it, or in its immediate vicinity.

(32) If the above principles are applied to the case of hydrochloric acid we should expect, when the gases are evolved by electrolysis, that the hydrogen would carry a positive charge, and the chlorine a negative charge. The subsequent change from positive to negative, in the case of the hydrogen, can be explained by supposing a small quantity of hydrochloric acid to be formed at the negative electrode, due to the action of the nascent hydrogen on the chlorine, and that the positive hydrogen atom is more active in forming a compound with chlorine than the negative. A similar explanation would apply to the change which occurs from negative to positive in the charge carried by the chlorine when platinum is used instead of carbon for an electrode, since a chloride of platinum is formed.

(33) The charged hydrogen given off from a platinum electrode, in the electrolysis of hydrochloric acid, has the same property of forming a cloud when bubbled through water as the highly charged gases obtained by other methods. The chlorine however has only a small charge, and moreover contains a quantity of hydrochloric acid vapour which would be difficult to remove, without making the charge too small to form a visible cloud. The hydrogen was treated in the following manner, in order to get rid of the acid vapour which would form a cloud when bubbled through water. The delivery tube from the cell was connected to the tube T_1 which dipped into a strong solution of caustic potash contained in a flask H_1, and the gas then passed along a tube containing glass wool soaked in caustic potash before it bubbled through the water in a flask H_2. The hydrogen which was evolved from the platinum electrode, after passing through this apparatus, was led into an insulated inductor connected to the electrometer and, with 10 ampères through the electrolyte, the spot of light on the electrometer scale was deflected 37 divisions per minute, and with 14 ampères, 62 divisions per minute, the

temperature of the cell being 36°. In both cases a dense cloud was observed above the water in H_2.

In order to test the efficiency of the apparatus, for removing the acid vapour, and at the same time to show that newly prepared hydrogen does not form a visible cloud unless it has a considerable charge, the gas evolved from the carbon electrode was led into H_1. It was found that the electrometer was giving only 3 divisions per minute and no cloud could be seen over the surface of the water in H_2, although the temperature of the cell, and current through it, were the same, as when 37 divisions per minute were obtained by using a platinum electrode. In weighing experiments were performed in order to find the ratio of the weight of the cloud to the charge, but it could easily be seen that the cloud was less dense than that which would be formed in similarly electrified gas evolved from a sulphuric acid electrolyte.

In addition to the above experiments, the effect of Röntgen rays in discharging the gas has been examined. It is hoped that the results of this investigation, and the experiments on conductivity alluded to in section 13, will be ready for publication in the near future.

In conclusion I desire to express my thanks to Professor Thomson for the valuable assistance his suggestions have afforded.

(3) *On a chemical effect produced by the impact of kathode rays.* By Professor J. J. Thomson and Mr S. Skinner.

Aluminium is rapidly evaporated from the kathode by an electric discharge in a highly exhausted vacuum tube in which air has been replaced by mercury vapour. The metal is condensed over the walls of the tube in the form of a bright mirror. An iron kathode gives a similar mirror in a mercury vapour discharge tube. When the aluminium coating is dissolved off the wall of the bulb by hydrochloric acid a gelatinous membrane remains which gives the reactions of silica, and the escaping hydrogen has an unpleasant odour from the small quantity of silicon hydride which it contains. The iron mirror dissolves completely in hydrochloric acid. From these observations it appears that aluminium driven off from a kathode by an electric discharge acts on the glass, forming a silicide, which on solution in hydrochloric acid yields aluminium chloride, gelatinous silica and some silicon hydride.

When potassium vapour is used in the place of mercury the glass opposite the aluminium kathode is roughened. In parts sheltered by screens from the discharge the glass is not attacked.

In potassium vapour the aluminium kathode is not evaporated to any marked degree. In this case the action on the glass may be attributed to the potassium which under the action of the discharge attacks the glass forming soluble potassium silicate; so when the bulb is washed out with water and dried a frosted surface is left.

Opposite the kathode both in the mercury vapour and potassium vapour bulbs a dark annular stain of the shape of the kathode is formed. This stain resists the action of strong hot hydrochloric acid, nitric acid, aqua regia and potash solution. The action of hydrofluoric acid removes it apparently by dissolving the glass. These tests indicate carbon rather than silicon, but the quantity of the stain is too small to make certain. The stain is also formed on screens of mica, quartz and calcite; and this supports the view that it is carbon fractionally evaporated from the aluminium and iron cathodes. The sheet aluminium and iron used for kathodes was not pure.

For comparison some pieces of an old incandescent lamp which was blackened inside with carbon, evaporated from the filament by long use, were treated with hydrofluoric acid and examined microscopically. The visible effects in this case were very like those observed when the stain was treated with the acid. The stain in each case was apparently in a state of tension and portions of it curled up when broken.

Monatomic gases appear to permit the evaporation of aluminium, as Professor Callendar has observed its evaporation in an argon vacuum tube.

(4) *On the effect of zinc and other metals on a photographic plate.* By Professor J. J. Thomson.

In the course of a discussion at the Cavendish Physical Society on Dr Russell's paper on the photographic effect produced by certain metals, Sir George Stokes suggested that possibly light might be thrown on the question as to whether these effects were due to radiation or to the vapour of the metals, if photographs were taken with a stream of air flowing between the metal and the photographic plate. In consequence of this suggestion a series of the photographs made by zinc and amalgamated zinc (1) with nothing but air between the zinc and the photographic plate, (2) when the zinc was covered with a film of celluloid, were taken both with and without an air blast. The photographs with the air blast were found in both cases to be distorted, which is in favour of the view that the effects on the photographic plates are due to the vapour of the metals.

Monday, 6 December, 1897.

MR F. DARWIN, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

Mr A. S. Ramsay, M.A., Fellow of Magdalene College; Mr W. M. Fetcher, B.A., Fellow of Trinity College.

The following were elected Associates of the Society:

Mr J. Zeleny, Mr H. A. Wilson, Mr G. H. Shakespeare, Advanced Students of Trinity College; Mr J. H. Vincent, Advanced Student of St John's College; Mr P. Langevin, Advanced Student.

The following Communications were made to the Society:

(1) *Notes on some Hawaiian Insects.* By R. C. L. PERKINS.

Amongst the most important representatives of the Order of Neuroptera in the Hawaiian Islands, are the dragon-flies of the genus *Agrion*. Several species are found on all the more important islands of the group, and the range of many of them extends over several islands; wherein they differ from the greater part of the endemic insects, which are for the most part confined to a single island; or to one or two of those which lie most closely together. At the same time, when a series of examples of a species from different islands is compared, certain more or less constant differences are often observable, especially as regards size.

But the most interesting facts relate to the earlier stages or nymphs, which are aquatic and carnivorous. Excluding these dragon-flies and a few water-beetles, the insect fauna of the streams and pools is almost non-existent. The Ephemeridae, Perlidae, and Trichoptera, usually so numerous, are entirely unrepresented in the Islands, although the mountain streams, rising at high altitudes, with their superb waterfalls, and various temperatures, appear admirably adapted for many of these. It is therefore not a little surprising to find the group of dragon-flies so well represented, and that the individuals are so numerous, being on the whole the most conspicuous of all the endemic insects. In the absence of the groups above mentioned, I believe that their main food-supply comes from without, consisting of such creatures as accidentally fall into the water. Under ordinary circumstances this is not great, but after rain, when the streams rise very quickly, food becomes abundant. When the streams, as is often the case, become nearly dry, large numbers of creatures resort to

the pools that are left, for the sake of the moisture, and the numbers that come to grief is often astonishing, the whole surface being covered with the drowned and drowning.

There are, however, other species the nymphs of which live under very different circumstances. These have given up their aquatic life, and live hidden at the bases of the leaves of a liliaceous plant—*Astelia veratroides.* Sometimes a little water is held by the plant around the stem, but more often there is merely a collection of damp earth and dead leaves. These nymphs would even appear to dislike the collections of water, for in wet weather they often crawl halfway up the leaves, instead of remaining at the base, where the water accumulates. They differ in some points from those which frequent the water; they are shorter and stouter, and much more sluggish, and the caudal appendages are very short and thick, differing therein greatly from some of the aquatic species, the appendages of which form beautiful tracheal gills.

On the whole they are without doubt better off as regards a food supply than the aquatic species, for there is generally abundance of animal life around them. A number of interesting beetles breed only in this plant, and minute young of molluscs and earthworms are generally abundant in the same, as well as the larvae of small moths. Moreover nymphs of various sizes often frequent a single plant, and if hard pressed for food the larger, no doubt, devour the smaller individuals.

In consequence of these habits, some of these species of dragon-flies, although their powers of flight are feeble, may often be seen in numbers in localities remote from water, and where they would not naturally be looked for.

These terrestrial nymphs are able to endure extreme drought. On one occasion when out shooting, having no more convenient receptacle, I carried a number for the greater part of the day in an envelope. In the evening, although very dry, they were still quite lively. They were then placed in a tumbler of water, where they remained on the bottom, not being able to crawl up the sides. Here they remained for a day, apparently as happily as on dry land, when they were taken out and preserved.

To turn to the Order of Coleoptera, which is more richly represented in the Islands than any other, by far the most conspicuous are the species of the genus *Plagithmysus.* With the allied genera *Callithmysus* and *Clytarlus* they form a special group of Longicorns, and no nearly allied form is known from elsewhere. Hence the country from which they may have been derived is quite uncertain. About 40 species are at present known. These beetles are remarkable in many ways, more particularly for the manner in which the wings are folded, there

being no transverse folding, so that they project beyond the elytra; secondly in the small development of the hind-body, at least in the males; and thirdly as being furnished with three distinct sets of stridulating organs. With regard to the latter the thoracic arrangement for stridulation is of the usual kind. The second mode by which sounds are produced, is by rubbing the bases of the hind femora against the margins of the elytra. These margins are specially sculptured much like a file, and the parts which rub them are roughened with minute tubercles. Altogether the arrangement reminds one not a little of the stridulating organs of the grasshoppers, and the *Plagithmysi* themselves have some little resemblance in general appearance to those insects. The third set of stridulating organs, which were discovered and pointed out to me by Dr Sharp, are on the basal joint or coxae of the middle and hind legs, and they may really be considered as two distinct sets, since the middle pair operate on the adjoining edge of the thorax, the hinder pair on a ridge on the basal segment of the abdomen. Moreover, in a specimen of *P. aequalis* examined, the striation of the coxae on either pair of legs was different, being evidently coarser on the intermediate pair.

It is highly probable therefore that by the use of these different organs a considerable variety of sounds can be produced, for the sound made by the legs when rubbing against the elytra is not the same as that produced by the prothorax rubbing on the scutellum, and the degree of pressure, or rapidity of movement, also causes a variation in the sounds. The insects run with extreme rapidity over the tree trunks, when the sun shines hotly on them, and often perform curious evolutions, running in short curves or zigzags and at times almost giving little leaps. It seems hardly possible that such movements can be made without stridulation, although the sounds are not audible unless the creature is held in the hand.

The small size of the hind-body may be connected with the frequent great disparity of size between the sexes, for the females of many of the species are often relatively very minute, and probably in all the species many such individuals would be found were sufficient material available for examination. While the minute males are not affected by the almost rudimentary condition of the hind-body, were this of the usual length, many of the large males would be quite unable to pair with a large proportion of the females. As it is, this is not the case, and male and female will often pair though the former is several times the bulk of the latter. Moreover it should be mentioned that both sexes are polygamous and pair many times. It is possible however that the rudimentary condition of the hind-body is in some way connected with the stridulating organs.

The extreme variation in size of a given species may be due to differences in the quality of the food of the larvae, which may be found in a tree which has only just fallen, or begun to die, while others are feeding on a tree the wood of which is in a much drier condition.

One of the most interesting points in regard to the Plagithmysi is the extreme variability of most of the species, which affects not only the colour but also the structure. The variation in the latter, however, is not so readily appreciated as the colour-variations, and I do not propose to refer further to it here. Variation in colour is shown in various parts of the insect but is more especially remarkable on the posterior femora.

On the Island of Hawaii there are four very closely allied species of the genus. Three of these, *P. varians*, *darwinianus*, and *lamarckianus*, are found in the same locality, but each is solely attached to the kind of tree on which its larva feeds. Where the different trees grow side by side the three Plagithmysi may be found side by side; but in some parts of the forest only one of the three trees is to be found, in which case only the one species whose larva feeds on it is also found. The fourth species, *P. blackburni*, feeds on the same tree as *darwinianus*, but occurs on the other side of the island.

All of these species are variable in the colour of the femora, but the variation as exhibited in a series is distinct in each. In some cases the variation is so great that the extreme forms would appear to be more distinct, than are some of the allied species from each other. I have examined a considerable number of individuals of each of these species which are here exhibited. Thus of *P. darwinianus*, which feeds on *Sophora chrysophylla*, an acacia, the 65 individuals all have the femora red, the apices being very rarely dark.

Of *P. blackburni* which feeds on the same tree but not in the same locality, 68 have these parts black, only 5 red.

Of *P. varians*, which feeds on *Acacia koa*, 41 have the femora black, 53 have the apices bright red, while 4 are nearly entirely of a reddish colour.

Of *P. lamarckianus*, which feeds on trees belonging to the Urticaceæ, the legs are wholly black or reddish, probably either variety being about equally numerous, the species being much less common than any of the preceding.

Thus on the whole each species divides into two or three varieties without true intermediate forms. It is true that if some of the individuals of *P. varians* be closely examined, the apices of the femora are seen to be less dark (i.e. slightly reddish), but these fall in naturally with the black-legged specimens, and are very far removed from the brightly coloured examples.

The only other species examined is *P. aequalis*, a species very distinct from any of the preceding, found on the Island of Kauai.

The variation in this species is similar in kind to that of *P. blackburni*, but the relative numbers of each variety are very different, for while in that species the red-legged variety is extremely rare, in the present one out of 181 examined no less than 70 are of this form. The large amount of variation exhibited is the more remarkable on account of the limited distribution of the species, for not one of the genus is known to inhabit more than one island, and many of them are certainly confined to a very small area of the island on which they are found.

It has been already mentioned that all the species are polygamous. Of selective pairing by the different varieties there is no trace. On one occasion when watching a number of *P. varians* on a fallen tree trunk, I noted that a female of the black-legged form paired successively with the brightly coloured variety, which has the apices of the femora red, then with one of her own colour, and shortly afterwards with the extremely rare variety which has the legs entirely dull red.—And instances of a similar kind can be noticed any day, where a species is plentiful. Were sexual selection operative there is no doubt that a much larger number of distinct forms would be produced. It is interesting to consider the conditions that have brought about the specific distinctions in these species so closely allied. Taking into consideration the fact that no species is found on more than one island, either geographical isolation, or a difference in food, or a combination of these two factors, seems to account sufficiently for the separation of the species. Of the species under consideration, three may be found side by side (although each of them is also found in places apart from the other two), yet each keeps entirely to the tree on which its larva feeds; and the other *P. blackburni*, although it has the same food as *P. darwinianus*, does not inhabit the same area.

So far as the native fauna is concerned these *Plagithmysi* have few enemies. It is doubtful whether any parasites attack them; if any do so, they are certainly of extreme rarity. Except on the Island of Maui, birds either leave them alone or perhaps are very rarely able to catch them. On that island, however, one of the Drepanididae—*Pseudonestor*—is extraordinarily modified in accordance with its habit of living on the larvae, pupae and immature beetles of this genus and the allied *Clytarlus*. Indeed its food consists chiefly of these creatures, which it is able to tear out from the hardest wood. The great scars, made on trees which are affected by the beetles, may often be seen as the result of the work of this bird, and when opened its stomach is usually filled with their larvae.

Like the other orders of Insects the Hymenoptera are represented by a very few types out of the enormous number that exist elsewhere. Thus the bees, excluding a few isolated forms, which have either certainly or probably been introduced by man, are represented by a single genus; the wasps also by a single genus, and the Fossors by two genera of Crabronidae, and two closely allied forms of Mimesidae. It is to the remarkable evolution of species of each of these types that the comparative wealth of the aculeate hymenoptera is due. Thus while there are a very few genera which can be looked upon as endemic, the average number of species to each genus is very great, this number being about 28. Most numerous of all in species is the genus *Odynerus* represented in the Islands by some 80 distinct forms. They are one of the most important elements in the fauna, and are sufficiently numerous to attract the attention of people not particularly interested in such things, the more so as one or two of the species abound around houses, and avail themselves of holes in woodwork or furniture for forming their cells. Although intimately related to one another, and extremely similar in general appearance, in reality they exhibit a very great variety of structure. If one collects these wasps over the Islands from Oahu to Hawaii, the general blackness of their colour is very remarkable. Some are more or less marked with red, the markings being rarely noticeable, unless the insect is caught and examined, and a few have one or two narrow bands of a pale yellow colour on the hind-body, which also are not as a rule easily noticed. Many are entirely black. If now one compares with these a collection of the species from the Island of Kauai a difference is at once seen. Excluding one or two species which frequent the coast and, scarcely modified, have a wide distribution over the Islands, all the species have two distinct yellowish abdominal bands, the second being nearly always evidently wider than the first. When the insects are on the wing, these bands are plainly seen. It so happened that in the course of my collecting over the Islands, Kauai was not visited until after the other islands had been collected over, so that it was no small surprise to find the wasps so distinct in superficial appearance. At the time I supposed that the Kauai species would turn out to be very closely related to one another, at least more closely than to those on the windward islands. This however proves not to be the case, for some of the most remotely allied species on the other islands, are represented on Kauai by species which are structurally only slightly modified forms of the same, although their superficial appearance is very distinct. For some reason or other it is clear that the *Odyneri* of Kauai have assumed an appearance almost unique amongst the great number of species occurring in the Islands—they have what are called 'warning

colours'. That these markings have any such significance is I think certainly not the case. There is no reason to suppose that the species on Kauai have any more need of their special coloration than the black species of Hawaii, but rather the reverse, as birds, which are their only possible enemies, are more numerous on the latter island. As a matter of fact, after the examination of the food of many individuals of nearly all the endemic birds, I have never found them to feed on any of these insects whether black or coloured, nor would the majority of them be the least likely to do so, on account of their habits and structure. For the most remarkable feature exhibited by the endemic land-birds is the specialization of form for the purpose of securing some special food, which has led to the extreme diversity of structure exhibited in a group so small as the Drepanididae. So extreme is this specialization of some of the birds, that their food is almost limited to one special form of plant or animal, as we have already seen in the case of *Pseudonestor;* while even in less specialized birds the range of food is very small. It is just these most highly specialized birds that are the first to disappear when their natural surroundings are interfered with, as they have already done on the Island of Oahu, which has been more upset than the rest of the group, by foreign settlers. It is not exaggerating to say that the extinction of a few species of beetles would exterminate both *Pseudonestor*, and the various species of *Heterorhynchus*, while the disappearance of three species of trees would do the same for the three genera Rhodacanthis, Chloridops, and Loxioides, and many of the other birds are equally dependent on a particular section of either fauna or flora.

The birds most likely to feed on the Odyneri are the fly-catchers of the genus Chasiempis, which do not appear to do so, but confine their attention for the most part to small Lepidoptera and their larvae. If they did feed on these, one would expect the Odyneri of Hawaii to have distinctive marks as well as those of Kauai, since the birds are equally common upon both. But there is every reason to believe that the colours of the insects have no such significance, and it is still further convincing to notice that the *Odyneri* of Molokai have a general tendency to red-markings—Molokai being an island where no fly-catchers exist,—while on Hawaii these insects, even in certain points of structure of secondary value, tend to greatly resemble each other.

The distinct style of coloration of the Kauai species is a point of some importance, for it has long been known that in other countries insects of this group have a great general resemblance to one another in the country they inhabit, as for instance the highly endemic species in Australia and Chili. No doubt, were they more widely known, these cases of Chilian and Australian

insects would be referred to as good instances of what are called 'warning' colours, but from the study of the Hawaiian species such a significance becomes very doubtful. It seems much more probable that they are due to the general condition of the country they inhabit—it may be climate or some such cause.

(2) *Remarks on a journey to investigate the habits and development of* Lepidosiren paradoxa. By Mr J. Graham Kerr, B.A., Christ's College.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Monday, 24 *January*, 1898.

Mr F. Darwin, President, in the Chair.

The following communications were made to the Society:—

(1) *A New Method in Combinatory Analysis with applications to Latin Squares and associated questions.* By Major P. A. MacMahon, R.A., Sc.D., F.R.S.

This paper is printed in the *Transactions*, Vol. xvi. Part iv.

(2) *Abelian Functions in connexion with two-dimensional fluid motions.* By H. F. Baker, St John's College.

1. Consider the portion of the upper half plane of the complex variable ζ which lies outside p circles, which do not intersect each other or the real axis. Then the function which gives the streaming motion in this portion of the upper half plane due to a doublet of arbitrary strength and position is already known (*loc. cit.* (β), (θ))*.

* The references given to the literature are by the Greek letters, as follows: (α) Riemann, *Ges. Werke* (1876), No. xxv., p. 413 (or *second edition*, p. 440); (β) Schottky, *Crelle*, lxxxiii. (1877); (γ) Schottky, *Crelle*, ci. (1887); (δ) Schottky, *Crelle*, ci. (1890); (θ) Burnside, *Proc. Lond. Math. Soc.* xxii., xxiii., and *Messenger of Mathematics*, xx. (1891); (κ) the writer's Abelian Functions (1897), Chap. xii.; (λ) Kluyver, *Acta Math.* xxi. (1897).

As we shall require the result, and to illustrate the point of view, we give a proof.

Denote the real axis by C_0; the points of the circles which are nearest to the real axis by $C_1', \ldots, C_p'$, and these circles themselves by the same letters, reckoning from right to left; denote the region bounded by $C_0, C_1', \ldots, C_p'$ by Ω_0; denote the circles which are the images of $C_1', \ldots, C_p'$ in the real axis by $C_1, \ldots, C_p$, the points of them which are nearest to the real axis being also denoted by $C_1, \ldots, C_p$; denote the region bounded by the $2p$ circles $C_1', \ldots, C_p'$, $C_1, \ldots, C_p$ by Ω, the axis C_0 being then disregarded. Then it is known (*loc. cit.* (β)) that there exists a function, G, single valued in Ω_0 save for real periods, real on $C_0, C_1', \ldots, C_p'$, and infinite at an arbitrary point $\zeta = t$ like $(P + iQ)/(\zeta - t)$, where P, Q are arbitrary real constants. Being real on C_0, this function will (Schwarz, *Ges. Abh.*, II. p. 66), have the same value at any point of the circumference C_r as at the point of the circumference C_r' which is its image in C_0, and will be infinite in Ω, beside at t, at $\zeta = t'$, where t' is the conjugate complex of t, namely like $(P - iQ)/(\zeta - t')$. Now the region Ω when considered as the locus for functions whose values at any point of one of the boundaries $C_1', \ldots, C_p'$ are equal to the values at the conjugate complex point of the boundaries $C_1, \ldots, C_p$, is a p-ply connected Riemann surface, for which the fundamental functions are well known (*loc. cit.* (κ)). It follows therefore from the theory of functions on such a surface that for proper values of the constants $A_1, \ldots, A_p, A$, the function desired is given by

$$G = -(P + iQ)\,\Gamma_t^{\zeta, a} - (P - iQ)\,\Gamma_{t'}^{\zeta, a} + A_1 v_1^{\zeta, a} + \ldots + A_p v_p^{\zeta, a} + A;$$

here A is an arbitrary real constant, and $A_1, \ldots, A_p$ are determined by

$$A_1 \tau_{1,r} + \ldots + A_p \tau_{p,r} = 2\pi i\,[(P + iQ)\,\phi_r(t) + (P - iQ)\,\phi_r(t')], \quad (r = 1, \ldots, p),$$

where $\phi_n(\zeta)$ denotes $dv_n^{\zeta, a}/d\zeta$; the point $\zeta = a$ is supposed taken on C_0.

By applying similar reasoning we find for the function giving the general circulatory motion in Ω_0,

$$\phi + i\psi = B_1 v_1^{\zeta, a} + \ldots + B_p v_p^{\zeta, a} + B,$$

where $B_1, \ldots, B_p$ are determined by the values of ψ on $C_1', \ldots, C_p'$; and infer, by the way, that $v_r^{\zeta, a}$ is real when ζ and a are real, and that $\tau_{r,s}$ is a pure imaginary, so that the constants $A_1, \ldots, A_p$ above are real (*loc. cit.* (θ), (λ)).

Therefore when ζ, a are real

$$(P+iQ)\,\Gamma_t^{\zeta,a}+(P-iQ)\,\Gamma_{t'}^{\zeta,a},$$

is real; as can be shewn from the explicit formulæ.

This function is single valued in Ω_0; it has its imaginary part constant on each of the circumferences $C_1', \ldots, C_p'$.

2. The problem of Riemann's paper (*loc. cit.* (α)), in which however the boundaries are not necessarily circular, is the determination of the solution, for the region Ω_0, of the potential equation

$$\frac{\partial^2 U}{\partial \xi^2}+\frac{\partial^2 U}{\partial \eta^2}=0,$$

which is finite within Ω_0 and takes on the boundary an assigned continuously varying value u. This is given, at $t=\xi+i\eta$, by

$$U=\int u P_t^{\zeta},$$

where P_t^{ζ} is the function real on the boundary of Ω_0, with only real periods, which is infinite at the interior point t like $\frac{1}{2\pi i}\log(\zeta-t)$. This function is known (*loc. cit.* (λ)), or follows as before from the theory of a Riemann surface, being given by

$$P_t^{\zeta}=\frac{1}{2\pi i}\Pi_{t,t'}^{\zeta,a}+A_1 v_1^{\zeta,a}+\ldots+A_p v_p^{\zeta,a}+A,$$

where t' is the conjugate complex of t, a is real, A is a real constant, $A_1, \ldots, A_p$ are real constants determined by the equations

$$v_r^{t,t'}+A_1\tau_{1,r}+\ldots+A_p\tau_{p,r}=0, \qquad (r=1, 2, \ldots, p).$$

3. Among the single valued functions for the region Ω_0, with rational infinities, there is one of particular interest—that namely which is real on the boundary and has a single pole of the first order at an assigned point of each of the $(p+1)$ separate boundary curves $C_0, C_1', \ldots, C_p'$. This function effects a conformal representation of the region Ω_0 upon an infinite half plane $(p+1)$ times covered, and is thus the direct generalisation of the function giving the conformal representation of a simply connected region upon a simple half plane.

In what follows we put down this function for the case when the circles $C_1', \ldots, C_p'$ are unrestricted (save by the inequalities which are necessary for the convergence of the series involved); and then give the very simple form taken by the function when the Abelian functions involved are those of hyperelliptic character.

The construction of a function, single valued in the region Ω, unchanged by the substitutions which transform the circles $C_1', \ldots, C_p'$ respectively into $C_1, \ldots, C_p$, and having $p+1$ poles of the first order within Ω, of arbitrary position, is known—being as follows (*loc. cit.* (κ)); let the poles be at $z, c_1, \ldots, c_p$, and let

$$\Phi(z, \zeta) = \sum_r \frac{(\gamma_r z + \delta_r)^{-2}}{z_r - \zeta};$$

let the additive arbitrary constant be determined by the condition that the function vanishes at $\zeta = a$; then, the function is given by

$$E \begin{vmatrix} \Phi(z, \zeta), & \Phi(z, a), & \Phi(z, a_1), & \ldots, & \Phi(z, a_p) \\ \Phi(c_1, \zeta), & \Phi(c_1, a), & \Phi(c_1, a_1), & \ldots, & \Phi(c_1, a_p) \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots & \ldots\ldots \\ \Phi(c_p, \zeta), & \Phi(c_p, a), & \Phi(c_p, a_1), & \ldots, & \Phi(c_p, a_p) \\ 1, & 1, & 1, & \ldots, & 1 \end{vmatrix},$$

where E is an arbitrary constant, and $a_1, \ldots, a_p$ are the analogues of a by the p fundamental substitutions whereby the circles $C_1', \ldots, C_p'$ are respectively transformed into $C_1, \ldots, C_p$. In the case under consideration the region Ω has the specialty that the circles $C_1, \ldots, C_p$ are the inverses of $C_1', \ldots, C_p'$ in regard to C_0—we proceed to prove that in this case, if z be upon the real axis C_0, and $c_1, \ldots, c_p$ be respectively upon the circumferences $C_1', \ldots, C_p'$ then, for a suitable determination of the constant E, the function is real upon each of the curves $C_0, C_1', \ldots, C_p'$. The necessary value for E is that of the inverse of the minor of $\Phi(z, \zeta)$ in the determinant (or any real multiple of this)—when E is so determined the function may be denotod by $\psi(\zeta, a; z, c_1, \ldots, c_p)$.

To prove this it is sufficient to prove that the function is real upon the real axis (Schwarz, *Ges. Abh.*, II. p. 66). Now it follows from Riemann's theory that the function, uniform in Ω, unchanged by the p fundamental substitutions, with poles at $z, c_1, \ldots, c_p$ respectively on $C_0, C_1', \ldots, C_p'$, and vanishing at $\zeta = a$, is given by

$$f(\zeta) = (P_1 + iQ_1)\,\Gamma_{c_1}^{\zeta, a} + \ldots + (P_p + iQ_p)\,\Gamma_{c_p}^{\zeta, a} + (P + iQ)\,\Gamma_z^{\zeta, a},$$

where P_1, Q_1, etc., are real constants whose ratios are given by

$$(P_1 + iQ_1)\,\phi_n(c_1) + \ldots + (P_p + iQ_p)\,\phi_n(c_p) + (P + iQ)\,\phi_n(z) = 0 \quad (n = 1, 2, \ldots, p),$$

where $\phi_n(\zeta) = dv_n^{\zeta, a}/d\zeta$. We have already seen that $\phi_n(\zeta)$ is real when ζ is real; hence these equations involve also, if c_r', c_r be conjugate complexes, the equations,

$$(P_1 - iQ_1)\,\phi_n(c_1') + \ldots + (P_p - iQ_p)\,\phi_n(c_p') + (P - iQ)\,\phi_n(z) = 0 \quad (n = 1, 2, \ldots, p),$$

which, in turn, shew that, a being real, the function

$$f_1(\zeta) = (P_1 - iQ_1)\,\Gamma^{\zeta, a}_{c_1'} + \ldots + (P_p - iQ_p)\,\Gamma^{\zeta, a}_{c_p'} + (P - iQ)\,\Gamma^{\zeta, a}_{z},$$

is a single valued function unaltered by the fundamental substitutions. The poles of $f_1(\zeta)$ in the ζ plane are however the same as those of $f(\zeta)$, since c_r', c_r are analogues of one another for the r-th of the fundamental substitutions—and the functions $f_1(\zeta)$, $f(\zeta)$ vanish at the same place a. Wherefore, considering the residues at the pole z, we infer

$$(P + iQ)^{-1} f(\zeta) = (P - iQ)^{-1} f_1(\zeta);$$

as we have already shewn that for conjugate complex values of ζ the quantities

$$(P_r + iQ_r)\,\Gamma^{\zeta, a}_{c_r}, \quad (P_r - iQ_r)\,\Gamma^{\zeta, a}_{c_r'},$$

have conjugate complex values, it follows that the function $(P + iQ)^{-1} f(\zeta)$ has the same property.

This proves the theorem in question.

4. The same argument can be used to establish the further result: Take $p + 1$ points entirely within the region Ω, so situated that those which are not upon the real axis C_0 consist of pairs occupying conjugate positions in regard to this axis. The uniform function unaltered by the p fundamental substitutions which has these points for poles of the first order is necessarily real upon C_0, C_1', ..., C_p' (and C_1, ..., C_p), or can be made so by multiplication by a proper constant factor. For the region Ω_0 this gives the theorem: take any m points within Ω_0 or upon one or more of the circumferences C_1', ..., C_p', and any k points upon C_0, so that $m + k = p + 1$; there exists a function uniform in Ω_0, real on C_1', ..., C_p', C_0, and infinite to the first order at each of the $m + k$ points taken in Ω_0.

It may be of interest to give the functions of §§ 3, 4 for $p = 1$. Let the fundamental substitution be

$$(\zeta' - B)/(\zeta' - A) = \mu\,(\zeta - B)/(\zeta - A),$$

and $\nu = |\sqrt{\mu}|$. Then if ω be any real quantity,

$$= \frac{C_1'B}{AC_1'}, \quad \nu = \frac{C_0B}{AC_0} \Big/ \frac{C_1'B}{AC_1'}, \quad \omega' = \frac{\omega}{i\pi} \log \nu, \quad u = \frac{\omega}{i\pi} \log\left(\frac{1}{\sigma}\,\frac{\zeta - B}{\zeta - A}\right),$$

forms which hold when C_0 becomes a circle, C_0 being the point where the straight line $C_1'C_1$ cuts the circle C_0), the general and a particular form of the function of § 3 are given by

$$\frac{\wp'(u) + \wp'(v)}{\wp(u) - \wp(v)} - \frac{\wp'(u) + \wp'(w + \omega')}{\wp(u) - \wp(w + \omega')}, \quad \frac{\wp'(u)}{\wp(u) - e_3},$$

where v, w are real; and the function of § 4 by

$$\frac{\wp'(u)+\wp'(t)}{\wp(u)-\wp(t)}-\frac{\wp'(u)+\wp'(t_0)}{\wp(u)-\wp(t_0)},$$

where t belongs to a point in Ω_0, and t_0 is the conjugate complex quantity.

The function of § 1, which is real on C_1' and C_0, and has only real periods, and has one arbitrary pole of the first order in Ω_0, is given by

$$(P+iQ)\frac{\wp'(u)+\wp'(t)}{\wp(u)-\wp(t)}+(P-iQ)\frac{\wp'(u)+\wp'(t_0)}{\wp(u)-\wp(t_0)}+4P\left[\zeta(u)-u\frac{\eta'}{\omega'}\right],$$

where P, Q are real quantities.

5. We consider now the case in which the circles $C_0, C_1', \ldots C_p'$ are all cut at right angles by another circle O. It is known that in this case the functions arising reduce themselves to hyperelliptic functions. The figure (1) drawn for $p=3$, will explain the notation we adopt.

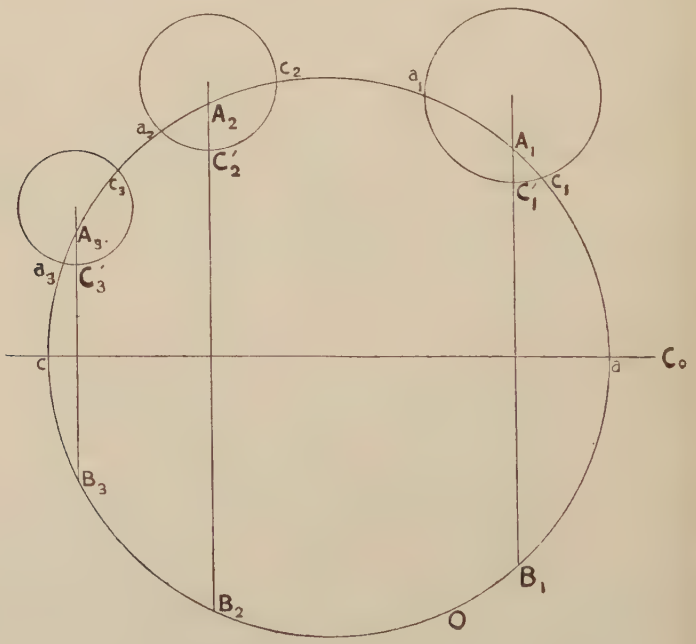


Fig 1.

In any case where the $2p$ circles C_1', C_1, ..., C_p', C_p consist o pairs of circles inverse to one another in regard to the real axis C

the singular points lie within the circles $C_1', C_1, \ldots, C_p', C_p$, and consist of pairs of points inverse to one another in regard to C_0. This is clear either because the substitution $\mathfrak{S}_r$ is equivalent to an inversion in regard to the circle C_0 followed by an inversion in regard to the circle C_r, or, what is the same thing, by remarking that if

$$\phi = \ldots \mathfrak{S}_1^{n_1} \mathfrak{S}_2^{n_2} \ldots, \quad \phi' = \ldots \mathfrak{S}_1^{-n_1} \mathfrak{S}_2^{-n_2} \ldots$$

be two substitutions in which the primary factors occur in the same order but are the inverses of one another, and, if z, z' be conjugate complexes, then also $\phi(z)$ and $\phi'(z')$ are conjugate complexes. But in the case in which the circles $C_0, C_1', \ldots, C_p'$ are all cut at right angles by another circle, O, it follows further that the singular points all lie upon this circle O.

Hence follows immediately another consequence; we have already seen that the imaginary part of the function

$$v_n^\zeta = \frac{1}{2\pi i} \sum_j \log \frac{\zeta - \mathfrak{S}_j(B_n)}{\zeta - \mathfrak{S}_j(A_n)}$$

is constant on each of the circles $C_1', \ldots, C_p', C_0$, being zero on C_0; it can now be seen that *the real part of v_n^ζ is constant on each of the arcs of the orthogonal circle O which are limited by the circles $C_1', \ldots, C_p', C_1, \ldots, C_p$.* For if j, j' denote two substitutions such as ϕ and ϕ', we can put

$$v_n^\zeta = \frac{1}{2\pi i} \sum_{j, j'} \log \frac{\zeta - \mathfrak{S}_j(B_n)}{\zeta - \mathfrak{S}_{j'}(A_n)} \cdot \frac{\zeta - \mathfrak{S}_{j'}(B_n)}{\zeta - \mathfrak{S}_j(A_n)},$$

and $\mathfrak{S}_j(B_n)$, $\mathfrak{S}_{j'}(A_n)$—as also $\mathfrak{S}_{j'}(B_n)$, $\mathfrak{S}_j(A_n)$—will be two singular points having conjugate complex positions upon O. As the real part of v_n^ζ depends only on the arguments of the factors

$$[\zeta - \mathfrak{S}_j(B_n)]/[\zeta - \mathfrak{S}_{j'}(A_n)],$$

the result follows at once.

From this, if barriers be drawn in Ω, consisting of the p straight lines joining $C_1'C_1$, $C_2'C_2, \ldots, C_p'C_p$, we find the equations

$$v_r^{a_n, c_n} = -\tfrac{1}{2}, \text{ or } 0, \qquad \text{as } n = r, \text{ or } n \neq r.$$

It will be sufficient to consider one case. Let D_1' be the inverse point of C_1' in regard to the circle O, and let K denote the real part of $v_r^{\zeta, a}$ at the point a_1; then since the function $-iv_r^\zeta$ is real on the arc ac_1 it has conjugate complex values along the arcs c_1C_1', c_1D_1'; so that we have on these arcs, respectively,

$$v_r^{\zeta, a} = -\tfrac{1}{2}\tau_{r,1} + \mu, \qquad v_r^{\zeta, a} = -\tfrac{1}{2}\tau_{r,1} - \mu,$$

where μ is a variable real quantity; again, since $-i(v_r^{\zeta, a} - K)$ is real on the arc a_1c_2, it has conjugate complex values on the arcs a_1C_1', a_1D_1'; so that we have on these arcs respectively

$$v_r^{\zeta, a} = K - \tfrac{1}{2}\tau_{r,1} + \lambda, \qquad v_r^{\zeta, a} = K - \tfrac{1}{2}\tau_{r,1} - \lambda,$$

where μ is a variable real quantity; therefore

$$v_r^{a_1, c_1} = v_r^{D_1', c_1} + v_r^{a_1, D_1'} = -v_r^{C_1', c_1} - v_r^{a_1, C_1'},$$

while, on the other hand,

$$v_r^{a_1, c_1} = v_r^{C_1', c_1} + \Pi + v_r^{a_1, C_1'},$$

where Π represents the value obtained in passing from one side to the other of the barrier through C_1', $= -1$ when $r = 1$ and $= 0$ when $r \neq 1$, it being supposed that, as in the figure, the arc c_1a_1 is in the counter-clockwise direction; hence we have by addition

$$2v_r^{a_1, c_1} = \Pi,$$

giving the result enunciated. This equation is also easily seen to be the condition that the function $v_r^{\zeta, a}$ be continuous at D_1'.

For these results cf. *loc. cit.* (θ); the proof here given is designed to bring out the identity with some corresponding results in the theory of a hyper-elliptic Riemann surface; it is upon this identity that an important result deduced below is made to rely.

Further, since the real part of v_r^{ζ} is constant upon the separate arcs of the circle O we obtain

$$v_r^{c_1, a} = -\tfrac{1}{2}\tau_{r,1}, \qquad v_r^{c_2, a_1} = \tfrac{1}{2}\tau_{r,1} - \tfrac{1}{2}\tau_{r,2},$$

$$v_r^{c_3, a_2} = \tfrac{1}{2}\tau_{r,2} - \tfrac{1}{2}\tau_{r,3}, \qquad v_r^{c, a_3} = \tfrac{1}{2}\tau_{r,3},$$

of which for instance the value of $v_r^{c_2, a_1}$ can be obtained by a method similar to that employed above, by adding the values obtained by passing from a_1 to c_2, first directly along the circle O, and then along the path $a_1C_1'C_1a_1'c_2'C_2C_2'c_2$, where a_1', c_2' are the images of a_1, c_2 on the circle C_0.

These equations shew that, for the definition of the half periods, the points a, c_1, a_1, c_2, a_2, ..., are exactly analogous to the branch places denoted by the same letters when the period loops of the hyperelliptic Riemann surface are drawn as in the annexed figure (2).

Hence from the theory of hyperelliptic theta functions we infer that *the theta functions* $\Theta(v^{\zeta, a})$, $\Theta(v^{\zeta, c})$ *vanish respectively in the places* $(a_1, \ldots, a_p)$ and $(c_1, \ldots, c_p)$.

Consider now the function

$$\lambda(\zeta, \mu) = \sum_r \frac{(\gamma_r \zeta + \delta_r)^{-1}}{\zeta_r - \mu},$$

where $\zeta_r = (\alpha_r \zeta + \beta_r)/(\gamma_r \zeta + \delta_r)$, and the summation extends to every substitution of the group. When one of the p fundamental substitutions of the group, as given by the positions of the circles on the ζ plane, is put into the form $\zeta' = (\alpha\zeta + \beta)/(\gamma\zeta + \delta)$, with $\alpha\delta - \beta\gamma = 1$, there is an ambiguity as to the signs of the quantities α, β, γ, δ, which must be settled beforehand in order that the

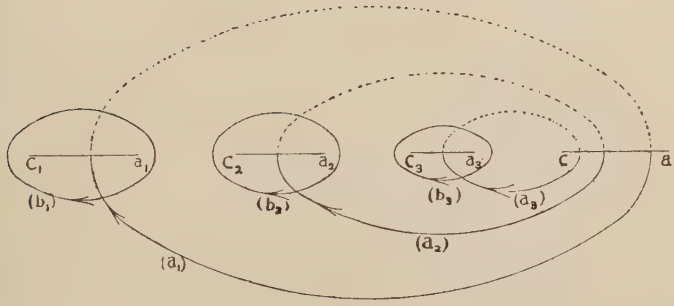


FIG 2.

function $\lambda(\zeta, \mu)$ may be definite. There are then 2^p functions $\lambda(\zeta, \mu)$, differing according to the conventions adopted in the case of the p fundamental substitutions. It is known (*Camb. Phil. Proceedings*, VIII (1895), p. 332, or *loc. cit.* (κ), p. 368) that any one of these functions has, in the region Ω, beside $\zeta = \infty$, p zeros $\mu_1, \ldots, \mu_p$ which are given by the equations

$$v_r^{\mu_1, m_1} + \ldots + v_r^{\mu_p, m_p} \equiv \tfrac{1}{2}(g_r + h_r), \qquad (r = 1, \ldots, p),$$

where $m_1, \ldots, m_p$ are the zeros of the theta function $\Theta(v^{\zeta, m})$, and

$$(g_1 + h_1), \ldots, (g_p + h_p)$$

is an aggregate of p integers, each of which may be taken to be either even or odd according to the conventions, explained above, adopted for the signs of α, β, γ, δ in the p fundamental substitutions.

Hence we deduce, *if the conventions be taken so that each of* $g_r + h_r$ *is an even integer, the functions*

$$\lambda(\zeta, a), \qquad \lambda(\zeta, c)$$

vanish respectively in $(a_1, \ldots, a_p)$ *and* $(c_1, \ldots. c_p)$. More generally, from the results developed above, such as $v_r^{a_r, c_r} = -\frac{1}{2}$, we have, *the*

p finite zeros of any one of the 2^p functions $\lambda(\zeta, a)$ are $(\mu_1, \ldots, \mu_p)$, where μ_r is either a_r or c_r, according to the convention adopted for the signs of $\alpha, \beta, \gamma, \delta$ in the p fundamental substitutions. A direct verification is given below.

Therefore, from what was proved in the general case, *a function which is uniform in the region Ω, unaltered by the substitutions of the group, real on the circles $C_0, C_1', \ldots, C_p', C_1, \ldots, C_p$, and infinite to the first order on each of these circles, is given by*

$$Z = \lambda(\zeta, a)/\lambda(\zeta, c);$$

with the notations explained *loc. cit.* (κ), save that c_r replaces b_r, this can be put into the various forms following, where μ is an arbitrary place, and a constant factor is disregarded,

$$Z = \frac{\Theta(v^{\zeta, a})}{\Theta(v^{\zeta, c})} e^{\Pi^{\zeta, \mu}_{c, a}}, \quad = e^{-\Pi^{\zeta, \mu}_{a, c} + \Pi^{\zeta, \mu}_{a_1, c_1} + \ldots + \Pi^{\zeta, \mu}_{a_p, c_p}},$$

$$= \frac{\varpi(\zeta, c)\, \varpi(\zeta, a_1) \ldots \varpi(\zeta, a_p)}{\varpi(\zeta, a)\, \varpi(\zeta, c_1) \ldots \varpi(\zeta, c_p)},$$

$$= \frac{y}{x} = \sqrt{\frac{[x - x(a_1)] \ldots [x - x(a_p)]}{x\,[x - x(c_1)] \ldots [x - x(c_p)]}}.$$

In the elliptic case, with

$$u = 2\omega v^{\zeta, a} = \frac{\omega}{i\pi} \log\left(\frac{\zeta - B}{\zeta - A} \Big/ \frac{a - B}{a - A}\right),$$

we have, with the convention of sign here adopted, save for a constant factor,

$$\frac{\lambda(\zeta, a)}{\lambda(\zeta, c)} = \frac{\sqrt{\wp(u) - e_2}}{\sqrt{\wp(u + \omega) - e_2}} = \frac{\wp'(u)}{\wp(u) - e_3},$$

the points c_1, a_1, c, a corresponding to the values e_3, e_2, e_1, ∞ of $\wp(u)$, respectively. It is easy to verify that this function has the desired behaviour.

Thus by the function $\lambda(\zeta, a)/\lambda(\zeta, c)$ the conformal representation required in Riemann's paper referred to, is effected.

6. We explain now how to obtain a direct verification from the series itself of the statements that have been made as to the zeros of the function $\lambda(\zeta, a)$. For this purpose we suppose the circle O, which cuts the circles $C_0, C_1', \ldots, C_p'$ at right angles, to be the axis of imaginary quantities, the point a being the origin, $\zeta = 0$, and the left side of the imaginary axis being the interior of the circle O. Then we may put

$$c_1 = ik_1, \quad a_1 = ih_1, \quad c_2 = ik_2, \quad a_2 = ih_2, \ldots$$

where k_1, h_1, k_2, h_2, ... are real positive quantities in ascending order of magnitude. It is then found that, for positive integral n, we have

$$\mathfrak{S}_r^n(\zeta) = \frac{\alpha_n^{(r)}\zeta + \beta_n^{(r)}}{\gamma_n^{(r)}\zeta + \delta_n^{(r)}} = \frac{\delta_n^{(r)}\zeta - h_r k_r \gamma_n^{(r)}}{\gamma_n^{(r)}\zeta + \delta_n^{(r)}},$$

where $\alpha_n^{(r)}\delta_n^{(r)} - \beta_n^{(r)}\gamma_n^{(r)} = 1$, and

$$\gamma_n^{(r)} = \frac{i\epsilon_r^n}{2\sqrt{h_r k_r}} \left\{ \left(\frac{\sqrt{h_r} + \sqrt{k_r}}{\sqrt{h_r} - \sqrt{k_r}} \right)^n - \left(\frac{\sqrt{h_r} - \sqrt{k_r}}{\sqrt{h_r} + \sqrt{k_r}} \right)^n \right\},$$

$$\delta_n^{(r)} = \tfrac{1}{2}\epsilon_r^n \left\{ \left(\frac{\sqrt{h_r} + \sqrt{k_r}}{\sqrt{h_r} - \sqrt{k_r}} \right)^n + \left(\frac{\sqrt{h_r} - \sqrt{k_r}}{\sqrt{h_r} + \sqrt{k_r}} \right)^n \right\},$$

the square roots $\sqrt{h_r}$, $\sqrt{k_r}$ being taken positively; here $\epsilon_r, = \pm 1$, is a quantity which is not determined by the positions of the circles, but must be assigned when the substitution $\mathfrak{S}_r$ is put into the form

$$\mathfrak{S}_r(\zeta) = \frac{\alpha^{(r)}\zeta + \beta^{(r)}}{\gamma^{(r)}\zeta + \delta^{(r)}},$$

with $\alpha^{(r)}\delta^{(r)} - \beta^{(r)}\gamma^{(r)} = 1$.

From these it immediately follows that if z be a pure imaginary quantity, and $z' = -z$ be its conjugate complex, $\gamma_n^{(r)}z + \delta_n^{(r)}$ is real and equal to $\gamma_{-n}^{(r)}z' + \delta_{-n}^{(r)}$, while $\alpha_n^{(r)}z + \beta_n^{(r)}$ is purely imaginary and equal to the negative of $\alpha_{-n}^{(r)}z' + \beta_{-n}^{(r)}$. Therefore if

$$\phi(\zeta) = \ldots \mathfrak{S}_1^{n_1}\mathfrak{S}_2^{n_2} \ldots (\zeta) = \frac{\alpha\zeta + \beta}{\gamma\zeta + \delta},$$

$$\phi'(\zeta) = \ldots \mathfrak{S}_1^{-n_1}\mathfrak{S}_2^{-n_2} \ldots (\zeta) = \frac{\alpha'\zeta + \beta'}{\gamma'\zeta + \delta'},$$

be two substitutions in which the factors formed from the primary substitutions follow one another in the same order, but for any factor $\mathfrak{S}_r^n$ in the first there enters, in the second, the factor $\mathfrak{S}_r^{-n}$, it follows, if z and z', $= -z$, are conjugate complex quantities, that $\phi(z) = -\phi'(z')$, and $\gamma z + \delta = \gamma' z' + \delta'$. These facts are clear enough from a consideration of the figure. For our purpose another fact is also necessary, namely

when $\epsilon_r = +1$, we obtain

$$\alpha^{(r)}c_r + \beta^{(r)} = -c_r = c_r', \quad \gamma^{(r)}c_r + \delta^{(r)} = 1,$$

when $\epsilon_r = -1$, we obtain

$$\alpha^{(r)}a_r + \beta^{(r)} = -a_r = a_r', \quad \gamma^{(r)}a_r + \delta^{(r)} = 1,$$

where c_r', a_r' denote the conjugate complexes of c_r and a_r respectively.

Hence, retaining the notation for $\phi(\zeta)$, $\phi'(\zeta)$, if

$$\phi'\mathfrak{S}_r(\zeta) = \frac{A\zeta + B}{C\zeta + D},$$

so that

$$A\zeta + B = \alpha'[\alpha^{(r)}\zeta + \beta^{(r)}] + \beta'[\gamma^{(r)}\zeta + \delta^{(r)}],$$

we have

when $\epsilon_r = 1$, $\quad Ac_r + B = \alpha' c_r' + \beta' = -(\alpha c_r + \beta)$,

when $\epsilon_r = -1$, $\quad Aa_r + B = \alpha' a_r' + \beta' = -(\alpha a_r + \beta)$;

we find also, in fact, in these two cases,

$$\phi(c_r) = -\phi'\mathfrak{S}_r(c_r), \quad \phi(a_r) = -\phi'\mathfrak{S}_r(a_r).$$

Thus it follows, that when $\epsilon_r = 1$, the series

$$\lambda(\zeta, 0) = \sum_m \frac{(\gamma_m\zeta + \delta_m)^{-1}}{\zeta_m} = \sum_m \frac{1}{\alpha_m\zeta + \beta_m},$$

vanishes for $\zeta = c_r$; for when $\zeta = c_r$ the term arising by the substitution ϕ, of the group, is equal and opposite to the term arising by the substitution $\phi'\mathfrak{S}_r$. Similarly when $\epsilon_r = -1$, the series vanishes for $\zeta = a_r$.

In a somewhat artificial way this result may be included in the statement—the series, $\lambda(\zeta, a)$, has, beside $\zeta = \infty$, the p zeros

$$\frac{a_1 + c_1}{2} - \epsilon_1 \frac{a_1 - c_1}{2}, \ \ldots, \ \frac{a_p + c_p}{2} - \epsilon_p \frac{a_p - c_p}{2},$$

where $\epsilon_1, \ldots, \epsilon_p$, each equal to ± 1, are the signs determining the p fundamental substitutions, as given by putting $n = 1$ in the formulæ written down above for the substitution $\mathfrak{S}_r^n$.

(3) *On the production of a cloud by the action of ultra-violet light on moist air.* By Mr C. T. R. Wilson, Clerk Maxwell Student.

If the light from an arc lamp be brought to a focus, by means of a quartz lens, within a vessel containing moist dust-free air, a bluish fog becomes visible in the course of a few minutes along the path of the light. The cloud particles remain in suspension for hours after the light has been cut off. The phenomenon is shewn even in unsaturated air, but the faint blue haze which then developes takes much longer to form. When the radiation is not sufficiently intense to shew these effects a dense fog can still be obtained by bringing about slight supersaturation by expansion. These clouds, unlike those obtained by Tyndall (*Phil.*

Trans., 160, p. 333, 1870) and by Aitken (*Edin. Trans.*, 39, I. p. 15, 1897) by the action of light on various vapours, are due to the ultra-violet rays alone; for if a thin sheet of glass or mica (substances which are opaque to these rays) be interposed, not a trace of fog or rain is formed even when a high degree of supersaturation is brought about by expansion. It is possible that the small particles to which the blue of the sky is due are the result of this action of the ultra-violet rays, of which sunlight, when it first enters our atmosphere, doubtless contains a plentiful supply.

(4) *On the use of logarithmic coordinates in Physics.* By Mr J. H. Vincent.

(5) *On the Diffuse Reflection of Röntgen Rays.* By J. J. Thomson, M.A., F.R.S.

It is well known that when Röntgen rays fall on a solid or liquid surface there is a diffuse return of rays from the surface; these diffusely returned rays we shall call secondary rays.

The consideration of the nature and manner of production of these secondary rays suggests some interesting questions, as the experiments of Sagnac and Langevin show that these secondary rays differ in their properties from the primary rays which excite them. So that it would seem that bodies exposed to Röntgen rays can themselves emit rays which possess many properties in common with the ordinary Röntgen ray. Thus for example Röntgen has recently shown that air through which Röntgen rays are travelling sends out rays into regions carefully screened from the direct action of the primary rays.

Sagnac, who has recently investigated the properties of the secondary rays diffusely reflected from metals, finds that though they resemble the primary rays in not being refracted, they are absorbed with very much greater rapidity, the secondary rays being practically extinguished after passing through a centimetre or so of air at atmospheric pressure. Sagnac found too that the penetrating power of the secondary rays depended on the nature of the metal from which they emanated; thus close to the surface of the metal the secondary rays were more abundant with zinc than with copper, at a short distance from the surface of the metal the proportions are reversed, showing that the rays from copper have more penetrating power than those from zinc. Sagnac arrived at his results by using the photographic method. Langevin

working at the Cavendish Laboratory has arrived at these and other results by the electric method.

The following investigations were undertaken in the hope of throwing some light on the way in which the molecules of a substance might be conceived to give out Röntgen rays.

Since there is no reflection of the Röntgen rays we cannot suppose that the secondary rays are produced by an action similar to that by which light is 'scattered' from small particles.

If however we adopt the theory described by the author in the *Philosophical Magazine* for Feb. 1898, that the Röntgen rays are thin pulses of intense electric and magnetic intensity such as is shown would be generated by the sudden stopping of the cathode rays, then one way in which the secondary Röntgen rays might be produced is as follows. Let us suppose that the atoms of the substance carry electric charges, then when the pulse of intense electric intensity which constitutes a Röntgen ray falls on these atoms it will suddenly change their velocities, this sudden change in the velocity of a charged atom will generate a secondary pulse of electric and magnetic intensity which on the above theory would constitute a secondary Röntgen ray. The nature of this secondary ray is as follows: if a sphere of radius a with a charge e is suddenly started with a velocity u parallel to the axis of X, then the state of the magnetic field can be shown to be as follows. If P is a point under consideration, O the centre of the particle, and $OP = r$, then H the magnetic force at P is zero when the time t which has elapsed since the particle was started is less than $(r-a)/V$, where V is the velocity of light through the medium surrounding the sphere: when $t > (r-a)/V < (r+a)/V$, then

$$H = \frac{ew \sin\theta}{ar},$$

where θ is the angle between OP and the axis of X; when $t > (r+a)/V$,

$$H = \frac{ew \sin\theta}{r^2},$$

the lines of magnetic force are circles with their centres along the axis of X and their planes at right angles to it. Thus between the times $t = (r-a)/V$ and $t = (r+a)/V$ a pulse of intense magnetic and electric force is passing over P.

Now consider a primary Röntgen ray travelling along the axis of z; let the electric intensity in the pulse make an angle ϕ with the axis of X, this intensity will give an impulsive velocity parallel to itself to a charged particle in the path of the pulse, let this

impulsive velocity be w. The magnetic force in the secondary pulse will in a direction making an angle ψ with the axis of z and at a distance r from the particle be equal to

$$\frac{ew}{ar}\sqrt{1-\sin^2\psi\sin^2\phi},$$

while the energy in the pulse will be proportional to

$$\frac{e^2w^2}{a^2r^2}(1-\sin^2\psi\sin^2\phi).$$

Let us suppose that instead of there being only one ray there are a large number in which the electric intensity is in different directions, but uniformly distributed round z, then the energy will be proportional to

$$\frac{e^2w^2}{a^2r^2}(1-\tfrac{1}{2}\sin^2\psi),$$

thus the energy when $\psi=0$ or π is twice that when $\psi=\pi/2$. So that if the secondary radiation arises solely from an action of this kind the intensity of the reflected radiation in the direction of the primary rays would be twice that in a direction at right angles to them.

The intensity of the diffuse reflection in different directions was tested by the following method. *ABCD* is a box made of

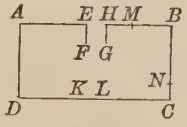


Fig. 1.

sheet lead, Röntgen rays pass into this through a small lead tube *EFGH* and are diffusely returned from the patch *KL*; pieces of photographic plates with screens of perforated zinc in front of them were placed at *M* and *N*, at equal distances from the middle of the patch *HK*; the diffusely returned rays from *HK* produced after an exposure of about two hours photographs on the plates *N*, *M*; the photographs at *M* and *N* did not show any appreciable difference in intensity though a large series were taken with exposures varying from 2 to 6 hours. Tests were made which showed that these photographs were made by rays coming from *HK* and not from the air through which the primary Röntgen rays passed. This was done by removing the bottom of the box, when the effect on the plates was very slight compared with that produced when the bottom of the box was in its place. Though the photographic method is not a very delicate test of the relative

intensity of the rays, the results show that the intensity at M where ψ is small cannot exceed that at N where ψ is nearly $\pi/2$ in anything like the ratio of 2 to 1, thus showing that the greater part at any rate of the effect does not arise in the way suggested.

Some experiments were made to see if there was anything of the nature of selective absorption of these diffusely returned rays. In these a plate in the position M was covered with paper, on this paper two crosses were placed, one of these crosses was made of thin platinum foil, the other by painting red lead on the paper; the thickness of the layer of red lead was altered until when the diffusely returned rays came from the lead sheet at the bottom of the box the depth of the shadow cast by the red lead was very nearly the same as that cast by the platinum foil. The bottom of the box was then covered by a piece of platinum and photographs again taken: the depth of the shadows seemed to be in the same proportion as when the rays were reflected from lead, showing that there is no marked selective absorption.

The absence of refraction in these diffusely returned rays was verified by the following experiment. A straight slit parallel to the bottom of the box was cut in the side BC of a lead box, the diffusely returned rays passing through the slit produced a bright line on a photographic plate placed outside the box, a thin wedge of aluminium was placed between the slit and the photographic plate and photographs were again taken, the line of light on the plate was found unbroken by the prism though faint in the part where the rays were intercepted. The equality of the radiation in different directions shows that these secondary rays do not originate by the method of which the theory is given above. The primary Röntgen rays ionize the substances through which they pass, and the rapid ionization in an electric field of a molecule consisting of positively and negatively electrified atoms will be accompanied by an electromagnetic disturbance lasting for only a short time. The general character of this disturbance may be represented graphically as in the following figures. In the first stage (a) the Faraday tubes in the external field and that connecting the atom in the molecule are uninfluenced by each other; in the next stage (b) the tubes bend towards each other, then they run together and break away as in fig. (c), and after this stage the atoms in the molecule are not connected by a Faraday tube and are dissociated. While these changes are in progress we have a very rapid movement of Faraday tubes in a space comparable with that between the atoms in a molecule. This movement of the tubes will give rise to an electromagnetic disturbance which in consequence of the intense radiation will be almost dead beat and so equivalent to a pulse of electromagnetic disturbance. The

adiation emitted from ionization will be more symmetrically istributed than that previously investigated and so there will be ess difference of intensity between the secondary radiation in the irection of the primary ray and that in a direction at right ngles to it.

Ionization (if sudden) may thus be expected to give rise to ays having properties similar to those of the secondary Röntgen ays. E. Wiedemann[1] has shown that an electric discharge gives se to rays to which he has given the name "Entladungstrahlen"; hese rays can pass through substances opaque to light and are not eflected by a magnet. Now the electric discharge is accompanied

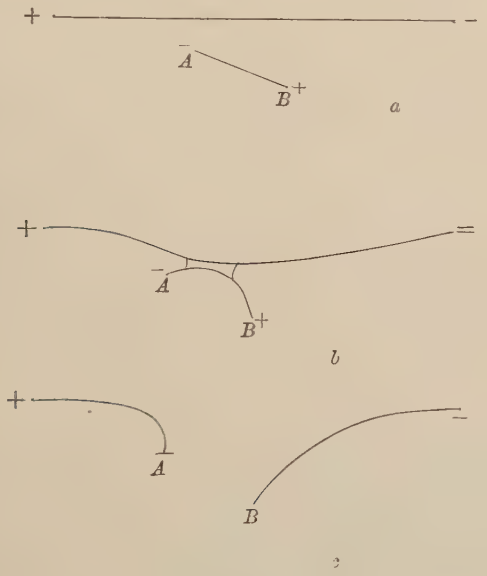


Fig. 2.

ionization of the gas through which the discharge passes and, ough this ionization is probably not of so sudden a character as at produced by Röntgen rays, Wiedemann's experiments seem to ow that it is rapid enough to give rise to rays having properties alogous to the secondary Röntgen rays. It seems not impossible at in the case of a complicated structure like the uranium atom grouping of the constituents of the atom may give rise to ctrical effects similar to those which occur in ionization and ght possibly be the origin of the uranium radiation.

[1] *Zeitschrift für Elektrochemie*, II. p. 159, 1895.

Monday, 7 February, 1898.

Mr F. Darwin, President, in the Chair.

The following were elected Fellows of the Society:

J. H. C. Dalton, M.D., Trinity College.
A. H. Ramsey, M.A., Magdalene College.
W. M. Fletcher, B.A., Trinity College.

The following communication was made to the Society:—

Some Zoological Results of a Voyage to Melanesia during th years 1894—1897. By Arthur Willey, D.Sc., Balfour Student

I. *Nautilus.* 1. The deposited eggs of *N. pompilius* wer obtained by me during the months July—September, 1897, i cages kept at a depth of some 50 to 60 fathoms in Blanche Ba New Britain. They differ very slightly in the ornamentation o the outer capsule from the eggs of *N. macromphalus*, the pectinat ridges in those of the former species being more frill-like.

2. The otolithic contents of the otocysts of *Nautilus* wer correctly described by Macdonald[1] in 1855 as consisting of in numerable minute otocones. Sometimes in preserved specimen the effect of the preservation appears to be to coagulate the flui contents of the otocyst, so that the otocones cohere to form solid otolith. This explanation will account for the erroneou interpretation recently published on this subject[2].

3. The region of the mantle which secretes the calcareou septa of the shell is marked off from the surrounding region b a thickened rim or contour line. The latter was described by m under the name of the septal contour and figured on p. 170 of th *Quart. Journ. Micro. Sc.*, Vol. xxxix., 1896, together with a repre sentation of the pallio-septal arteries. The septal contour ha been redescribed by L. E. Griffin (*loc. cit.*) as the "posteri ventral aponeurotic band" without referring to my account, a though the paper containing it is included in his bibliography.

4. The animal of *N. umbilicatus* has not, so far as I a aware, ever been examined or even seen by a zoologist befor I was fortunate enough to obtain one specimen which had bee taken from the surface in the neighbourhood of the D'Entr

[1] *Phil. Trans.* 1855, p. 277. Macdonald supposed his specimen, which w obtained off the Isle of Pines, to be N. umbilicatus; but that it was really macromphalus is abundantly evident from his description of the animal even if were not already known that he had mistaken the species.

[2] L. E. Griffin, "Notes on the Anatomy of *Nautilus pompilius*," *Zool. Bulle* Vol. i. p. 147. 1897.

asteaux Group, to the east of New Guinea. While the animals f *N. pompilius* and *N. macromphalus* are almost, if not quite, ndistinguishable from one another, that of *N. umbilicatus* differs narkedly from both, especially in the surface texture of the hood, vhich is coarsely areolated, the polygonal block-like areas being eparated by deep valleys.

5. In any attempt to interpret the morphology of the tentaular appendages of *Nautilus* and the arms of the Dibranchiate Cephalopods, if function, innervation, and development are reected as evidence of their pedal origin, the only way left open o approach the subject seems to be by taking into consideration he general phenomena involved in cephalogenesis. In all other nimals which possess a compound head, the latter is not prouced by elaboration of the pre-existing simple head with its ræoral lobe, but by the incorporation of other parts of the body nto the cephalic complex. Thus while in Arthropods we have he *cephalothorax*, in the Cephalopod Molluscs we have, viewed rom the standpoint here adopted, the *cephalopodium*.

II. *Ctenoplana*. This remarkable animal appears to be the xisting representative of the type from which the Ctenophora nd the Plathelminthes diverged; the former being its pelagic nd the latter its littoral descendants. The importance of *Ctenolana* lies in the fact that it presents a transition from biradial o bilateral symmetry, the transitional feature being its dorsoentrality.

Some authorities consider *Ctenoplana* only as a slightly modified Ctenophore. From this point of view, however, *Ctenoplana* loses nuch of its interest and all of its importance; but I do not think nat there is sufficient justification for such a depreciatory attitude. t may indeed be said that *Ctenoplana* is no more a Ctenophore n the one hand or a Planarian on the other than *Peripatus* is a hætopod or a Myriapod. For further details on Ctenoplana I nay refer the reader to my paper in the *Quart. Journ. Micro. Sc.* ol. XXXIX., p. 323.

III. *Enteropneusta*. In Spengel's great Monograph of this roup there are two theoretical conclusions to which I wish to efer. The first is that the Enteropneusta have no Chordate finities; and the second, that the genus *Balanoglossus* (*sensu ricto*) is the most primitive type of the group. As this genus the one which is least capable of being compared, in any etail, with *Amphioxus*, the latter conclusion, if true, would tend confirm the former.

In the section *Chlamydothorax* of the genus *Ptychodera* the ll-slits open freely to the exterior throughout the greater part of

their length, in *Pt. flava* throughout their entire extent; in other words, the branchial sac is freely exposed to the surrounding water.

Observations made on living specimens of *Pt. flava* at the Isle of Pines led me to suppose that this condition of the branchial sac, with its skeletal supports, which include synapticula or cross-rods, was more primitive than that in which the primary bars are fused with the body-wall and the external openings of the gill-slits are reduced to minute pores. This latter condition would naturally tend to render the skeletal structures more or less superfluous, and, in fact, as shown by Spengel, there are no synapticula in *Glandiceps* and *Balanoglossus*.

That this is a secondary loss rather than a primary deficiency seems to me to be extremely probable, and this view is further confirmed by the organisation of an Enteropneust which I have described as a new genus, *Spengelia*[1]. This genus, apart from its own peculiar features, exhibits close affinity to Glandiceps, but it has synapticula.

Another point which Spengel regarded as evidence of a primitive nature was the absence of a circular layer of muscles in the body wall of *Balanoglossus*. Spengel himself could not avoid calling attention to the improbability of this defect in the structure of a soft-bodied animal like *Balanoglossus* being primitive, even while vindicating his position. In a species of *Ptychodera* from Funafuti, recently described by J. P. Hill[2] as *Pt. hedleyi*, Hill found that there was no circular layer of muscles in the wall of the trunk-region, except at the extreme posterior end, where it forms the anal sphincter. As this species belongs to that section of the genus *Ptychodera* which bears most external resemblance to *Balanoglossus*, there is obviously every reason to suppose that the absence of circular muscles is also a secondary loss.

Such considerations as the above are adduced for the purpose of proving that *Ptychodera* (*Chlamydothorax*) is the most primitive and *Balanoglossus* the least primitive type of Enteropneusta.

IV. *Peripatus.* Last year I obtained thirteen specimens of a species of *Peripatus* in New Britain, which constitutes a new (Melanesian) type equivalent to the types represented respectively by the Neotropical, Australasian and Ethiopian species. In the position of the generative orifice behind the last pair of legs it resembles the Cape species, in the absence of crural glands it resembles the New Zealand species, while the facts that the males have a less number of claw-bearing legs than the females (22 pairs in the former and 24 in the latter) and that embryos in all stages

[1] *Q. J. M. S.*, Vol. XL. p. 623.
[2] *Mem. Austral. Mus.* III. Pt. 5. 1897. p. 335.

f development are to be found in the uteri of one female, indicate very striking correspondence with the Neotropical species.

The embryos are provided with a large ectodermic trophoblastic rgan apparently formed by the dilatation of the primary body-avity. Of the thirteen specimens only two were males, but the xamination of the older embryos taken from the larger females howed that the number of legs in the respective sexes was con-tant. It is probable that the embyros of this species will throw side-light on the "placental" development of the Neotropical pecies as described by *Kennel*.

Monday, 21 *February*, 1898.

Mr F. Darwin, President, in the Chair.

The following communications were made to the Society:

(1) *On some differential equations in the theory of symmetric lgebra.* By Professor Forsyth.

This paper is printed in the *Transactions*, Vol. xvi. Part iv.

(2) *The Discharge of Electrification by Ultra-violet Light.* y E. Rutherford, M.A., B.Sc., 1851 Exhibition Scholar New ealand University, and Coutts Trotter Student, Trinity College, ambridge.

The general action of ultra-violet light on the discharge of lectrification has been investigated by many different experi-nenters. Hertz[1] in 1887 first drew attention to the action of ltra-violet light on a spark gap. Wiedemann and Ebert[2] showed nat the kathode was the seat of this action and investigated the eneral effect on high potential discharges. Hallwachs[3] and ighi[4] observed the fact that zinc and other metals became ositively electrified under the action of ultra-violet light. These esults were extended by Elster and Geitel[5], who have published a eries of papers on the effect of ultra-violet light in causing ischarge under various conditions and have also[6] investigated

[1] *Wied. Annal.* xxxi. p. 983. 1887.
[2] *Wied. Annal.* xxxiii. p. 241. 1888.
[3] *Phil. Mag.* xxvi. p. 78. 1888.
[4] *Phil. Mag.* xxv. p. 314. 1888.
[5] *Wied. Annal.* xxxviii. p. 40, xxxviii. p. 497, xxxix. p. 332, xli. pp. 162, 166, l. p. 433, lv. p. 684.
[6] *Wied. Annal.* xli. p. 166. 1890.

the action of a magnetic field on the discharge at low pressures Stoletow[1] investigated in detail the relation between the current and electromotive force for the discharge at low voltages and at different pressures.

Most of these papers have dealt with the general character of the discharge, but the subject of the nature of the conduction and of the carrier that discharges the electrification has not been specially attacked. In a very interesting paper Lenard and Wolff[2] investigated the effect of a surface, on which ultra-violet light fell, on the condensation of a steam jet in the neighbourhood, and their results led them to the conclusion that many bodies were disintegrated under the action of ultra-violet light and that the particles torn off became nuclei for the condensation of the steam jet. In the light of more recent experiments these results are however capable of other interpretations.

R. v. Helmholtz[3] has shown that a steam jet is acted on when chemical action is going on in its neighbourhood. Richarz[4] has shown that Röntgen rays produce condensation in a steam jet and Wilson[5] has recently observed that ions produced under the action of Uranium and Röntgen radiation become under certain conditions nuclei for the condensation of water around them. He has also demonstrated the important fact, which appears to have been overlooked, that ultra-violet light produces clouds in ordinary moist air quite independent of any solid body in its neighbourhood. The presence of this effect must have complicated the effects of Lenard and Wolff, and the more general results on the properties of ions in producing condensation seem to show that possibly their results may be ascribed to the presence of free gaseous ions rather than disintegrated particles of metal.

It is the object of this communication to give results of investigations on the nature of the carriers of the negative electrification produced under the action of ultra-violet light, and to show that probably the greater part of the electrification is carried by gaseous ions produced at the surface of the negatively electrified plate.

In order to obtain a discharge with ultra-violet light, the light must fall on a negatively electrified surface. There is no discharge produced by allowing the light to fall between two plates without impinging on either. In this respect the action of ultra-violet light is very different to Röntgen and Uranium radiation, which produce a volume ionisation of the gas through

[1] *Journal de Russ. Phys.* XXI. 1889. *Journal de Physique*, IX. p. 468. 1890.
[2] *Wied. Annal.* XXXVII. p. 443. 1888.
[3] *Wied. Annal.* XXXII. p. 1. 1887. XL. p. 161. 1890.
[4] *Wied. Annal.* LIX. p. 592. 1896.
[5] *Camb. Philos. Soc.* Vol. IX. Pt. VII. 1897.

which they pass. The question whether there is any volume ionisation of a gas through which ultra-violet light passes was investigated by Henry[1] who tried the vapours of iodine and methyl iodide both of which are very powerful conductors under Röntgen rays but with a negative result. The result of Wilson that a cloud is formed in moist air with strong ultra-violet light renders it possible that there is a slight volume ionisation of the gas through which the light passes, but the effect appears to be too small to be determined by electrical means, and in all later experiments it is assumed that the surface of the negatively charged plate is the seat of the action of the ultra-violet light discharge.

If we allow ultra-violet light to fall on a negatively electrified surface, *e.g.* a polished zinc plate, since the body is slowly discharged, it seems probable that if the discharge is due to the convection of the charged particles, these charged particles can be blown away by directing a sufficiently rapid blast of air across their path. This has been experimentally shown by Zeleny[2], who showed that a negatively charged gas can be obtained by blowing past the negatively charged plate when the ultra-violet light was acting. This gas has similar properties to the charged gas[3] obtained by the separation of ions in Röntgen ionisation, for they readily give up their charge and refuse to pass through a plug of cotton-wool.

I had independently observed the same fact, and had also investigated the effects of blowing currents of air by electrodes, especially from the point of view of determining the velocity of the carrier of electricity.

Before entering on the general results it is necessary to draw attention to the phenomena observed by Blondlot and Bichat[4]. They found that if an insulated zinc plate was acted on by ultra-violet light, when all the conductors in the neighbourhood were connected to earth, the potential to which it could be raised was increased 6 or 7 times by allowing a blast of air to impinge on the plate. They found that this action was independent of dust and moisture. It is easy in this way to raise the potential of a plate of amalgamated zinc to 15 volts in a few minutes, although the potential to which it could be raised under the action of the light alone was less than 2 volts.

I have also found that the rate of leak of a body charged negatively is much more rapid when a blast of dust-free air is directed against it. A plate of polished zinc, charged to -8 volts,

[1] *Proc. Camb. Phil. Soc.* Vol. IX. Pt. VI. 1897.
[2] *Phil. Mag.* March, 1898.
[3] Rutherford. *Phil. Mag.* April, 1897.
[4] *C. R.* XVII. p. 29. 1888.

placed at a distance of 10 cm. from a neighbouring plate gave a rate of leak 12 times as fast as under the action of the ultra-violet light alone. The blast seems to assist the electromotive force in removing the negative charge. The presence of this action must be taken into account in all cases where currents of air impinge on negatively electrified surfaces.

The following figure (Fig. 1) shows the general arrangement

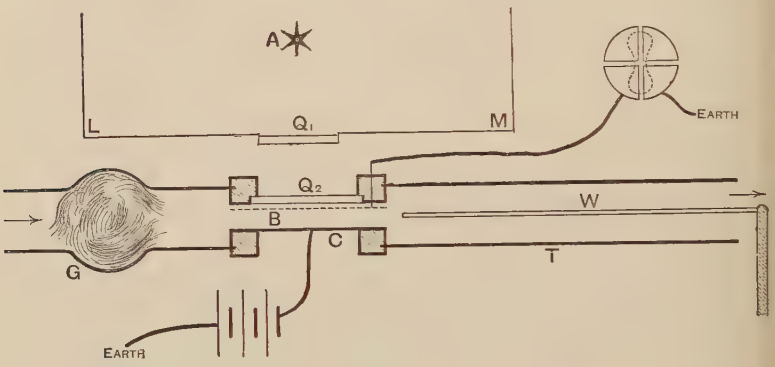


FIG. 1.

of the experiment to show the effect of blowing a current of air by a negatively electrified plate on which ultra-violet light is allowed to fall.

A blast of air from a bellows or a gasometer passed between two parallel plane electrodes B and C, and then through an insulated aluminium cylinder T. The air before reaching the plates was free from dust as far as possible by passing through a bulb G tightly packed with cotton-wool. An arc light A was used as a source of ultra-violet light. The light after passing through a quartz plate Q_1 which covered an opening in a metal screen LM surrounding the arc, passed through a second quartz plate Q_2, through the fine wire-gauze B and then impinged on the metal plate C, which was generally of polished or amalgamated zinc. The broken lines show the position of insulators by which the gauze B, the plate C, and the tube T were all insulated from one another. An insulated wire W passed centrally down the tube T.

In a particular experiment the plate C was 4·2 cms. long, 1·5 cms. wide and ·8 cm. from the gauze B. Since the area of the rectangular orifice through which the air passed was only 1·2 sq. cms., velocities of the air of 300 or 400 cms. per sec. could be readily obtained.

Experiment 1.

C was connected to the negative pole of a battery of 8 volts, the other pole being connected to earth. The gauze B was connected to one pair of quadrants of an electrometer, the other pair being connected to earth. All other parts of the apparatus were earthed. When the arc light was acting the plate C lost a negative charge which passed over to the gauze B and the electrometer needle showed a movement corresponding to

60 divs. per min.

When a rapid current of air was directed between the plates, the leak to the electrometer was completely stopped. This showed that the carriers of the negative charge, which had left C, had been blown out by the rapid current of air.

If C was charged to -24 volts the rate of leak from C corresponded to

170 divs. per min.,

but where the blast was in action the rate of leak was reduced to

8 divs. per min.,

or less than $\frac{1}{20}$ of the charge which escaped from C reached the gauze B; when C was charged to higher voltages and the blast kept in action, a still greater proportion of the charged particles reached the gauze without being blown out. The number of carriers that reached the gauze could be raised or lowered by diminishing or increasing the velocity of the blast, the other conditions remaining the same.

Experiment 2.

We have seen that a whole or part of the charged carriers can be prevented from reaching the gauze B. It is now necessary to show what becomes of the carriers after being blown out from between the plates. The plate C was charged to -24 volts and the gauze connected to earth. The aluminium cylinder T was connected to one pole of a battery of 30 volts, and the wire W connected to the electrometer. There was no deflection of the electrometer if the arc alone was acting. If T was charged negatively, then when a current of air was sent between the plates the wire W became charged negatively. If T was charged positively there was no appreciable leak to W. This shows that a negatively charged gas has passed into the cylinder which in the first case lost its charge to the central wire and in the other to the tube T. A very convenient means of testing the whole charge conveyed with the current of air is to place a plug of cotton-wool inside the tube, which has the property of completely discharging the electrification carried with the air.

Experiment 3. *Discharging power of a metal tube.*

If the aluminium tube T which was 30 cm. long and 1·1 cm. diameter was connected to the electrometer and the central wire removed, it was found that the gas gave up part of its charge to the tube in its passage along it. In an experiment where the velocity of the air along the tube was about 150 cm. per sec., $\frac{1}{3}$ of the gas was discharged in passing along the tube. Since the electrified particles tend to repel one another to the side such a discharge is to be expected. If the volume density of the electrification were uniform over the cross section of the tube, an experiment of this kind on the discharging power would allow us to calculate the velocity with which the carrier moves under its own repulsion, but this condition is here not fulfilled.

Experiment 4.

All the electrification blown out between the electrodes C and B can be collected. In one experiment the rate of leak of C was observed with the blast in action. Part (1) of the charge which left C was given up to the gauze B, a part (2) was discharged on the tube T and the part (3) which escaped was caught in a cotton-wool collector. The sums of the rates of leak to (1), (2) and (3) were found to be very nearly equal to the rate of leak of C.

Velocity of the Carrier.

The general experiments on the effects of a current of air between two electrodes when the ultra-violet light is acting may be simply explained by supposing that the negatively charged particles, which escape from the surface C, travel towards the gauze B with a velocity proportional to the electromotive force between the two plates.

Let u = velocity of the charged particle for a potential gradient of 1 volt per cm.,

d = distance between the plates,

l = length of the plates,

v = difference of potential in volts between the plates.

The time t taken for the carrier to pass from one plate to the other is therefore given by

$$t = \frac{d}{u \cdot \frac{v}{d}} = \frac{d^2}{uv}.$$

Now if P = velocity of blast, the time t_1 taken for the current of air to pass between the electrodes is given by

$$t_1 = \frac{l}{P}.$$

Assuming that the velocity of the air blast is constant over the cross section of the plates, if t_1 is less than t none of the charged carriers which leave C can reach the gauze B, but they will be all blown out with the current of air. We may suppose the carrier describes a diagonal path between the plates due to the resultant of the two impressed velocities at right angles to each other, and unless this diagonal path cuts the gauze B the carrier will escape.

Let AB, CD (Fig. 2) be the two plates. Suppose the carriers to be produced uniformly along CD by the action of ultra-violet light. A carrier starting from C travels along the diagonal path CL and gives up its charge to the plate AB. Draw BP parallel to LC, meeting CD at P. We see that a carrier starting from P

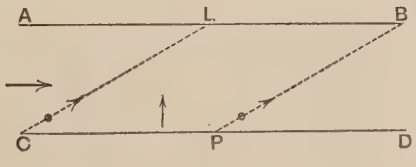


Fig. 2.

will just reach B. All carriers starting from the right of P will not reach AB but will be carried out by the current, while those to the left of P will give up their charges to AB. The ratio ρ of the number of carriers blown out to the total number that leave CD is given by

$$\rho = \frac{PD}{CD},$$

the distance $PD = P \,.\, t$, where t is the time taken for the carrier to cross over between the plates;

$$\therefore\ PD = P \times \frac{d^2}{uv},$$

$$\therefore\ u = \frac{P}{l} \cdot \frac{d^2}{v\rho}.$$

The ratio ρ is determined experimentally and P can be measured, therefore the value of u can be at once obtained.

Experiments performed in this way give a value of the velocity of the carrier at normal pressure of about

1·5 cm. per sec.

for a potential gradient of 1 volt per cm. This method is however probably not capable of accuracy, on account of the variation of the velocity of the air across the cross section of the plates and accidental disturbances due to the violent eddying motion of the air when velocities of the order of 300 or 400 cm. per sec. are used. The method is also practically restricted to the case of air on account of the large amount of gas required for an experiment, so that I was led to devise a more general and satisfactory method of determining the velocity of the carrier.

In Fig. 3 the general arrangement is shown: a glass bell-jar

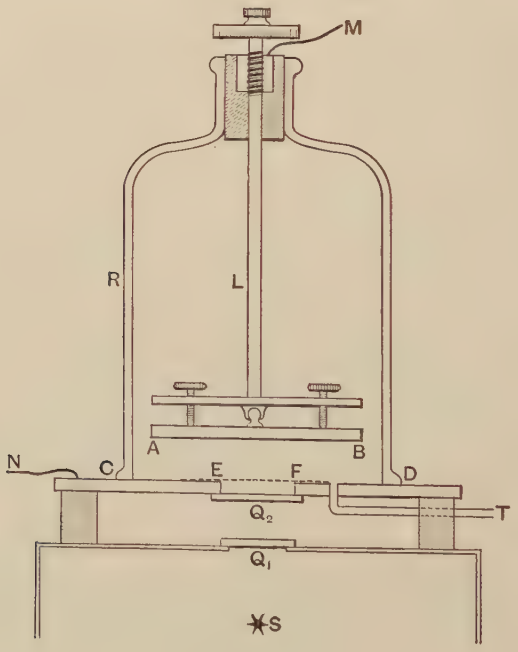


Fig. 3.

was fixed on a base plate *CD* of zinc through which a circular opening *EF* was cut. In the top of the vessel there was fixed an ebonite stopper through which passed a rod *L* carrying at one end a polished metal plate *AB*. This plate *AB* was fixed to a ball-and-socket joint, and could be levelled by screws passing

through a plate fixed to the rod. The plate AB could be raised or lowered from the outside of the vessel by a screw. The bell-jar was fitted down to the base plate with sealing-wax, and the whole was placed on insulating blocks over a source of ultra-violet light S, which was either an arc light or a spark gap. The ultra-violet light first passed through a quartz plate Q_1, fixed over an opening in a large metal screen, then through the quartz plate Q_2 covering the opening EF, and then fell on the metal plate AB. The wire-gauze performed the double function of allowing part of the light to pass through and yet acting electrically as a plane metal surface. The plate AB was generally of polished zinc and was accurately levelled to the base plate. A delivery tube T was let into the base plate and the whole vessel was made air-tight to allow of exhaustion.

In order to determine the velocity of the carrier, the rod L which was insulated by means of the ebonite stopper was connected to one pair of quadrants of the electrometer by the wire M, the other pair being to earth. The base plate CD was then connected to one terminal of a 100 volt transformer, worked from the town mains, the other terminal of which was connected to earth. When the base plate is charged positively, the plate AB is charged negatively by induction, and the negative carriers, set free under the action of the ultra-violet light, start travelling towards the base plate under the influence of the electromotive force acting between the two plates. If the plate AB is close to CD, a large number of the carriers are able to reach CD and give up their charge before the electromotive force is reversed. All the carriers distributed between the plates at the instant of reversal travel back to the top plate, and since the top plate is charged positively by induction no more carriers are produced during that half alternation. Experimentally it was found that there was no leak to the top plate when the base plate was negatively charged and the ultra-violet light was in action. We see therefore that when the plates are close together the plate AB loses a negative charge. This rate of leak will evidently decrease as the distance between the plates is increased, until a certain distance is reached, when the plate AB shows no loss of charge, although the ultra-violet light and alternating E.M.F. are both acting. When the plate is at this distance the first carriers liberated when AB becomes negative by induction are able to travel nearly to the base plate, but before any can give up their charge, the E.M.F. is reversed and they travel back to the plate from which they came. All distances greater than this give us no rate of leak, but it is the object of the experiment to determine the *shortest* distance between the plates for which AB shows no loss of charge. This point is in general fairly sharply defined, as

the table below shows, which gives the rate of leak at different distances from the base plate.

Turns of Screw	Deflection of Elect. in 30 sec.
1 turn	off scale
3	290 divn.
5	220
7	120
8	35
9	2
9·5	0

9 turns of the screw is approximately the shortest distance between the plates for which the loss of charge is negligible, and the numbers show how rapid is the decrease of deflection between 7 and 9 turns.

Let u = velocity of the carrier for a potential gradient of one volt per cm.

d = shortest distance between the plates for which the leak of AB is zero.

T = time of complete alternation of the transformer.

It is assumed that the E.M.F. at any time of the alternation is given by $E_0 \sin 2\pi \frac{t}{T}$, where E_0 is the maximum value of the alternating E.M.F.

The distance dx traversed by an ion in the time dt is given by

$$dx = \frac{E_0}{d} . \sin 2\pi \frac{t}{T} . u . dt.$$

Now the distance passed over by the carriers which first set out is equal to d in the time $\frac{T}{2}$.

$\therefore$ integrating both sides we obtain

$$d = \frac{E_0}{d} \cdot \frac{T}{\pi} \cdot u,$$

or

$$u = \frac{d^2\pi}{E_0 T}.$$

The distance d is determined experimentally, and E_0 and T are constants of the transformer circuit, so that u is known. Since the electrometer circuit is insulated before the E.M.F. is applied to the lower plate, the true potential difference between the top and base plate is less than the potential of the lower plate on account of the induction effect between the two plates. In consequence of the rapidity of the alternations the electrometer needle shows no movement due to the induction effect, so that it is necessary to determine the correction to be applied when the needle of the electrometer remains at rest. This may be simply done as follows:

The two quadrants of the electrometer are insulated from each other, and then a steady E.M.F. is applied to the base plate. The electrometer needle is deflected, but is brought back to its original position by applying a suitable E.M.F. to the other pair of quadrants. Let r be the ratio of the applied potential to the potential V of the base plate. The potential of the electrometer circuit is rV and the true difference of potential between the plates is

$$(1 - r)\, V.$$

The correction is thus made by putting $(1 - r)\, E_0$ instead of E_0 in the equation that determines the velocity.

The following is an example of a determination of the velocity of an air ion travelling through air,

$$d = \cdot 8 \text{ cm.},$$

$$T = \tfrac{1}{91} \text{ sec.}$$

Reading of alternating voltmeter 95, the maximum value of the E.M.F. E_0 is therefore given by

$$E_0 = 95\sqrt{2},$$

$$u = \frac{d^2\pi}{E_0 T} = 1{\cdot}37.$$

The correction for induction between the plates was 5 per cent., so that the final value is

$$u = 1{\cdot}44 \text{ cm. per sec.}$$

A large number of determinations have been made for the velocity of the air ion, and the mean of the results gives a velocity of about

1·45 cm. per sec.

In these experiments two sources of ultra-violet light have been used, viz. an arc lamp and a spark between two zinc terminals. For the spark two large Leyden jars were charged up by a Rumkhorff coil and discharged through a spark gap of about ·8 cms. In order to get rid of electrostatic effects, the jars, coil, and spark gap were all placed inside a metal tank connected to earth. The opening above the sparking terminals was tightly closed by a quartz plate, on both sides of which fine wire-gauze, connected to earth, was stretched. This arrangement was found to screen thoroughly from electrostatic disturbances.

For experiments on the velocity of the carrier the arc light is not nearly so satisfactory a source of ultra-violet rays as the spark discharge. The arc as a rule gives out more intense radiation but it has the drawback of rapidly raising the temperature of the plates and of the adjacent air. In consequence of this the velocity of the carrier is changed, and the determination of the least distance of no appreciable loss of charge cannot be made with certainty. In this respect the spark light is far more satisfactory and it is also a more constant source of rays, but it has the disadvantage of being an intermittent source of light.

A source of error in the determination of the velocity, which is difficult to avoid, lies in the irregularity of the E.M.F. from the town supply. During the daytime, the load on the machines was light, and there were often rapid alterations in the E.M.F. and period of the alternations. Another source of error also lies in the assumption that the electromotive force curve of the alternator is a sine curve. It is intended to continue these experiments and to use instead of the transformer an alternating electromotive force produced by reversing the sign of a steady E.M.F. by means of a suitable revolving commutator. It is hoped in that way to obtain a very accurate value of the velocity under varying conditions of pressure and temperature.

Effects of different metals.

If the theory that the discharge of electrification is due to the disintegration of negatively charged particles is true, we should expect to obtain different velocities of the carrier according to the metal on which the ultra-violet light falls. This point was tested by replacing the zinc plates by similar sized plates of aluminium, lead, copper, amalgamated zinc and a zinc plate covered with mercury sodium amalgam. Although there were differences in the rate of leak of these metals due to their different degrees of sensitiveness to ultra-violet light, it was found that the distance of no loss of charge was nearly the same in each case. The arc light was used for this series of experiments, and allowing

for errors of experiments it seems to be true that the velocity of the carrier is *independent* of the metal on which the light falls.

This seems to show that the carrier is produced from the gas near the plate and not from the metal itself.

Effect of electromotive force.

The velocity of the carrier, *i.e.* the velocity for a potential gradient of one volt per cm. is independent of the potential of the surface from which the carrier sets out.

Experiments were also made to determine the value of the velocity for different values of the E.M.F. To test this the ultra-violet light from an arc was allowed to shine through a wire-gauze on to a metal plate which was much larger than in the experiments in the bell-jar. By using a suitable transformer, potential differences, which were measured by a Thomson's Multicellular Voltmeter, of 365 and 700 volts beside the usual 95 volts were obtained.

Voltage	*d*	Correction for induction
95 volts	·775 cms.	14 per cent.
365 „	1·53 „	10 „ „
700 „	2·07 „	7 „ „

When the corrections for induction are made, the radius of the velocities obtained for unit potential gradient are

Voltage	Velocity
95	1·46
365	1·43
700	1·33

We see from the above table the velocities are very approximately proportional to the electromotive intensity between the

plates. The actual velocity of the carrier when the E.M.F. was 700 volts was 450 cms. per sec.

From considerations based on the kinetic theory of gases it can readily be shown that a carrier of molecular dimensions carrying an atomic charge attains a limiting velocity in a uniform field after a very short interval of time and it then moves uniformly. No correction therefore need be made for the interval that elapses before the carrier reaches its limiting velocity.

Effect of pressure on the velocity of the carriers.

The effect of pressure on the velocity was investigated by means of the apparatus shown in Fig. 3.

The following table shows the results obtained for air:

Pressure	Velocity
765 mm.	1·4 cm.
323	3·36
162	7·3
140	7·8
95	11·9
58	20 3
34	33·6

These results show that down to a pressure of 34 mm. the velocity of the carrier is *inversely* proportional to the pressure.

As Stoletow has shown, it was found that the rate of leak for a given E.M.F. increased with the diminution of pressure so that the results were capable of fair accuracy even at low pressures.

The plates were never separated by more than 1·5 cm. but the electromotive force employed was much smaller for the lower pressures. In the experiment for the pressure of 34 mm. only $\frac{1}{10}$ of the voltage of the transformer was used.

If the law that the velocity is inversely proportional to the pressure holds for low pressures at a pressure of 1 mm. with a potential gradient of 1000 volts per cm. the carrier would travel with a velocity of $1{\cdot}4 \times 760 \times 1000 = 10^6$ cms. per sec.

These experiments on the variation of the velocity with the pressure afford very strong evidence that the carrier of the charge is of molecular dimensions.

According to the kinetic theory of gases the velocity of a charged particle of molecular dimensions varies inversely as the pressure[1]. If however the particle is large compared with a molecule, the velocity of the carrier is dependent only on the viscosity of the gas which is independent of the pressure within limits. The conclusion to be drawn from these results is that the carriers must be very small and comparable in size with a molecule.

Many of the interesting results of Stoletow[2] on the relation between the current and the E.M.F. for different pressures on ultra-violet light conduction can be generally explained on the hypothesis that the rate of production of the carriers is a function of the pressure, and that the rate of escape of the negative ions is dependent on the velocity of the carrier.

It is not my intention here to enter into a detailed discussion of his valuable results but the fact that the current through the gas at very low pressures (about ·002 mm.) is independent of the pressure seems to show that either there is a slight disintegration of the electrodes by ultra-violet light or that the presence of mercury vapour is responsible for the action.

Velocity in different gases.

The velocity of the carrier depends on the gas surrounding the plates. Hydrogen and carbonic acid were used and were well dried before passing into the apparatus.

Gas	Velocity	Ratio, Air = 1
Air	1·4	1
Hydrogen	3·9	2·8
Carbonic acid ..	·78	·56

These results were obtained with the use of the arc light when the heating of the gas prevented the velocities being obtained with as much accuracy as desired. The hydrogen ion in hydrogen

[1] J. J. Thomson. *Brit. Assoc. Report*, 1894.
[2] *Journ. de Phys.* IX. p. 468. 1890.

travels nearly three times as fast as the air ion in air, and 5 times as fast as the carbonic acid ion in carbonic acid.

It has been shown that the carriers which are produced under the action of ultra-violet light have very similar properties to the ions which are produced by Röntgen radiation. It is also an interesting result that the velocity of the air ion in Röntgen conduction is not very different from that obtained in ultra-violet light conduction. In a previous paper[1] it has been shown that the sums of the velocities of the positive and negative ions in air, hydrogen and carbonic acid are 3·2, 10·4 and 2·15 cms. respectively. Assuming for the purpose of comparison that the velocities of the positive and negative ions are equal, the table below shows the relative values of the velocities for the negative ions:

Gas	Velocity in Röntgen Conduction	Velocity in Ultra-violet light Conduction
Air	1·6 cms.	1·4
Hydrogen	5·2 „	3·9
Carbonic acid ...	1·07 „	·78

Considering the widely different methods used for the determination of the velocities in the two cases, the comparative agreement shows that possibly the carrier is the same in the two cases or in any case not greatly different.

The results obtained in this paper seem to show that the gas near the surface of the negatively electrified plate is ionised under the action of ultra-violet light. The positive ion gives up its charge to the plate and the negative ion is repelled from the plate. This ion travels through air at normal pressure and temperature with a velocity of about 1·4 cms. per sec. for a potential gradient of one volt per cm. The velocity of the ion varies inversely as the pressure and is different for different gases.

It is intended to continue these investigations on the velocity of the ions and on the general phenomena of ultra-violet light conduction.

[1] *Phil. Mag.* Nov. 1897.

Cavendish Laboratory,
Feb. 21, 1898.

(3) *Röntgen photographs of metallic alloys.* By Mr C. T. Heycock and Mr F. H. Neville.

The authors exhibited and described photographs taken by means of the Röntgen rays through plates of alloy. As the two metals forming the alloy possess different degrees of transparency to these rays the photographs show the separation of the metals that has taken place during the solidification of the alloy. For example, alloys of gold and sodium containing less than 30 per cent. of gold are seen to consist of well-developed, very transparent crystals, which must be pure or nearly pure sodium, imbedded in a mother substance which solidified last, and which from its comparative opacity evidently contains the gold. On the other hand, alloys containing more gold show very opaque needles of gold imbedded in a less opaque mother substance. This mother substance was the same as that in the first-mentioned alloy; it solidified after the needles of gold had been formed. Photographs of alloys of aluminium and gold and of aluminium and copper were shown which exhibited similar phenomena. The crystals of aluminium in the alloy with copper were perfect rectangular crosses several millimetres in diameter. The gold-aluminium alloys showed a precipitate of Roberts-Austen's compound of the formula $AuAl_2$; the crystals were well-marked cubes and octahedra.

MONDAY, 7 *March*, 1898.

PROFESSOR NEWTON, VICE-PRESIDENT, IN THE CHAIR.

The following Communications were made to the Society :—

The Coral Reefs of Funafuti, Rotuma and Fiji together with some Notes on the Structure and Formation of Coral Reefs in general. By J. STANLEY GARDINER, M.A., Gonville and Caius College, Cambridge.

INTRODUCTION.

HAVING been enabled by means of a grant from the "Balfour Fund" to join the "Coral Reef Boring Expedition" to Funafuti, I had the pleasure of spending upwards of three months on that atoll with Prof. Sollas, who suggested to me the propriety of comparing its reefs with those of Fiji and other islands in the vicinity. To Prof. Sollas I am under deep obligations, and any value that my observations may have is largely due to him.

I am also much indebted to Capt. Field and the officers of H.M.S. *Penguin* for their great courtesy and assistance at all times.

Returning to Fiji in the middle of August, I left in the *Penguin* at the end of the month for Rotuma, where I remained until the middle of December. Here I was much beholden to Mr Leefe, the Resident Commissioner, for his assistance and especially for the loan of his boat and crew.

Coming back to Fiji the second time, after a tour round Ovalau, I proceeded along the north coast of Viti Levu to Nathilau. Crossing from there to the Rewa river, I obtained two canoes and in five days reached Suva, where I remained for some days, making excursions to the Navua river, the Mbenga barrier reef and into the neighbouring country. I then spent a fortnight in Wakaya on the kind invitation of Capt. Langdale, late R.N. Having received from the Hon. J. Stewart, Colonial Secretary, an offer of a passage round the Lau Group in the government S.S. *Clyde*, I gladly accepted, visiting the Exploring Group, Lakemba, Fulanga, Ongea, Namuka and Kambara. On returning I went to Taviuni, where I spent a fortnight, riding or walking round the greater part of that island. Ratu Lala, the native chief, then sent me in his whaleboat to Rambi and on to Lambasa, which we reached in four days. I next crossed to Nateva Bay, down which I proceeded, partially by canoe, and reached Savu-Savu Bay on the third day, whence I proceeded to Levuka, where I obtained ship to Sydney.

In Fiji I was greatly assisted by Capt. W. W. Wilson, Harbour Master of Levuka, who took the *Clyde* round the Lau Group especially to help and direct me. As Capt. Wilson probably knows more than any man living about the reefs and anchorages in the Central Pacific, and most generously placed freely at my disposal his journals and the mass of information he has collected, I cannot sufficiently express my indebtedness to him. I wish also to express my thanks to the Hon. W. L. Allardyce, Native Commissioner; Mr Berry, Commissioner of Works; Mr Turner, Chief of Customs, Levuka; Capt. Langdale, late R.N.; the late Capt. Puttnam; Ratu Lala; Mr C. Thomas; Mr W. Reay; Mr W. Chalmers and many others for their hospitality and aid.

In England I am under great obligations to Prof. Newton, Mr Adam Sedgwick, Mr W. Bateson, Mr A. C. Seward and especially to Prof. Hickson, for their encouragement and counsel. In particular I must thank Mr J. J. Lister for much valuable criticism and advice.

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PART I.

THE ATOLL OF FUNAFUTI.

THE atoll of Funafuti was chosen as a typical atoll by the Coral Reef Boring Committee of the Royal Society in 1896. It belongs to the Ellice islands, which lie to the north of the Fiji Group, and extend in a broad line from lat. 11° S. to lat. 5° 30″S., running almost north-north-west and south-south-east. In the same line to the north at a distance of about 150 miles lie the Gilbert islands, which are themselves continued north by the Radack and Ralick chains of the Marshall islands. All are of coral origin and many of atoll form.

In the bathymetrical charts, published in the *Report of the Challenger Expedition*, the Ellice islands are seen to lie on a plateau of about 1500 fathoms with the Fijian, Tongan and Samoan Groups, the whole in a deep of over 2000 fathoms. The soundings of Captain Field, H.M.S. *Penguin*, in 1896 proved, however, that the Ellice islands are separated from this plateau by depths of over 2000 fathoms, and showed also that Funafuti rises as an isolated reef in this deep. Further soundings of the *Penguin* by Nui, Nukulailai and towards Nukufetau from Funafuti indicate that these atolls probably rise in a similar manner so

that there appear to be in the Ellice Group a number of isolated islands rising separately from a depth of over 2000 fathoms.

The atoll of Funafuti (Fig. 1) has a lagoon 12 miles long by 8 miles broad, lying almost north and south. It is of a roughly oval shape, extending south into a "pocket" about 3 miles deep, much of the bed of which is uncovered at low tides. Round this lagoon is a more or less continuous reef, varying in breadth from $\frac{1}{4}$ to $\frac{1}{2}$ a mile in accordance largely with the portion, which has been turned into land. On the east side is an island $6\frac{1}{2}$ miles long, continued north by two more small islands, Amatuku and Mulitefala, with narrow breaks in the land between; this main island bends to the westward about 2 miles from its south end. Continuing it to the south-west are three other islands, Fatata, Funangongo and Funamanu, joined to one another and to the main island by the reef, which can easily be crossed at low tide. The next island to the west, Falefatu, is separated from these by a broad ship's channel with a minimum depth of four fathoms; on the west side of this island there is another similar channel with a depth of five fathoms. The reef is here continued into the "pocket," which extends about $3\frac{1}{2}$ miles to the south, and is largely surrounded by land. On the west side of the atoll the reef is very broad, nevertheless it has only a few small islands. To the north-west is another broad ship's channel through the reef with a minimum depth of $4\frac{1}{2}$ fathoms. On the west side there are also three channels of 4 fathoms and two narrow somewhat tortuous passages of 11 and 23 fathoms. The north side of the lagoon has two small islands, Fualifeke and Pava, but while surrounded by the reef has otherwise no land.

The direction of the wind from March to November is from the east-south-east, veering completely round in the three summer months, December, January and February, to the west-north-west; but while the easterly wind is steady during the greater part of the year, the westerly wind in the summer is more irregular and less strong with considerable periods of calm. The current is due probably entirely to the wind and accordingly comes from the same direction; I was informed, however, by several trading captains in Fiji that the set in the summer months westward is never very great and in calms is lost, giving place to a slow set eastwards. In agreement with the wind the reef is, save for the two ship's channels to the south, continuous round the east and south sides of the atoll and largely converted into land. The "pocket," the projecting southern horn of the atoll, is exposed fully to the south and east and is almost completely surrounded by land. The flood tide into the lagoon sets in strongly through the ship's channels; but at the same time a large amount of water is received from the breaking of the sea over the reefs

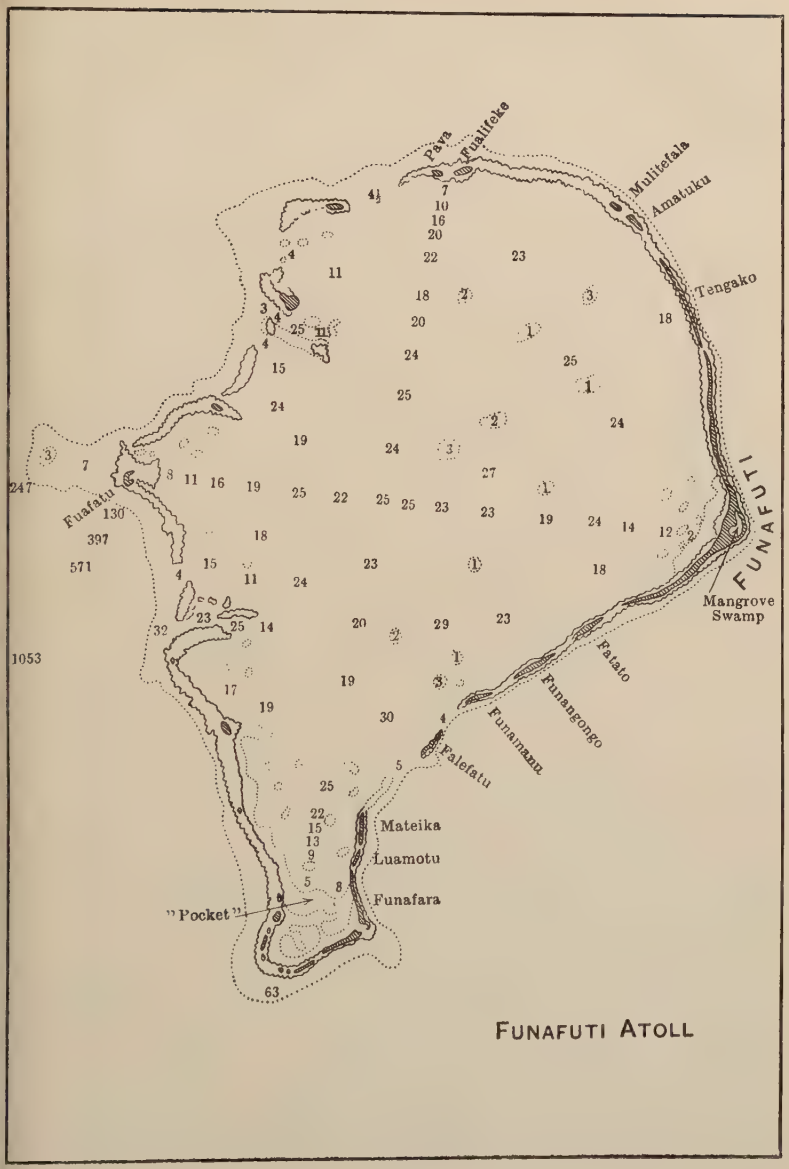

Pava
Fualifeke
Mulitefala
Amatuku
Tengako
FUNAFUTI
Mangrove Swamp
Fatato
Funangongo
Funamanu
Falefatu
Mateika
Luamotu
Funafara
"Pocket"
Fuafatu
247
130
397
571
1053
63
FUNAFUTI ATOLL

FIG. 1.

between the different islands. The ebb tide, however, can receive little assistance by the sea water of the lagoon escaping in this way, so that the out-going currents are much stronger; for some time indeed after low tide there is still a noticeable current outwards in these channels. The current too in the southerly channels naturally sets in stronger, and goes out weaker than in the westerly ones, as it is to some extent the resultant of the forces of the tidal and wind currents.

The islands on the reef are generally covered with loose blocks of reef-rock between which coconut and other trees have taken root; in most places there is little or no soil. In some parts the surface shows a flat platform of hard rock with scanty, or no vegetation; the largest lies in the middle of Fualifeke, and there is another on Funafara to the east of the "pocket." Usually the presence of this flat smooth platform is concealed by boulders, among which the trees grow, but I found it existing very generally on all the islands to the south of the atoll, and by the sides of the "pocket." To the east such a flat does not exist, the whole surface seeming simply to be a mass of loose blocks, the skeletons of different species of corals and masses of reef-rock.

The breadth of the islands varies from 50—300 yards, but of course at their ends they narrow down to mere ridges; their general breadth is scarcely more than 120 yards. The main island in its extreme eastern part is considerably broader than any of the other islands of the atoll, by the village being about 600 yards across. Here there is, however, for the greater part of the breadth on the lagoon side a deep sandy soil, formed of the ordinary beach sand, which is found in the lagoon in the vicinity, mixed with a certain amount of mould from the vegetation, which grows especially luxuriantly on it. Here too there is what Professor Sollas has called the "mangrove swamp," a bare area in the middle of this part of the island, covered with water at high tide; to it the sea gains access from the seaward face of the atoll through three tunnels at its northern end. Its south end is covered by a fine mud, varying up to 3 feet in depth, with boulders imbedded in it; these are often of large size and project in places above the surface of the mud. Partially imbedded in this mud Professor Sollas has traced a crescent-shaped[1] "dead coral reef, constituted almost entirely of two species, one a massive *Porites* and the other *Heliopora coerulea*. For a great part of the day this floor lies bare and dry, the frayed ends of the *Heliopora* standing like broken reeds 6 inches above its surface, and the great clumps of *Porites* forming a series of stepping stones of equal height. Neither of these corals stands long

[1] Report on the Coral Reef at Funafuti. *Nature*, Vol. LV. 1897, p. 376.

exposure to the air; on Funafuti they require constant submergence and we are thus led to regard their upper surface as marking what was at one time the level of low tide in the swamp." The species of *Porites*, referred to, very closely resembles *Porites arenosa* (Esper), a very common species on the shoals in the lagoon; living *Heliopora* was only obtained once at Funafuti, and then from a depth of over 35 fathoms off the southern reefs of the atoll. The rest of the swamp is sparsely covered with mud, being formed of a hard consolidated rock, which closely resembles the rock forming the reef. North of the mangrove swamp is a marshy pool more or less filled with brackish water, and there are in the middle of the south end of the same island a number of such pools; they seem to me to be directly comparable to the "barachois" of Diego Garcia[1], described by Bourne.

From the sea the mangrove swamp is separated by the "hurricane beach," a structure which extends to some extent round the seaward face of all the islands, but varies considerably on account of the position of the islands in respect to the atoll, and possibly from other causes. It is the extreme edge of the land against which the sea, after passing over the reef, expends its force. The height of its summit above the sea level at high tide varies up to 7 feet, but a greater height than 4—5 feet is uncommon. Its top is covered with boulders torn from the reef facing it, often much rounded and worn by the water. Near its summit this beach slopes down to the sea at an angle of about 30°, but towards its base, where it is washed by the tide, and formed of a hard consolidated coral-rock, it passes gradually into the flat reef. Its formation along the whole of the main island is precisely similar to the above, but in the islands to the south-east of the "pocket" there can scarcely be said to be any such beach at all, and the land is broken into, and in places partially submerged at spring tides. On the leeward side of the atoll generally this beach, or a sudden fall from the land to the high tide level, exists, but at its base the solid rock cannot be seen; instead the whole is covered by small rounded fragments of reef-rock, coral, shells, etc., forming in fact a pebbly beach. From the hurricane beach the land usually slopes towards the lagoon with a drop of two or three feet in a few yards, and then a very gradual slope. To windward of the atoll the islands on their lagoon sides have usually no sharp fall, the high-tide level at "springs" being marked by the extension of the vegetation. By the village, however, the wash on the sandy shore has caused a well-defined drop of one to two feet. To leeward of the atoll

[1] "The Atoll of Diego Garcia and the Coral Formations of the Indian Ocean," by G. C. Bourne. *Proc. R. Soc.* 1888, XLIII. p. 440.

there is a very distinct beach on the lagoon sides of the island similar to the hurricane beach of the same islands.

A hurricane beach, such as is found on the windward face of the atoll, could indubitably have been formed by masses of coral and reef-rock, thrown up by the waves, subsequently consolidating underneath by the deposition of carbonate of lime on the evaporation of the water at low tide, but under such circumstances the strata should dip towards the ocean, which piled up the fragments in the first place. The stratification is not generally visible, but at the ends of the islands, especially to the south-east of the atoll, it appears rather to be horizontal. At the north end of Amatuku there is a section clearly visible of a small cliff about 6 feet high; the strata are horizontal, but this island is here rather broad and the cliff stands well back from the sea, continuing for about 50 yards along the lagoon shore. The three islands, Funamanu, Funangongo and Fatato, have the hurricane beach much steeper than the main island; indeed the centre one, Funangongo, has towards the ocean a precipitous fall, in places almost a cliff, of 5 feet, succeeded by a steep boulder-covered beach. On the top it is not formed of a firmly consolidated rock, but of somewhat loosely aggregated blocks of coral and reef-rock; the extreme height here above high tide I measured at 6 feet. The action of the sea on the coast is plainly visible in fallen shrubs and trees undermining the beach, and from the loose nature of the land above continually causing slips. The beach is strewn with boulders, which extend considerably below high tide level, and between tide marks these seem to be joining on to the rock below by the deposition of carbonate of lime on the evaporation of the water left on the beach, when the tide falls. On the island lying north-east of the "pocket," a somewhat similar hurricane beach is found, but it is not so steep; it is, however, consolidated to a higher level than in these islands, and the horizontal character of its strata is more distinctly visible.

Under the hurricane beach on the windward side of the atoll there is an area, rarely more than 15 yards broad, of very rough ragged rock with shallow pools of water here and there at low tide. Outside this is the flat of the reef and its raised outer edge: these latter parts vary on this face in combined width from 40—80 yards. As I shall have occasion to refer to these zones constantly, I shall call them the *rough zone*, the *reef-flat* and the *rim*. The rock of all these three zones consists of a consolidated mass of the skeletons of corals and nullipores with shells, sand and the remains of various marine animals; it is exceedingly hard and rings to the hammer. On the surface where, exposed to the air between tide marks, it has time to dry, it is almost structureless to the unaided eye, but pieces, obtained from under

water by means of a chisel, show clearly its constituents. In the pools of the rough zone, referred to above, and on the reef-flat slabs may be here and there raised by a crowbar, used as a lever. These are found to have been fixed on to the rock below by means of the living bodies of Tunicata and Porifera, growing to both, while the spaces between them abound with Polychaeta Gephyrea, Mollusca and Echinodermata. These slabs, and indeed the solid rock elsewhere, are in most places much bored by worms, so that this formed perhaps my most prolific collecting ground for animals; a considerable amount of muddy sand, formed by these animals, under the rocks is a general feature. Some of these slabs have been but recently thrown into their present positions, and are being consolidated on to the rock below them. Many others, on the contrary, which could only be raised by breaking the rock itself, seemed to be due rather to the inwash of the sea, perhaps removing from under the harder coral, or nullipore remains, a layer of sandy or softer rock, which had formerly fixed them to the rock below. Probably both actions are going on at the same time, the former in the pools of the rough zone, lying immediately under the hurricane beach, and the latter on the reef-flat outside its limits. That the effect of this latter may in time be very considerable was indicated by the whole beach on the weather side, after a strong easterly gale, being to some extent strewn with slabs, which, on investigation, I found to have come from the reef-flat close to the rough zone.

The ordinary range of the tide at springs is $6\frac{1}{4}$ feet. At high tide the hurricane beach at its base is covered with 4 feet of water; in the rough zone there is a fall of about one foot, and a sharp fall from this zone of one foot to the reef-flat. The reef-flat is itself hollowed out to some extent so that it usually has about 3 inches of water on it, but is at the same time cut off from the sea by the rim of the reef, which stands up about one foot above low tide. The outer edge is much broken up by fissures (Fig. 2), which often run for 20 yards or more straight into the reef: these are joined together to some extent by cross fissures, so that the rim at low tide appears to consist of a number of great more or less rectangular flat masses projecting 1—2 feet above the reef-flat. These are arranged so that there is a distinct edge running along fairly evenly, but there are outside this on the weather face of the atoll a number of similar masses extending outwards for 8—10 yards with a depth over them of 2—3 feet at low tide. These masses can be clearly seen at low springs, after the backwash which precedes a breaker, and form the proper edge of the reef. Off this line there is a sudden fall to a depth of 3—4 fathoms; in places, however, there is no such fall and no proper edge, but a succession of masses and buttresses at

greater depths further and further seawards. Such a succession was clearly visible off the south end of the "pocket," and in many places to windward a gradual slope, such as the above, was indicated by the waves breaking at considerable distances outside the rim. The fissures (Fig. 2) vary greatly in breadth and shape: some are only a few inches broad while others are five or six yards. Their depth varies up to 3 fathoms, the average being at low tide about $1\frac{1}{2}$ fathoms where they run through the rim of the reef. In all, the edges overhang the sides considerably and are very thin; indeed, the fissures seemed as if they were being enclosed by the edges growing out above and meeting one another, while the channel below was being kept open by the scour of the tide. In some the edges have absolutely met, and the presence of the fissure is merely indicated by one or more blowholes, out of which at low tide is driven a cloud of spray with each in-rushing wave. Although I carefully examined a number of these fissures after storms, I never saw any signs of their edges being broken, and I think that there is no doubt that the flat surface of the reef extends by their means. The blowholes can be found almost completely closed, and indeed every intermediate stage, from what might be termed a boat channel to the solid reef, formed in such a manner as I have suggested, can be seen (Fig. 2). At the bottom of these fissures a few small boulders often lie with a certain amount of sand around and between them. Occasionally the sides are found to approach one another, and even to have fused, cutting off from the fissures pools, which, when the tide recedes, are left full of water.

At the edges of these fissures the reef shows a somewhat sharp rise of $1—1\frac{1}{2}$ feet from the reef-flat (Fig. 2, *a*). It is here much pitted on the surface and has a somewhat mammillated appearance; in the smaller pits Echini live in large numbers, and crabs and other animals take refuge in them. Over the surface there grow small green seaweeds a few inches high, and a small slate-coloured species of *Zoanthus*. Both of these might have to be carefully searched for, as generally nearly the whole is covered by white, grey, brick-red and dark green coloured incrusting nullipores. The importance of these cannot be over-estimated; the lime they secrete is always very dense, and apparently structureless. Its thickness for a single plant is often very considerable, and nearly the whole reef at the rim I consider to have been formed by them. It is by means of the growth of these nullipores that the fissures are bridged, and indeed, by the use of the chisel, small spaces of a few inches in diameter can be easily found completely surrounded by these organisms. The reef-rock of the surface in this position, when broken into, shows no trace of a coral origin, but simply consists of the lime formed

by these nullipores. Generally the horizontal thickness of the lime formed by each plant, as it were, does not seem to be very great while the total mass is enormous; the thickness of the lime of some large massive nullipores, obtained by the use of Priestman's grab outside the reef, was between 3 and 4 inches, but from the reef itself I never saw a nullipore with a greater thickness than 2 inches. What I have termed before the rim of the reef is really then an area 3—4 feet broad at the sides of these fissures, but facing the sea about 5 yards across. The waves break a short distance outside the rim and rush over it, or to some extent wet it every time they break; the water also rushes up the fissures, and so great is the momentum behind it, that it wells over their sides in every direction. The rim generally extends back all round the fissures, but from the ends of some, which run especially deeply into the reef, it is absent, these serving to take the return water, which is poured over the edges of the others, back to the sea.

Between these several fissures, and between the rim and the rough zone under the hurricane beach, the reef-flat, before mentioned, lies. The surface of this zone is very flat and smooth and covered over completely in its outer parts by nullipores and other algae. Its level is such that at ordinary low tide the greater part of its surface is covered over by 8 inches to a foot of water; its level below the rim round even the most open fissures is also such that at the lowest spring tides most parts of it are concealed by a few inches. Nullipores do not thrive on it well, perhaps on account of the hot sun at low tide, and most of it is covered by small, delicate, filamentous algae, which give it a very smooth slippery surface. Holes in it a few inches to several feet below its general level occur, and in these and on its outer part a few species of corals are sparingly found. Of these *Pocillopora* and *Stylophora* are by far the most numerously represented, and in many parts were the only corals obtained from this area. The species of these genera represented were *Pocillopora suffruticosa*, *P. brevicornis*, *P. clavaria*, *P. squarrosa*, *P. meandrina*, *P. glomerata*, *Stylophora palmata*, *S. pistillata*, *S. digitata* and three species which I have named *S. compressa*, *S. lobosa* and *S. rugosa*. These grow for the most part in clumps, and in this situation are rarely more than 6—7 inches in diameter, while 3—4 inches with a height of 1—2 inches for the clump is more usual. Their distribution nowhere is such that they could for a moment be considered to have any effect practically on the growth of this part of the reef. The only other species of coral at all common was a *Prionastraea*, which was however very local in its distribution.

The fissures for the reef present the most abundant signs of

coral life: their sides, where they do not overhang excessively, often have growths of *Pocillopora grandis* which markedly shows positive heliotropism, growing outwards at first from the sides and then straight towards the light. Their bottoms in any parts, where they are raised above the general level, and so are above the scour of the sand, may be covered with corals. From this position I obtained, as well as various species of *Pocillopora*, two species each of *Coeloria* and *Favia* and one species each of *Prionastraea, Heliastraea,* and *Montipora*; in addition to the above, I also obtained *Madrepora surculosa* and *Madrepora haimei* from the north end of the main island, but did not find any other species of this genus on the windward side of the atoll, although in Fiji I subsequently found it to be by far the most abundantly represented on the outer reefs. It would be a misnomer to speak of any of these corals as being abundant on the outer reef, the conditions requisite in the fissures being by no means common; in any area to be visited there could only be anticipated with any certainty a few species of *Pocillopora* and *Stylophora* and *Montipora incognita* (Bernard). *Millepora,* which is generally supposed to grow where the break of the waves is strongest, I only obtained in this area in a well-protected situation on the leeward side of the atoll. Under the overhanging sides of the fissures corals do not grow, though the conditions as to food supply should be eminently favourable; this may be due to the scour of the water along the sides, but I think it may also be partially accounted for by the absence of direct sunlight, the growth in general of the species of *Madreporaria* found in these seas showing, according to my observations, markedly positive heliotropism.

The lagoon shores of the islands on the rim of the atoll have either a hard pebbly beach corresponding in position, though not in height or extent, to the hurricane beach with a platform running off from it into the lagoon, or else a sandy beach usually tailing off with a similar lagoon platform, or reef. Opposite to the village there is no such reef, but the sand beach slopes off gradually into the lagoon, the bottom of which is here covered by a dense coating of sand. Professor Sollas, in his report[1] to the Royal Society, especially draws attention to the sand: "As regards the nature of this sand it is important to observe, that it does not consist of coral débris; this material and fragments of shells forming but an insignificant part of it; calcareous algae are more abundant, but its chief constituents are large Foraminifera which seem to belong chiefly to two genera (*Orbitolites* and *Tinoporus*). It covers a considerable area of the islands, and has accumulated during the memory of the inhabitants to such an

[1] *Loc. cit.* p. 375.

extent as to silt up certain parts of the lagoon. This and the abundant growth of corals and calcareous algae, such as *Halimeda*, lead to the belief that the lagoon is slowly filling up." I shall have occasion later to refer again to this paragraph; here I am merely drawing attention to it as showing the nature of the sand. It is only in this one situation that I failed to find a solid gently sloping platform of rock extending from the islands into the lagoon; in some places its nature is to some extent disguised near the shore by a deposit of sand, but its presence is everywhere clearly indicated. Its surface is smooth, but somewhat broken up into holes, which generally increase in size with the distance from the land; it is also in places strewn with boulders. After a gradual slope, to a depth of about 6 feet at low tide, it commonly ends in a cliff, or wall, soundings from the edge of which give depths of 2—5 fathoms, or even more. Such parts of it, as lie below the low tide level, are covered with green algae, *Sarcophytum*, sponges and foliaceous nullipores; corals too are fairly abundant on its extreme outer edge. Denser nullipores are also found belonging for the most part to species with upright plates, or round mamillated projections; great bunches of the genus *Halimeda* are very common. *Sarcophytum* is found in great flat spreading masses, but its distribution and that of corals over this area depend largely on the position of the lagoon-reef in respect to the openings in the atoll-reef, currents, etc.

The outer reef varies to an extraordinary extent, but everywhere the rim and the reef-flat present similar features to those already described for the windward side. The inner rough zone on that side is generally narrow and fairly even in height, but in places on its inner part just at the base of the hurricane beach isolated pinnacles of rock occur. These clearly form part of the reef-rock below them, and consist of similar constituents. The largest have a diameter of 6×6 feet with a similar height; their tops often to some extent overhang their bases. At high tide they are washed by the waves, but stand up for 1—2 feet above the tidal level. By this wash of the waves they are sometimes eaten into in such a way that the softer rock between the harder coral skeletons has been removed, leaving the corals freely exposed. The genera *Millepora* and *Madrepora* can commonly be identified; I also found the laminae of *Heliopora*. The branches or laminae of these corals generally grow straight upright in any position, in which they are well covered at low tide. Their skeletons, in the few places where I could trace them satisfactorily, I found invariably standing on these pinnacles in the same position. Such would not be the case were these pinnacles formed in a hurricane beach by the consolidation of fragments and the subsequent solution of the parts of the beach round them.

Hence I can only conclude that these pinnacles are really due to an elevation, and are the remains of a part of an old raised reef, which formerly extended for some distance outside the present hurricane beach. If this same zone is examined, on the three islands to the south of the main island, it will be seen to get progressively rougher with more pinnacles. The middle island, Funangongo, has no pinnacles, but across this zone there run out from the hurricane beach great buttresses of coral with fissures between, up which the waves rush at high tide. These buttresses are essentially a part of the hurricane beach behind them, and run out almost to the edge of the rough zone: at ordinary high tides the crests of most are just awash. The scour of the tide between these buttresses is very considerable, and accordingly they are to some extent undermined at their sides and bases. On the next island to the south, Funamanu, the same characters are still further accentuated, cross channels sometimes occurring between these fissures leaving pinnacles outside the buttresses. Corals in the position of growth are found plentifully on the tops of these buttresses. The rock forming their greater part is to the eye structureless, but in places sand and slabs may be seen, indicating horizontal stratification. The buttresses are continuous with the lower part of the hurricane beach behind, the upper part of which is formed by piled-up fragments from the reef in front. The resemblance of this part of the dry reef to the extreme outer edge of the living reef, as it at present exists, is very extraordinary. Every stage, from the pinnacles on the extreme windward face to this structure, can be traced, and I do not think there can be any doubt but that these pinnacles once formed part of a similar structure on the weather face, the greater portion of which has been washed or dissolved away.

On Mateika, Luamotu and Funafara, an almost exactly similar rough zone to that of Funamanu was found; but to the south of the "pocket" this zone is about 70 yards broad and very much as described for the windward side of the atoll, save that it is rougher and more strewn with boulders, which the seas have apparently not been powerful enough to roll up together to form a hurricane beach, as in fact none such can be said to exist. On the outer sides of the leeward islands the same zones are found, but generally the outer rim is less well defined, while the rough zone is exceedingly broad and rugged. Commonly the reef-flat is 30—40 yards across, the rough zone being about twice as broad.

Horse-shoe island, or Fuafatu, on the leeward side exhibits all these characters in its reef very well. The rim is distinct, but its fissures are less numerous and deep than to windward; the reef-flat is about 35 yards in breadth and the rough zone about double

that amount. Then there is a steep beach of rounded pieces of coral-rock, shells, etc. On the island itself there is a considerable amount of sand with a very small, though distinct, sandy cliff by the lagoon with a steep beach of shingle and sand. The lagoon reef has a maximum breadth of about 120 yards with a steep drop into about 4 fathoms of water. The sandy beach is not nearly so well marked in the bay, which gives it the name of Horse-shoe island, here being formed rather of a consolidated breccia, which is not on the surface very firm. It can be seen to consist of coral fragments, sand, and a large number of *Tridacna* and other shells. Neither the coral skeletons nor the shells are in the position that they naturally live in, but both dip towards the lagoon, and no doubt there is a formation of rock going on here between tide marks by the consolidation of these fragments. The greater part is not exposed even at the lowest spring tides, so that we have here the consolidation of a breccia both between tide marks and under the water.

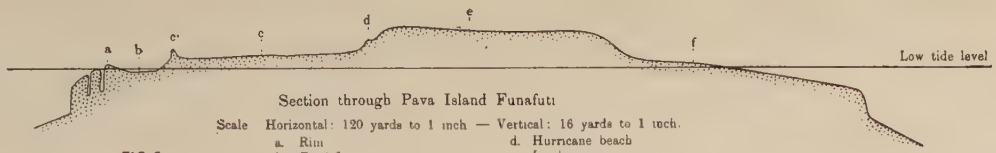
Small pinnacles are by no means uncommon on the outer rough zone to leeward: they are exactly similar to those of the windward face, but are situated generally close to the outside of the zone, which differs also in being much broader and often strewn to a great extent with massive boulders. Some of the pinnacles are not so well defined as belonging essentially to the rock, of which this zone is composed, and seem to be rather masses of the conglomerate, now being formed outside the reef, thrown up on it by strong gales or hurricanes; such are commonly called "negroheads," and have been well described by Dana. One part of the reef between two of the islands, which I was unfortunately prevented from visiting owing to a rough sea, appeared to have a considerable number of such fragments. It is remarkable that if there are "negroheads" on the leeward reefs, there should be none on the reefs to windward, against which the force of most of the gales is broken. Either they are only thrown up in bad cyclones, or hurricanes (which always strike to leeward), an almost impossible supposition, or the growth of these masses is firmer to windward. I consider that such masses would be worn away, and broken up much quicker to windward of the atoll than to leeward, and I think, too, that the growth of the reef outwards goes on faster likewise, so that the masses of reef forming outside should be fixed more strongly, and be more firmly built to windward.

To the north of the atoll, are the two islands, Pava and Fualifeke, separated by a narrow channel from one another, but at low tide their rough zones are continuous. Towards the other each sends out a sand bank, but these are separated by a pool with about three feet of water at low tide. Off these islands

(Fig. 3), the rough zone is extremely broad being rarely less than 100 yards, while the reef-flat is only about 30 yards. There is here practically all along a sudden drop of two feet from the rough zone to the reef-flat, and along this edge there is a line of small pinnacles. The hurricane beach is formed of coarse shingle and shows very distinctly in its abrupt fall the height to which the tide rises. Towards the lagoon the beach is sandy; below this the lagoon platform is distinct. In the small lagoonlet, between the two islands, there is found a sandstone, the strata of which dip slightly towards the lagoon. It is apparently a true beach sand-rock formation and only occurs in this position. I have appended a section (Fig. 3) across the island of Pava, showing the general features of the rim of the atoll here.

The reef, except where it is broken by the deep channels before mentioned, is continuous round the whole lagoon, and connects all the islands with one another. The rim and reef-flat show no discontinuity, the reef-flat, continuing between the islands, and a little lagoonwards ending in a straight cliff exactly similar to that of the lagoon platform of the islands. The reef between the islands slopes a little towards the lagoon, so that, even at extreme low tide, a small quantity of water makes its way over it into the lagoon instead of flowing out again through the fissures, which run through the rim. Probably owing to the difficulty of fixation found by the larvae, few corals grow on the reef in such a position. To leeward the reef is often very broad, as indeed are the inter-spaces between the several islands, and is generally exposed slightly at extreme low tides. Often its lagoon edge extends continuously with that of the lagoon platform of the islands, and is much broken up into knolls and patches, on which corals and nullipores flourish luxuriantly. South of Fuafatu, the reef has a distinct channel in its middle a few fathoms deep, extending along for some distance it consists of an outer reef with a distinct row of pinnacles marking its position at low tide, and an inner reef, a broad flat platform, a continuation of the lagoon reef of the island itself. South of this both reefs have a distinct but narrow passage to the ocean beyond; this passage through the outer reef is nothing more than a long fissure with a depth of about $2\frac{1}{2}$ fathoms, and is just broad enough to take a boat through. North again of the same island and generally to the westward, the reef has a depression in the centre, but it is not usually more than a few feet deep.

The shoals in the lagoon commonly on their upper surface, resemble very closely the inner parts in relation to the atoll of the lagoon reef, and the same species of coral are found on both. About the centre of the lagoon is one large shoal, and there are many others, which lie mostly on the windward side and towards

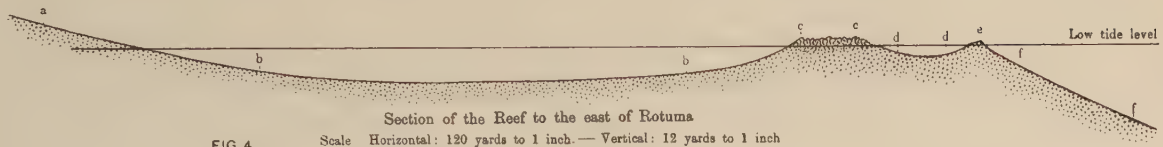


Section through Pava Island Funafuti

Scale Horizontal: 120 yards to 1 inch — Vertical: 16 yards to 1 inch.

a. Rim
b. Reef flat
c. Rough zone with pinnacle c'
d. Hurricane beach
e. Land
f. Lagoon platform

FIG. 3.

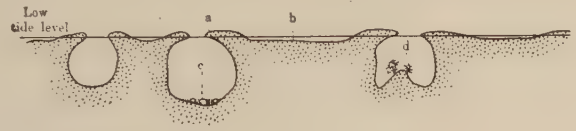


Section of the Reef to the east of Rotuma

FIG. 4

Scale Horizontal: 120 yards to 1 inch. — Vertical: 12 yards to 1 inch

a. Beach
b. b. Boat channel
c. c. Boulder zone
d. d. Reef flat
e. Rim
f. f. Outer slope

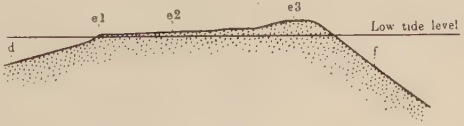


Section across three fissures in the outer reef of Funafuti.

Scale. 12 yards to 1 inch.

FIG. 2

a. Rim continued round the fissure
b. Reef flat
c. Boulders and sand
d. Growing corals



The rim in fig 4 enlarged.

Scale. Horizontal: 8 yards to 1 inch. — Vertical: 4 yards to 1 inch

d. Reef flat
e1. Sudden rise to rim
e. 2. Gradual slope of rim
e. 3. Crest of rim
f. Outer slope

FIG. 5.

the "pocket." The greatest depth of the lagoon is 30 fathoms. The bottom of the lagoon, if the shoals were removed, and the whole elevated, would be a great plain surrounded by a ridge sloping steeply up to a line of perpendicular cliffs broken only at the few deep ship's channels; on this plain the greatest heights would be from 20—30 feet. The shoals in the lagoon do not vary greatly in depth; indeed most can be easily discerned by the break of the sea on them at low tide, if the wind is at all fresh. In dredging I searched especially for deep shoals, to find out if they had any growth of organisms which might be expected, in course of time, to bring them up to the surface; I passed over several, both with the dredge and tangles, but I obtained nothing from them, nor were my dredges ever caught up, as they always were every few yards on the shallow shoals. This shows that their surface is smooth and regular. They seemed rather to be gently sloping eminences on the bottom of the lagoon than shoals growing up to the surface. One near the large ship's channel to the south was perhaps an exception, as the dredge had to be tripped several times, when I went over it, and subsequently I lost a large tangle on it in 11—12 fathoms of water. Most of the shoals have, however, only a few inches of water and are fairly flat topped, without any central depression; they extend less to leeward than to windward, or in other words such summits, as they have, are to leeward. They slope gently at the sides to 3—4 fathoms, succeeded by a cliff to a depth of 10—12 fathoms, the normal depth of the lagoon being reached within 100 yards of their edges, close up to which I have often obtained soundings of 14 and 16 fathoms. In the shallower water to windward, these shoals can often be seen to overhang their bases, and to be mere mushroom heads. Corals grow most luxuriantly at the sides where they slope down before the cliff is reached, but many also flourish on their summits. *Pocillopora* is very rare, but two varieties of *Millepora* flourish, one near the openings in the reef, and the other, a massive brown lamellar variety, on nearly all the shoals of the lagoon. There is never on any shoal a great variety in species, though on some the total quantity of coral is very considerable, and often exceeds in amount the nullipores growing on the same shoal. A considerable part of the surface is to be found on all the shoals covered by species of *Sarcophytum* and *Halimeda*. Of corals, the genera *Favia* and *Madrepora* are the most abundant in species, while *Porites arenosa* (Esper) with several varieties is the most common coral, and in many places forms large masses several feet in diameter and of great thickness. The genera *Montipora, Coeloria,* and *Astraeopora* are very local in distribution, often assisting very materially.

Between the patches and knolls of rock outside the lagoon

platform, the bottom is generally covered by a sand which is similar in all respects to the sand described by Professor Sollas. It does not extend far into the lagoon off these reefs, and the arming of the lead usually comes up clean, or only contains a few of the segments of different species of *Halimeda*. To leeward of all the windward islands, sand is found most abundantly, but only by the main island does it extend more than a couple of hundred yards into the lagoon where, however, it runs out about $1\frac{1}{2}$ miles in a bank with about 17 fathoms of water, the breadth of which a mile out from the shore is only about 250 yards. It seems to me to be deposited by an eddy meeting here, caused by the rising and rushing of the tide over the reefs to the north and south of this island. These currents sweep up and down along the coast, the lagoon reefs surrounding which are notably free from mud and sand. The sandy part of the island by the village, I think, has been formed by the deposition and washing up of sand by this means, and, if any reliance can be placed on the reports of the natives, the lagoon is here evidently silting up.

In dredging, I took bearings across the lagoon from the village in three directions, besides trying off certain islands and in particular localities; I also took a series of soundings at the same time to find out the nature of the bottom over which I was passing. The dredges were made by the Zoological Station at Naples, the one with a net and the other a canvas bag. The result showed that the bottom generally must be a flat surface of hard rock, as repeatedly nothing but a few segments of *Halimeda* were brought up, and the dredges, while they could be felt on the rope working along the bottom, met with no hindrances. Near the shoals masses of dead coral were occasionally obtained, but living corals never. I cannot agree with Professor Sollas that there is any evidence that the lagoon of the atoll is filling up, except in the one position opposite the village. Although I dredged over a great part of the surface, I found no traces of coral growth on the bottom and no shoals, with a few fathoms of water which might be supposed to be coming up to the surface, and I can confidently assert that there are no considerable banks of sand in the lagoon other than the one already referred to. Undoubtedly there should be in some parts a considerable deposition of the skeletons of such an alga as *Halimeda*, but of such I found no traces. Either it must be consolidated with the bottom, or it must be washed out of the lagoon, or it must be dissolved. I have seen no evidence of such a consolidation as this taking place anywhere in the islands of the Central Pacific, and I believe it does not occur; if it is possible for it to take place at all, the other forces must act before it has had time to consolidate.

There is no doubt that many of the islands on the rim of the

atoll are being washed away. I have already noticed the undermining of the pinnacles on the outer reef, and the action of the sea on this reef generally. Some of the breaks, between the different islands, seem to have been but recently formed, and the cliff on Amatuku, before referred to, shows that the action is still going on. Tengako is a part of the main island to the north joined by a low, bare sand-bank about twenty yards broad, across which in a big gale the waves would break. The natives state that this area was once covered by coconut trees. Additional evidence is furnished by Afualiku a small island west of the north entrance; it has now only one living coconut tree, but shows the remains of several others which have fallen over through the approach of the waves.

Reference has been made before to the character of the reef just outside the rim. Masses of reef-rock with fissures and cross channels extend out for several fathoms; in places, down to depths of 10—12 fathoms, the bottom is similarly formed. These masses are covered with encrusting nullipores, which seem gradually to be joining them on to the rim and extending the reef further seawards. I could find no signs of their having been torn up, or washed away, except in a few negroheads on the leeward reefs of the atoll, and it necessarily follows, as they are completely protected from solution by growing organisms, that they must be stationary, or growing upwards and outwards. We have thus the islands of the rim of this atoll being washed away, while the reef round them is gradually extending itself further and further seawards.

Professor Sollas has given, from the soundings taken by the officers of the *Penguin*, a section[1] of the external slope of the atoll, but the fall between 15 and 40 fathoms is by no means always so rapid. Off Falefatu, between the two southern entrances, the fall from 15 fathoms was fairly gradual down to 40 fathoms, and I was able to take a haul with the swabs without going into a less depth than 35 fathoms. There was also off Fuafatu a broad flat with a depth of about 8 fathoms, on which the *Penguin* anchored on several successive nights. A considerable number of species of coral were obtained on the outer slopes in depths down to 35 fathoms, but it was impossible to use dredges on this area without a greater force than the steam launches of the *Penguin* possessed, and so no knowledge of encrusting or massive species could be obtained. From 35 fathoms I obtained one small piece of *Heliopora*, a discovery of great interest as, while its corallum is exceedingly abundant in the mangrove swamp, it was only obtained alive at Funafuti on this occasion;

[1] *loc. cit.*, p. 375.

its general habitat seemed to me in Fiji and Rotuma rather to be on the tops of the reefs, and in the boat channels of fringing reefs, than in the deeper water outside the reef, or in the lagoon.

From the steep outer slopes of the reef, between 40 and 100 fathoms, the swabs brought up only a few solitary corals with a considerable number of fragments of several species of *Gorgonacea*. Nearly all these hauls, from the outer slope, produced a number of loose pieces of stone, most of which had encrusting nullipores growing on them. In one haul, made up the reef by the *Penguin* to 75 fathoms, there were upwards of 30 such pieces none of which showed any signs of fracture, and nearly all were to some extent encrusted by these algae. From these dredgings the steep slope would seem to be formed by the talus of the reef, its angle 75°—80° being apparently the angle at which shingle will lie at such a depth under the water. The presence of nullipores growing at depths greater than 75 fathoms is of extreme importance; the observation, however, requires further confirmation.

In all, the swabs were hauled eight times at a greater depth than 40 fathoms; the result of six of these hauls I examined and in no less than four of them encrusting nullipores were present in the living state on stones brought up; the fifth haul was to a depth of 105 fathoms, and the sixth only brought up a few pieces of *Gorgonacea*.

How such an atoll as Funafuti has been formed can only be a matter of conjecture in the present state of our knowledge; the bathymetrical limit of any single species of coral is unknown, and our knowledge is limited with regard to the multifarious conditions affecting coral and nullipore life. I have succeeded, I believe, in showing that there has been, in the case of Funafuti, an elevation. Professor Sollas, mainly from the evidence of the *Heliopora* reef which he very carefully examined in the mangrove swamp, suggests an elevation of at least four feet. I have pointed out that smooth flats almost bare of vegetation exist in different islands of the atoll; now, unless these were originally part of a reef-flat, I do not see how they could have been formed. In the raised reefs of Viti Levu, Fiji, I could find no such consolidation of fragments to flat, hard platforms, such as these, having taken place above water, and reefs covered over by a fathom or more of water at low tide do not exhibit any signs of such a platform. As some of these flats are 5 feet above high tide and the existing reef-flat is 5 feet below, these would require an elevation of the whole atoll of at least 10 feet. The edge of the hurricane beach is simply the edge of these flats, perhaps close to the rim of the former reef, where it abutted against the sea; the term, hurricane beach, is then clearly a misnomer, as none, such as is found on

other atolls, can properly exist. Considerable denudation and wearing away by tidal action has certainly gone on, and I think that its amount is shown by the extent of the rough zone, the pinnacles and buttresses left being the remains of harder, viz. coral-formed parts of the reef. Sand enters considerably into the formation of the islands to leeward, and atoll reefs generally, such as I suppose the atoll of Funafuti to have been before it was elevated, are not so high to leeward as to windward, so that I see no difficulty in the width of this zone there. From the position of the pinnacles it is necessary to suppose that the animals, or plants with the hardest and most durable skeletons, i.e. the corals, were then living on the extreme edge of the reef in the full force of the breakers, as they are declared by Darwin to have been doing on the reefs of Keeling Atoll, when he visited them. The formation of the present reef would of course be subsequent to this elevation.

The evidence from the facts, bearing on the theory of formation of coral reefs, collected at Funafuti admit of four conclusions:— (1) The atoll has been elevated about 10 feet. (2) The land is now being washed away, the extent of the erosion being shown to some degree by the breadth of the rough zone. (3) The reef seems to have been mainly formed by the growth of nullipores, which are now building up masses outside the rim and adding them on to the reef, causing its extension seawards. (4) The lagoon does not seem to be filling up to any appreciable extent by the growth of shoals, etc., and solution is indicated by the characters of its slopes and floor.

PART II.

Rotuma.

The island of Rotuma, situated in lat. 12° 30′ S. and long. 177° 15′ E. is about 260 miles distant from the Great Barrier Reef of Fiji, the islands of which group form its nearest high land. It rests on a plateau 1500 fathoms in depth with the Fijian and Tongan islands, extending to the west to the Santa Cruz Group. The island is volcanic with hills ranging up to 900 feet in height, and there are no less than seven craters, the appearance of some of which leads me to believe that they have not been long inactive[1]. From these hills broad lava streams can be traced to the sea and along the beach, except where they are covered by a deposit of

[1] *Vide* "The Geology of Rotuma," *Quar. Jour. Geo. Soc.* LIV. pp. 1—11.

beach sand; the angle of their slope for 500 yards inland is rarely more than 3° or 4°. The coast is fairly even with a complete absence of the "long points and deep fiord-like bays," which, according to Dana, would on a volcanic island give indubitable evidence of subsidence. Two miles from the west end the island narrows down to an isthmus about 100 yards broad, completely covered with sand. The rock to the east, a recent lava, slopes gradually down to the beach, but to the west the isthmus is bounded by the perpendicular cliffs of two basaltic hills, Kugoi and Kiliga; these are undermined considerably at their bases owing to wave action at the present tide level, proving that the isthmus at one time did not exist. There is also a deep pool, 300 yards across, with 4—6 fathoms of water and precipitous walls, within the reef to the north, directly in the line joining the deepest passages through the reefs to the north and south of the island in this situation. I have no doubt that there was once a deep channel across here, which has closed up, as there could have been no current to keep it open.

The only other rock found besides the recent lava and basalt is a volcanic ash, which forms the islands on the reef and three hills at the west end, of which Sororoa alone abuts on the sea. Where the basalt of Kiliga and the ash of Sororoa and the reef islands face the sea, they end nearly perpendicularly, the cliffs of Sororoa being over 700 feet high. On the island I failed to find any marine deposit, or any evidence of any change of level, so that, to explain the manifold conditions of its reef, recourse to such would, I believe, be irrelevant. I consider too that the extensive beach sand-flats could only have been formed in a stationary period. Every variety of fringing reef is found, and I shall endeavour to indicate by a brief sketch of the characteristics of the reef in different situations the method of formation of a fringing reef and the first stages in the change from such to a barrier condition.

About three miles to the west of Rotuma lie three islands Hoflewa, Hatana and Uea in a line from south-west to north-east; of these Hoflewa and Hatana are of recent lava while Uea, which is 900 feet high and surrounded by cliffs, is formed of ash-rock. Between Uea and Hatana is a shoal Hofhaveanlolo, awash at low tide; its top is flat and completely covered with incrusting nullipores with no growing corals. The short distance in which the breakers rise, together with the deep-coloured water around, show that its walls are almost precipitous, and indeed with a 50-fathom line I could obtain no soundings about 50 yards from its edge.

Hoflewa is a precipitous horse-shoe shaped rock open to the west-south-west with a cleft, extending from the summit to the

sea, about 4 yards broad at the base. There is no shallow reef of any sort and, although the rock at the sea level is covered with nullipores, these have nowhere attained a thickness of more than a few inches. The concavity, which is very shallow, is covered with great masses of rock with deep fissures between, leaving a channel to the cleft through which the water rushes with great force. These masses can still be seen for a short distance outside the mouth of the horse-shoe, but while the surface of the inner masses is studded with corals, here their summits are completely covered with nullipores, only a few colonies of *Pocillopora*, *Madrepora* and *Millepora* growing on their sides. Around the island to 15 fathoms these masses and ridges continue, but after 25 fathoms in a few yards no bottom was obtained with 50 fathoms; the slope, however, is not quite so rapid along the line of the three islands.

Uea has no reef, but, where the cliffs pass into the water, nullipores grow freely and have in many places formed a platform at the low-tide level, extending out for a few yards, dropping off almost perpendicularly into 20 fathoms. In one position where a stream has cut its way down to the sea, a broader platform has been formed by these algae, which do not seem in any way affected by the fresh water.

Kiliga Point and the islands of Solkopi, Sol Onau and Afaga present in places the same features as Hoflewa and Uea, but off Afaga the reef broadens. At its north end there is no platform of any sort; then to the south at low-tide level a flat, smooth platform succeeds, covered with nullipores. Having attained a breadth of about 10 yards, it commences on its inner side to be slightly hollowed out, forming a channel. The platform continues to the south, retaining a constant breadth of about 9 yards but with its outer edge raised about 8 inches above the low-tide level, while the channel increases with the breadth of the reef, giving rise to a pool with about 3 feet of water at low tide between the island and the platform, in which fissures have now appeared, running in for a few yards.

Off the cliffs of Sororoa the reef is only about 25 yards broad, and consists of a channel about 4 feet deep and a platform 8 yards broad; but, while the channel off Afaga is cut off from the sea, it is here directly connected by a deep, broad fissure, to which a strong current sets at ebb tide. Perhaps owing to this, it is devoid of all coral growth and more or less covered with nullipores, while the pool off Afaga has the same species of coral as were found on the *reef-flat* elsewhere. The cliffs behind are in no way undermined, being indurated with carbonate of lime and extremely hard.

Round the most westerly point of the island the reef is similar

in all essential respects to that off Sororoa, but its breadth is about 50 yards and its channel has a depth of 6—9 feet. The platform attains a breadth of about 12 yards, and on it an outer rather higher part can be distinctly recognised. No fissures reach through the platform and, as each breaker drives a large amount of water over it, there results a strong current to passages to the east and south. A few *Prionastrea* and *Madrepora* grow on the platform among incrusting nullipores, but there are none in the channel and few nullipores, most of the surface being covered with low green algae.

As the edge of the shallow reef lies further from the shore, the platform gets broader, as is well seen off Lopta. A distinction into two parts is now marked, an outer 5—6 yards broad about 9 inches above and an inner at the low tide level. The division is still more obvious in the reef between Solkopi and Pepji, where the outer is about 8 yards broad and the inner 30 yards. These parts, as they seem homologous to the rim and reef-flat at Funafuti, are accordingly so named here. As the reef-flat broadens, fragments, torn from the rim or the reef outside, are thrown on its inner side so that it is itself divided into two parts, the outer of which is equivalent to the reef-flat proper of an atoll, and the inner to the piled-up masses of reef-rock, which in an atoll might be supposed to form the land; the latter is termed the *boulder zone*. A section (Fig. 4) then of the reef to the north of the island or off Noatau, where it is about 500 yards broad, presents four zones which with their characteristics are as follows:—

(i) A BOAT CHANNEL (Fig. 4 *bb*) extending out for about 250 yards from the shore with a greatest depth of 8 feet at low tide. Its bottom is covered by a thin layer of sand and mud over a hard well-consolidated rock, formed of nullipores, corals and sand. The sand is found principally near the beach and is the same as the beach sand, while the mud, which lies towards the outside, is much finer, being formed mainly by boring worms and molluscs. *Halimeda* is very abundant, but massive nullipores are not found; corals do not grow near the beach, but the remainder of the channel is studded with large clumps up to 20 feet in diameter. Of these *Heliopora* and a variety of *Millepora* are very common in clusters 5—6 feet across, consisting of parallel lamellae, arising from broad and firm bases. *Symphyllia, Coeloria, Favia, Astraeopora, Porites* and *Montipora* form large masses, which after they have reached the surface die in the centre, but extend at the periphery so as to overhang for several feet. They are then hollowed out by various boring animals, but their edges extend evenly outwards a few inches below the low tide level, so that even at spring tides the living polyps are not exposed.

(ii) Outside the boat channel is the BOULDER ZONE (Fig. 4 *cc*) formed of fragments, torn off the reef outside and piled up for 1—2 feet above low tide level with isolated boulders 2 feet higher. It is not generally well-defined except on the windward face, its greatest breadth here being about 70 yards. It is in reality a layer of boulders, lying on the edge of the reef-flat, which here commences and up to which the boat channel gradually for some distance shoals; it is separated merely for convenience as it should properly be described with the next zone.

Undoubtedly these boulders have been broken off and piled up by the sea, but after a severe easterly gale the examination of a part, well hunted over on several occasions, showed but few additions[1].

(iii) The REEF-FLAT (Fig. 4 *dd*) consists of a smooth platform of reef-rock with a depth varying from 1—3 feet below low tide level. As it is cut off by the boulder zone from the boat channel and by the rim from the sea, it always retains at low tide approximately the same depth of water. Its bottom varies enormously, in some places being smooth and almost bare of corals, but in the position of the section, where it is about 60 yards broad, growing corals cover about 25 per cent. of its surface. *Madrepora* are extremely abundant, adapting themselves to the depth of the water by spreading out horizontally; of this genus I obtained 9 species from this area, of which *Madrepora rotumana* is by far the most abundant. Other genera represented include *Pocillopora*, *Stylophora*, *Pavonia*, *Coeloria*, *Montipora*, *Prionastrea*, *Plesiastrea*, *Heliastrea* and *Favia*. Incrusting nullipores are absent except for a few yards on the outer side.

(iv) The last zone slopes up to the RIM (Fig. 4, *e* and Fig. 5) but is separated from it by an abrupt rise of 6 inches, for a few yards above and below which the reef is covered by white nullipores. A platform, gradually sloping upwards, succeeds, here about 6 yards broad with upwards of 5 per cent. of its surface covered with *Prionastrea*, *Pocillopora* and *Stylophora* of low growing species, between which the surface is almost hidden by white, pink and red nullipores. Fissures commence at the outer side and run through the crest of the rim, which is about 4 yards broad and presents the same appearance as the extreme edge of the reef at Funafuti. The fissures, however, are narrower and in no way drain the reef-flat; their sides, too, do not overhang.

Outside the rim the reef has a slope of 10°—15° for about 15 yards, but is much broken up into buttresses, masses and ridges, which extend as deep as the bottom can be seen. Sound-

[1] I was informed by the natives that about 1881 most of the boulders were swept away in an hurricane, and that the zone has only been gradually reforming since.

ings in 20—40 fathoms showed that they further occurred to this depth, as after resting the lead constantly fell off into a depth greater by 2—3 fathoms. These masses to a depth of about 16 fathoms have a most striking appearance to windward from the brilliantly coloured nullipores, which cover them, corals being rarely seen. North of the island, where the breadth of the visible zone is greater than elsewhere, corals are more abundant, growing principally on the sides of the buttresses, between which the bottom is covered with sand. Off the western islands and to windward there is a sudden drop from about 35 fathoms, in a few yards 50 fathoms giving no bottom. The angle, however, again decreases, as off Afaga there is at least 600 yards between this sounding and the 100 fathom line, which runs very evenly round the whole island, including also the western islands. It in no way curves in towards the cliffs, and hence I consider affords strong presumptive evidence against any subsidence.

The passages in number and size are proportional to the extent of the reef surface, and are the natural outfalls for the tide, which probably keeps them open by its scour. At Noatau and Lopta are perennial streams of fresh water, but there are no passages especially attributable to them. The depth of the boat channel varies, but does not exceed 2 fathoms. Between Husia, a small island between Noatau and Afaga, and the main island are depths of 10—11 feet, but the tidal currents set strongly there and are gradually washing Husia away; off Malaha and by the isthmus depths of 7—9 feet are not uncommon. To the north of the isthmus is the deep pool of water, previously referred to, which I consider to have been formed by the reef growing up around.

At Noatau, Oinafa, Pepji and many places along the coast the volcanic rock is shut off from the sea by the formation of a sand flat, of which the isthmus also is formed. At Noatau it is about 500 yards broad and 1 foot above high tide level, rising for 2—3 feet higher near the beach. I dug into it in places to the depth of 6—12 feet, but only found loose sand with fragments of corals and nullipores. On the beach between tide marks it ends with a beach sandstone formation in strata, 4—7 inches thick, dipping down at an angle of 5°—10° to the sea. On the beach the ends of 6—7 layers may be seen. As these are removed the sand is found to be less firmly consolidated, until in 3—4 feet it is quite loose.

The top of a layer, which is the part most exposed to the air and waves, is the hardest. If it is removed, as is constantly done by the natives for gravestones, it seems to be rebuilt, growing up as it were from the broken edge; fresh layers on the outside seem to be formed too in a similar manner. These sand flats are very

common in Fiji, and many show clearly the sandstone formation on their outer sides; I have no doubt that many have been formed by layer after layer of this rock being added to their outer faces. It then a few feet back breaks down into sand, the rain water perhaps, having plenty of carbonic acid gas in solution from decaying organic matter, readily dissolving the finely divided carbonate of lime by the deposition of which the sand was in the first place consolidated. In the limestone islands in the Lau group of Fiji the sandstone is much less broken down, some recently dug wells in the flat showing the strata clearly; here, however, the mud, caused by the wash of the sea on the limestone cliffs, assists in binding the sand more closely, giving rise to a much finer grained rock.

Mr Jukes, in his narrative of H. M. S. *Fly's* survey of the Great Barrier Reef of Australia, has especially referred to this beach sandstone formation. He in particular noted one island in the Capricorn Group, where it was being washed away, probably owing to a change of current from some reef growing up. After dismissing the idea of elevation as a "difficult and gratuitous hypothesis," he suggests that "the present structure must have been produced in the interior of a mass of loose sand by the infiltration of sea or rain water, or some other cause of which we are ignorant." "After the interior of such a mass of sand had been consolidated, the loose exterior may have been washed away and the solid rock exposed." In fact Jukes considered that this rock is formed in the interior of a mass of sand, the formation gradually proceeding outwards, while it seems to me patent that it is rather formed on the outside and broken down within.

At Oinafa the sand flat has a height over most of its surface of a few inches above high tide level, but 100 yards from the coast opposite the anchorage there is a brackish swamp 1—3 feet below this level about 800 yards long by 300 yards in greatest breadth with in places pools 6 feet or more deep. On it more or less buried in the sand are small blocks of coral strewn about, and a crowbar cannot be forced down far without meeting the coral-rock, so that the flat here appears to have been formed on the inner edge of the reef.

From a consideration of the foregoing facts and the evidence, collected at Rotuma, it appears to me probable that the reef has been formed entirely at the present level. A gradual outgrowth from the shore, due principally to nullipore growth, took place. When it had reached a certain breadth, masses began to be added to the rim, as at Funafuti, causing it to extend more rapidly. A boat channel was then formed, partially by solution and partially by the scour of the tide. As the whole broadened, this deepened and sandstone flats were formed on its inner parts.

PART III.

The Reefs of Fiji.

The Fijian Group has within its limits, as many observers have pointed out, every variety of reef from the narrow fringing to the well-formed atoll. Although there are many atolls in the Lau, or Eastern Group, there is not one with any considerable portion of its rim converted into land. To the east of the Ngele Levu lagoon is a small island, divided by two channels into three parts, but these are not more than 2 miles in length, while the reef is over 40 miles in circumference and except for passages to the south-west everywhere awash. The Nukusemanu and Nanuku reefs in the Ringold islands are the north and south ends of an atoll-reef over 80 miles in circumference, the greater part submerged to a depth of 6—8 fathoms. Wailangilala is not properly one of the islets of an atoll, being too high for such; it is, however, formed of coral rock, and is probably the remains of an upheaved coral island, the rest of which has been washed away. Other atoll-reefs have only a few rocks but such are scattered on all reefs, freely exposed to the seas, and are what I have before called negroheads. These are especially abundant on the reefs to the south-east of Viti Levu, and are simply masses of reef-rock which have been torn from the outside and deposited in their present positions, often several yards from the edge of the reef in hurricanes, or other bad storms; some measure 6—8 feet in height and must weigh many tons[1]. Barrier reefs too in Fiji have no islets, a possible exception being Ono-i-lau, which has a number of small islets to the east; two of these appear on the chart to have low hills and probably all are the remains of a larger island, which has been washed away.

Another characteristic is the extreme regularity of the 100 fathom line round all reefs, freely exposed to the ocean. Off prominent points it may be a little further out, but there are no stretches of sea of any extent off the reefs with a less depth than this; different atoll and barrier reefs, however near they may lie to one another, are not connected by a less depth. The section outside all is nearly the same, a gentle slope to about 40 fathoms and then a sudden steep. There is further no essential difference off those parts of Taviuni, which have no reef but the sea directly washing their shores; the bottom here slopes to 40 fathoms in less

[1] Capt. Wilson estimated that one, thrown up on the reef at Mbenga in the hurricane of 1879, weighed 40 tons.

than 200 yards, and in the next cast a few yards further out I could obtain no soundings with 60 fathoms. Captain Wilson pointed out to me that a ship can often get better anchorage to windward than to leeward outside reefs, in other words that the slope down to 40 fathoms is often more gentle to windward than to leeward. This is distinctly so to the south-east of Taviuni, where there is good anchorage, 300 yards outside the reef; south of Cape Undu, to the north-east of Vanua Levu, I found that the steep lay about 350 yards from the reef, while to the north it began within 200.

The regular wind in the group for nine months sets from the south-east to the east-south-east; in the remaining months calms and north to north-west winds occur. The current varies, being largely affected by the wind, but from information, obtained from skippers trading in the group, there seems to be always a definite set to the west-north-west, which, however, with a wind from this direction may be disguised by a merely superficial current in the opposite direction. On an absolutely calm day the sea always breaks on the windward reefs, while there may be no break on the leeward, though the reef, as off Wakaya, is quite as high there. The same phenomenon I observed too off the north-east point of Vanua Levu, the reefs of the two sides of which I visited at succeeding low tides without being able to appreciate any difference in height; there was a distinct break for every wave at high tide on the south-east reefs, but only one out of every five waves broke on the northerly ones. Early in 1896 a timber ship was wrecked on a reef off Vatoa to the south of the Lau Group; some of the timber was recovered on the islands of this group, but the greater part was washed to the south and east of Viti Levu, showing a distinct current to the north-west.

Taviuni displays every type of reef from a mere fringe a few yards wide to a well-formed barrier. The strength of the tide through the Somosomo Strait varies up to 5 knots, the ebb flowing to the north-east up by Rambi, while the flood sets along the north end of Taviuni and to the south-west. Perhaps this strong current together with a rather precipitous slope in places is sufficient to account for the fact, that off a great part of this side there are no reefs of any sort. From Wairiki to Wailanga there is in places a hard platform extending out for about 50 yards above which the hills are usually separated from the sea by a sand flat 50—100 yards broad. This platform is formed of reef-rock destitute even at its extreme outer edge, where it ends precipitously, of corals or nullipores. I believe that along this part of the coast a broad fringing reef formerly existed, but after growing outwards for some distance was checked by the current; this gave time for the sand to accumulate, its wash finally killing

all the reef-forming organisms. Off this platform, after a sudden drop to 3 fathoms, the slope is gradual to 40 fathoms, at which the steep commences; the bottom is of sand, in most places without masses or ridges. North of Taviuni 40 fathoms is often reached in 100 yards, but then a flat commences with 40—50 fathoms and a breadth of 2—3 miles; its edge generally falls abruptly, but there are traces of a barrier in some reefs almost awash with shallower soundings between. In the middle of the strait is a small volcanic island, Koro Levu, which to the west and south has a well-formed barrier but to the north-east a narrow fringing reef; sheltered by it are a number of small isolated patches. These reefs have all narrow, flat tops, covered with incrusting nullipores and almost precipitous walls to 8 or 10 fathoms.

Off Vuna Point to the south-west of Taviuni a barrier reef runs out for 2 miles, following the line of the coast, so that it is at once suggested that it is formed on a bank piled up here by the current through the strait; it is well protected too from the east by a cape to the south, which is itself destitute of any reef. The lagoon of this barrier has a depth of 10 fathoms and a sandy bottom, the reef falling to it precipitously while the slope up to the land is quite gradual. Off the south-east of Taviuni there is usually some sort of a fringing reef, but only off Salia Levu is it more than about 20 yards across, attaining here a breadth of 100—150 yards and having a well-defined rim, reef-flat and a boat channel with a depth of 5—9 feet at low tide. The rim is completely covered with nullipores, and corals are rare both on the reef-flat and in the boat channel, although the shore is strewn with their branches. Outside the rim, which is similar in every respect to that of Funafuti, masses almost as high extend for some yards, the sea breaking on them; further out the bottom is covered by masses and ridges with a fairly abundant coral growth.

The south-west of Viti Levu is fringed by a narrow reef, unbroken practically save where the Singatoke flows into the sea. To the east it is succeeded by a reef of the barrier class, which extends up round the east coast, until it joins the Ovalau reef; it nowhere lies more than 2—3 miles from the shore, and there are large passages in it opposite the mouths of the Navua and Rewa rivers and a small passage at Suva. The lagoon between Navua and Suva is broken by a stretch of fringing reef, but on to the Rewa river it is deep though often very narrow. The outer part of the reef does not along this area differ materially from that of Rotuma, but the channels inside the reef are practically devoid of corals. The reef to the lagoon slopes steeply but not precipitously, and by the land there is commonly a broad platform. The lagoon

varies in depth up to 20 fathoms, having a bottom of dark grey muddy sand with which the shore platform is also covered; in many places too the reef has its breadth considerably increased by the same deposit on its landward face, and shoals are covered by it.

The delta of the Rewa forms the whole south-east corner of Viti Levu; here the reef fringes and is as much as 3 miles broad. This great breadth is apparently due to the sand with the earth carried down by the river having filled up the lagoon. By the island of Nukulau is the largest passage; it has a bottom free of mud and the bottom of the river too is for some distance clean and hard, appearing to be formed of reef-rock. It seems to me that the reef here formerly extended up under the delta of the Rewa, and by the deposition of mud was gradually killed, growing up afresh outside. The bottom is distinctly visible in the Nukulau passage; it is by no means bare of corals and nullipores, and has further a small reef awash in the middle. Yet the fresh water and detritus of the river must have a considerable effect or there should be patches and shoals of coral-rock in the lagoon.

Muddy sand is found on all the inner reefs and patches to the east of Viti Levu and the inner reefs of Ovalau are all coated by it; that it forms often merely a thin layer over the reef-rock can be plainly seen, but in many places it is undoubtedly filling up the lagoon by its deposition. It varies largely in its composition with its proximity to rivers and streams, but shells of Foraminifera, fragments of Mollusca and Echinodermata, segments of such algae as *Halimeda* and sponge spicules can usually be found. It occurs further on all the inner reefs and some of the outer reefs to the north of Viti Levu and Vanua Levu. Under water it sets firm and hard, but does not apparently consolidate; sponges and green algae grow on it luxuriantly, but corals and incrusting nullipores are never found.

Round the north of Viti Levu from Ovalau for some distance there is a much broken barrier reef with one or two sandbanks, like supplementary barriers between it and the shore, which always has some sort of a platform. These inner barriers are covered by the same muddy sand, which is of a lighter colour owing to a less admixture of river washings. Thakau Moi, a shoal to the north of Viti Levu, is a part of this barrier reef; its landward side has a steep face, succeeded by a broad flat with a central depression, which is covered by the same sand. The true reef is a narrow strip on the outside, and it is only on the outer face of this that corals, or reef-building nullipores, grow in any luxuriance. I shall have occasion to refer again to this sand, when noticing the raised reefs of Viti Levu; it enters largely into

the composition of some, and many have been, I believe, formed by an analogous deposit.

The barrier reefs of Vanua Levu show all the characteristics of the similar reefs elsewhere in Fiji, an abrupt rise from a sandy lagoon, a reef-flat of variable breadth, raised rim with fissures not generally very deep, a slope to 40 fathoms in 300 yards and then the usual steep. Fringing reefs only occur to the south-east and to the north of the entrance of Nateva Bay; they are fairly broad with a distinct boat channel, devoid of corals. Nateva Bay has on the west a broad shore reef covered with sand; outside is then a clear stretch of water about 1300 yards broad bounded by a line of reefs, on which corals grow in great luxuriance. The rest of the bay is full of shoals and patches, on which also corals abound, many seeming to have been formed by them alone. The Great Barrier Reef of Fiji, extending through $2\frac{1}{2}^{\circ}$ of longitude, lies to the north of Vanua Levu; to the west in the charts it seems to be continued on by the fringing reefs of the Yasawas, but there is really outside these a line of reefs and shoals, which connect it round the west of Viti Levu to the fringing reefs of that island. Off Waia the continuation is ill-defined; there is a broad flat 40 fathoms in depth studded with shoals but soundings to the outside show that a barrier is probably growing up. The small island of Viwa alone stands outside the 100-fathom line of this great reef; its rock is limestone, and it is probably a raised coral island.

The Great Barrier Reef is in many places double, seeming to consist of two reefs closely apposed, leaving only a narrow channel between with 2—3 fathoms of water. In crossing it to the north-west of Vanua Levu, we sailed through an outer reef about 200 yards broad into a channel of about the same breadth extending as far as we could see with a sandy bottom; we then passed through another reef into the lagoon. Both reefs near the passage were covered with corals and nullipores and had flats falling precipitously to the small channel between. The reef here is at a considerable distance from the shore, and, while it is exposed to the ocean to the north, it is beaten on its south side for the greater part of the year by the waves of the trade wind; thus it is quite conceivable that it should grow up with a face against both and a lagoon between, since the growth of reefs everywhere seems to be greatest, where exposed to the heaviest seas. The broad lagoon within the Great Barrier Reef has an almost uniform depth of 35—45 fathoms with a bottom of hard sand; it is studded with patches and small reefs, having precipitous walls to 20 fathoms and summits almost awash at low tide. Near the land are a number of larger reefs, some with depressions a few fathoms deep in their centres and so somewhat simulating atoll-reefs. Thakau

Levu[1] off the west of Vanua Levu has a lagoon, 4—5 fathoms deep, with a sandy bottom; the reef around is of sand except on the extreme outer face where corals abound.

Between Viti Levu and Vanua Levu is a pool about 300 fathoms deep with a sandy bottom. It has not yet been properly surveyed, but my information leads me to believe, that there is against it on all sides a much broken barrier reef. In it is a true atoll reef, the Charybdis, with its eastern side awash at low tide, but to the west much broken up by passages and deep channels. The current runs strongly through the Vatu-ira Channel and then spreads out in all directions while the usual direction of the wind is east-south-east, so that it is those parts of the reef, which lie to windward and against the current, which have grown up.

To the south-east of the Vatu-ira Channel lie a number of volcanic islands, which resemble one another, with the exception of Wakaya, in their hills sloping fairly evenly to the sea without cliffs. Yet their reefs vary enormously; they fringe or are barrier, and have passages on all sides quite irrespective of the wind and current. Mbatiki has a fringing reef, continuous except to the north, where there are a large number of small patches; in it are impounded pools of water 5 fathoms in depth. Round Matuka the reef also fringes, but it has a well-marked boat channel with 2 fathoms of water, and is broken by two large passages to the west and east, leading into pools of 25 fathoms. Moala has a barrier 2 miles from the shore except to the north-west, where the reef fringes; the lagoon has an even bottom at 20 fathoms. The passages lie to the west, south-east and east (2) while the reef to the north consists of a series of patches. Ngau has a fringing reef except to the west and south-west, where there is a barrier, enclosing a lagoon 30 fathoms deep and 3 miles across. The fringing reef varies much in breadth and has a number of deep fissures where the streams from the land flow into the sea, while the barrier has three passages to the north-west. South of Ngau lies the Mumbolitha, an atoll-shaped reef with a shallow lagoon and rim awash on all sides, very abundantly studded with negro-heads. Nairai is completely surrounded by a barrier reef enclosing a lagoon, 18 fathoms deep, with passages to the east, west and north-west. Totoya is a crater, the south side of which has apparently blown out. Its reef is a barrier having passages to the north and west, and a small channel to the south; the lagoon is 1 mile broad with a depth of 25 fathoms outside and 30 within the crater.

The reef of Wakaya is of the barrier class, distant 3—5 miles, except to the south-west, where it fringes but has impounded

[1] My information about this shoal is principally derived from native bêche-de-mer fishers.

several pools of water, 5 fathoms deep, with precipitous walls. The fringing reef is about 500 yards broad with no channel, but it is full of holes up to 2 fathoms deep with their sides covered with true corals, *Millepora* and *Tubipora*. I obtained from this part in a few days more than 50 species of true corals of 21 genera, and certainly both for variety and luxuriance of coral growth it was the most prolific area I saw in Fiji. Outside this is a reef-flat about 40 yards broad, covered with nullipores with a few species of *Pocillopora* and *Prionastrea*; the rim is similar to that of the Funafuti reef, but its fissures are not so deep. The barrier reef is about 200 yards wide with but few corals; its passages lie to the east and west. The depth of the lagoon varies up to 36 fathoms, but to the south depths of over 20 fathoms are rare and the lagoon is much filled up by shoals with precipitous walls. As if in correspondence with the reef, the island is bounded to the west-south-west by precipitous cliffs, while the slope to the east and north is similar to that of the volcanic islands to the south. The north-west corner of the island is a sand-flat with a beach sand rock, similar to that of Rotuma; on it are five lava hillocks, each about 50 feet high and nearly all showing signs of wave-action. To the east are also areas of beach sand, but off the whole of this side lies an inner fringing reef, varying in breadth up to 200 yards and in places exhibiting almost the typical structure of such a reef exposed to the ocean; corals, especially *Millepora*, are very abundant on its outer face, which falls precipitously to a sandy bottom with 5—6 fathoms of water, but incrusting nullipores are not commonly found. To the north the barrier reef joins on to that of Makongai, an island which slopes down evenly on all sides from one main central peak. Its reef is of the barrier class except to the east, where it is fringing, 900 yards broad with pools of deep water, its lagoon is about 20 fathoms deep with passages mainly to the north. Ten miles off the south end of Wakaya lies the Horse-shoe Shoal, an atoll-reef with rim awash on all sides except the north-west.

Most of the islands of the Lau Group are formed of limestone and so the living reefs of this group are treated together with their raised reefs. The island of Koro in the centre of the whole group has steep hills and slopes; its reef to the east is of the broad fringing type, but to the west is wanting. Kandavu has a well-marked barrier reef off its south-east, east and extreme north sides, but off the rest of the island the reef is either wanting, fringing, or represented by a few shoals. North-east of Kandavu the North Astrolabe Reef is the barrier of an isolated rock about 10 feet high in its centre; the lagoon is about $1\frac{1}{2}$ miles broad but only has a depth of 15 fathoms. There are two passages to the north, where the lagoon is fairly clear, while to the south it is full

of patches and shoals. The barrier reef of Mbenga, including four volcanic islands, is well marked to the south-east but much broken to the north; this perhaps is accounted for by the strong tidal current which sets through the passage between this island and Viti Levu.

The mention of the foregoing facts is designed to point out the great variability of the reefs in the Fiji Group and round even its separate islands. It will further be evident that they bear little or no relationship to the slope of the land, while the peculiarities of some may be explained by the consideration of the currents and winds prevalent in the group. The variability is so great indeed that probably no single method of formation would explain all. The further consideration, however, will be deferred to Part VI.

PART IV.

The Raised Reefs of Fiji.

SECTION I. Viti Levu.

The main branch of the Rewa river is fed by several streams from the Kauvadra, a range of volcanic hills to the north of Vitu Levu, which, rising from the coast of Viti Levu Bay, runs due west for about 20 miles, whence it continues to the south with spurs to the west, and divides the island into two halves. The eastern half which is drained by the Rewa, has no long ranges of hills, but is studded with isolated peaks, many of which are characterised by having precipitous slopes and flat summits. These are formed of limestone or of sandstone, in places obviously overlying the volcanic rock, which is generally either a consolidated volcanic mud, or a vesicular lava. The hills above Nailega on the Upper Rewa show clearly the characters of the limestone hills, which reach their maximum height above sea-level here; the sandstone resembles the limestone in its slopes, and it is hard to find the one rock without the other near at hand. The hill of Koro Mbasanga, as a type, has a flat summit, which on three sides is bounded by cliffs of 150—250 feet; on the fourth the slope is extremely steep for 100 feet, but then there is a broad flat terrace to which succeeds a slope at about 70° for 200 feet. Below these cliffs the slopes are gradual except towards the Rewa, to which the hill falls at an angle of about 30°; the height above the river is about 700 feet, but, as the Rewa falls at least 200 feet below this, the total height is about 900 feet.

In the quarry at Suva two kinds of rock are found, a limestone usually called *coral-rock*, and a sandstone to which the term *soapstone* is locally applied. The coral-rock derives its name from the presence of corals, many of genera still found living on the reefs in the vicinity; shells of Echinoidea and Mollusca, principally bivalves, are abundant, while sponges occur locally. The remains of organic life, mentioned above, does not, I should consider, exceed 10 per cent. of the total mass of the rock, the greater part being of a coarse grained, friable nature, apparently formed by a sand composed principally of rounded nodules and shells of Foraminifera. In places the remains of calcareous algae can be recognised; much of the rock, being of a very fine texture, round and between masses of coral, implies from its position that it has been formed by incrusting nullipores, as it has no appearance of any organic origin, and loose sand is sometimes found underneath. The rock then is a highly fossiliferous soft limestone. Between Nailega and Soutoka on the Rewa crops out a dense, white, finely crystalline limestone, closely resembling Carrara marble, only harder. It is absolutely destitute of any trace of organic origin to the unaided eye and its fractured surface has always the same pure white colour. I received specimens too of a similar limestone from some cliffs near the mouth of the Singatoke river.

The so-called soapstone is a fine grained sandy rock in which the shells of Foraminifera are very common, and those of Lamellibranchiata can occasionally be found; its colour, when dry, varies from light grey to pure white. According to Brady this rock contains about 50 per cent. of silica, 18 per cent. of alumina, 3—8 per cent. of ferric oxide, 16 per cent. of water and traces of potassium carbonate, etc.[1] When first exposed in the quarry, it has almost a blue lustre; then, as it dries, it gets a soapy feel, and finally can be seen to be a very fine grained, friable, sandy rock. Where it is covered by the water in the beds of streams, it remains hard and firm, but by the banks there is a tendency for its surface to flake off.

From Viti Levu Bay to Navatu on the Rewa the rock is all soapstone, but below this village there is an outcrop of volcanic rock which it overlies. By the village of Nailega, lower down, both soft limestone and soapstone rocks are found, the former resting on the latter. Then the banks of the river show soapstone overlying the crystalline limestone, of which there is a small outcrop on the right bank. A volcanic conglomerate is next found, to be succeeded on the right bank by several low lying hills, formed apparently entirely of crystalline limestone. From this point soft limestone and soapstone rocks alone occur down to the delta of the Rewa.

[1] Note on the so-called "Soapstone of Fiji." *Quart. Jour. Geo. Soc.* Feb. 1888.

In the quarry at Suva, which runs into the edge of a cliff, there is in the section exposed about 2 feet of earth on the top of a layer of soapstone, 6—10 feet thick, while it is nearer 30 feet in most parts of the cliff. The line of junction of the soapstone with the coral-rock below it is horizontal, and along this line there is a layer about an inch thick of unconsolidated sand; in the soapstone it is immediately above this layer that most bivalve shells are found. The coral-rock is about 65 feet thick on the face of the cliff. Its upper few feet are very granular, being formed of small nodules and sand, with few corals or shells. Twenty feet below at the bottom of the quarry it is much coarser on account of numerous fossil shells and corals. Vertical fissures a few inches across run in from the edge of the cliffs; their sides are coated with crystalline carbonate of lime. Under the coral-rock the soapstone is again found and extends into the sea, its thickness being at least 150 feet. On top of it is a layer of pebbles, varying from 1 to 2 feet in thickness, formed both of soapstone and of a volcanic conglomerate; in size they vary, many being several inches across. Fragments of coral, some apparently water-worn, are found in the same layer, and bivalve shells are more abundant than in any part of the coral-rock above.

The village of Kalamba about 6 miles from Suva is situated on the flat round top of an almost precipitous hill, rising suddenly to a height of 220 feet. Round one side a tributary of the Tamavua river flows, and to join this a small stream has cut itself a tunnel through the hill; this is about 250 yards long and 70—100 feet high with a fall of about 60 feet. The rock exposed in the section can be seen to be very distinctly stratified horizontally, and I distinguished the following layers:—

1. Earth about 2 feet.
2. Soapstone „ 60 „
3. Coral-rock „ 50 „
4. Soapstone „ 30 „
5. Sand „ 2 „
6. Coral-rock „ 12 „
7. Soapstone „ 75 „
8. Black vesicular lava in the bed of the larger stream.

The soapstone, in all its layers, has its typical appearance but the two layers of coral-rock are very different. The upper is coarsely grained, plentifully strewn with pebbles of soapstone, bivalve shells and sponges but with no corals; its upper 10 feet almost form a distinct shell layer. The fifth layer is a very fine pebbly rock with red and black volcanic nodules. Corals are plentiful in the sixth layer, which closely resembles the coral-rock of the

Suva quarry but has fewer shells; a distinct pebbly layer forms its lower foot, and separates it from the soapstone, which is of a much finer grain than either of its upper layers. The lava also forms the greater part of a hill on the opposite side of the stream, where it distinctly overlies the soapstone, and is as distinctly overlain by it.

The soapstone nowhere shows any great variations, whereas the coral-rock in places by the Rewa consists mainly of corals and sand; I nowhere found it as a hard firm rock like that of a growing reef, although it is much harder on its outer face, where exposed to the weather. In some parts it consists of masses of almost structureless, fine grained rock, which sometimes lie in layers. Large and deep caves and small cavities are very common in it[1]. The greatest thickness near Suva is about 80 feet, but by the Rewa I found beds of at least 200 feet. I have no doubt whatever that this coral-rock was formed in the same way as coral reefs are now being formed; its great variability I consider to be due to the different positions with regard to exposure to the ocean, in which its reefs were built up.

Of the soapstone Brady examined specimens from the shore road, from an elevation of 100 feet or more and from an intermediate point at Suva. He found that they varied considerably in their species of Foraminifera, specimens from the first and second localities having only 36 species in common out of 92 obtained in all; he also found the remains of a few Ostracoda, but did not apparently obtain any bivalve Mollusca. Of the Foraminifera, Brady points out that 87 species still live in the neighbouring Pacific, the other 5 being very rare. Comparing the species from this rock with lists from material, collected by the Challenger Expedition, Brady remarks that "it is found to include several forms, not recorded from depths of less than 129 fathoms, and certain others, of which the minimum depth is about 150 fathoms." He then concludes:—"Nevertheless judging from its general facies the Rhizopod fauna is one that I should expect to find in a deposit forming from 150 to 200 fathoms (rather more than less) in the neighbourhood of any of the volcanic islands of the Pacific."

Should Brady be right there would be an almost incomprehensible series of elevations and subsidences required to form such a hill as Kalamba, unless the coral-rock be also supposed to have formed at depths of 150 to 200 fathoms. I do not assert that this is impossible, but such evidence as there is points to its being exceedingly unlikely, and, if so, there should certainly be

[1] Capt. Langdale, late R.N., informed me that up the Wai Mala branch of the Rewa there is a large hill, formed entirely of this rock, which is pierced through by several ramifying tunnels.

the remains in it of the rest of the fauna of this depth. The inne reefs, described with the living reefs, are round the large island coated with a fine muddy sand, in which the shells of many specie of Foraminifera occur. I would suggest that the soapstone reall had a similar origin, and that it is a shallow water deposit; o these reefs few Mollusca, Echinodermata and other animals an plants are found, such as might be supposed to leave traces o their origin. The deposition of this muddy sand can be at n time very rapid, as it depends largely on the amount of eart carried down by the rivers. The surface is hard and firm, so tha trees and other vegetable matter from the land would no ordinarily have time to get imbedded before they decomposed o were destroyed.

The formation of such a soapstone at 200 fathoms is quit possible; in this case it should form flat banks 1—2 miles wid opposite the rivers, which carry down to it its silica and alumina It is quite inconceivable, when one considers the rugged nature o the soapstone country of Viti Levu, how it could have been forme from these flats. There would indeed require to have been whole series of elevations over small areas close to one anothe since it is quite impossible to ascribe the forms of most of th hills to rain and river action alone. These inner reefs on th other hand have steep slopes, vary enormously in size, and ar distributed over all the lagoons within the barriers of Viti Lev and Vanua Levu; the distribution of living corals varies, and i often completely changed over large areas owing to local changes o current. I think the section of the raised reef at Suva shows ho it has itself been formed. There was a sandy reef at first near th mouth of some river or stream, when owing to some small chang in current pebbles began to be deposited. These pebbles of cours gave a firm foundation for the coral larvae, and a reef began t grow up; but, before this reef reached the surface, a deposit o sand on it, possibly from another change of current, killed th corals, and then a further deposit of muddy sand took place on it surface, perhaps raising it almost to the sea-level, before it wa upheaved to its present height.

The crystalline limestone does not form any ranges of hill sufficiently large for any theory of a coral reef origin to be pu forward. It is overlain by the soapstone, and was more probabl formed from a chalk, which by the infiltration of water has meta morphosed.

SECTION II.

The Lau Group.

The Lau, or Eastern Group of Fiji, consists of a line of islands and atoll-reefs, extending from lat. 16° 30″ S. to lat. 21° S. with a breadth of about 1° of longitude; the line is continued 1° northwards by the Ringold islands, and properly includes Thikombia-i-ra as well. Of reefs, every description is to be found from the narrow fringing to the perfect atoll-shaped.

The islands may be divided into three classes in accordance with the rock of which they are formed:—

1. Limestone.
2. Limestone and volcanic.
3. Volcanic.

Even at a considerable distance, it is possible with almost absolute certainty to say whether the rock is volcanic or limestone. The vegetation of the former consists of coarse grass, reeds and small bushes; the hills are usually conical, the gradients easy, and at the base there is invariably a more or less extensive flat. The limestone on the other hand is densely covered with heavy timber, grass and reed being absent owing to a lack of surface soil; the hills are flat-topped with precipitous slopes, which end seawards in overhanging cliffs, appearing extremely white, where recent falls of rock having taken place, against a light grey ground colour.

1. Limestone Islands.

To the south Fulanga (Fig. 6) presents an almost complete ring of land, surrounding a central lagoon; its diameter is about 4 miles in a north-west to south-east direction, and at right angles to this 4. The ring is complete except to the north-east, where for 3 miles it is broken up into separate islands between which a coral reef extends except for a ship's channel, about 3½ cables long, 60 yards broad and 6 fathoms deep with a tidal current of about 5 knots. The land varies from a few hundred yards to ¼ mile in width to the south-west with an average breadth of about 800 yards. The enclosed lagoon, 3½ by 2 miles, has a depth of 5—6 fathoms, but to the north-west 10 fathoms are found. It is studded with rocks and islets varying in height up to 100 feet, a distinct row extending right across to the south of the ship's entrance. The above ring of land is very uniform in elevation, ending precipitously at either end where the break occurs; its general height is about 210 feet, but Quoin Hill to the

north is 260 feet, and to the south-west are two hills 240 fee high. There are, however, three or four narrow gaps with precipitous sides from 30—50 feet below the general level of the ridge.

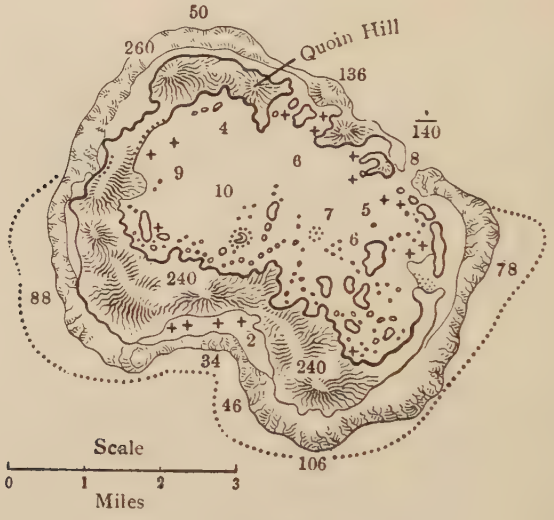


Fig. 6.

Towards the lagoon the rock ends in low, overhanging cliffs except in the bays, which generally have sandy beaches. From these the land rises gradually for some distance, followed by an almost precipitous cliff 30—40 feet high, reaching to the summit of the ridge, where there is a flat plateau, which varies extremely in width, in some places being at least 250 yards broad, but in general little more than 100; beyond this, towards the ocean there is usually an almost precipitous fall. The north side of Quoin Hill is a precipice 260 feet high, with a sea-washed base; the shore too along the whole of the north side is very similar. To the south-south-west the fall is about 80° generally; outside this there is an extensive sand flat, upwards of 400 yards broad. This flat is composed of a very fine grained, almost muddy, white sand consisting of the remains of Foraminifera and calcareous algae, especially of the genus *Halimeda*, but mainly of the limestone very finely pulverised. It is about 4 feet above high-water level and wells, after passing through the beach sand rock, reach it 8 feet what appears to be the reef-rock; their water is brackish, percolating through from the sea.

In the limestone are occasionally large crevices several fee

across, with caves here and there and numbers of small tunnels. Where the rock is weathered, it is grey, rough, sharp and much pitted, while its fracture is white, not uncommonly streaked with red; the appearance presents no trace of organic structure. When broken into, holes are often found, a few inches to some feet in diameter, having rough and rugged sides; other parts show pockets of a red hard amorphous sandy rock, apparently very largely consisting of ferric oxide. Low down, especially where directly acted upon by the tide and spray, the rock is much more crystalline, its rough projections being frequently entirely formed of transparent, yellow, prismatic needles, or shorter and thicker crystals.

Where exposed to the action of the waves, the rock is worn away to an extraordinary extent, so that many of the rocks in the lagoon are mushroom-shaped. The height at the edge above low water is from 7—15 feet, depending largely on the waves and hence on the exposure, the stretch of water that the wind may have for raising a sea. On the seaward face of the island it is generally low, as much wave action is impossible, owing to the reef breaking the waves; within the lagoon near the passage it is greatest. The cliffs by the sand flat to the south-west have the same undermining, proving that it was formed by the sea piling up the sand. In the lagoon there is usually a flat platform at low tide level, running from the base of the cliff to the outer edge of the overhanging part, where it drops directly into 2 or 3 fathoms of water; on its surface are a few calcareous algae, a little weed and some worm tubes. The heads of these mushroom-shaped rocks in the lagoon are often very evenly convex; occasionally one can be seen by the wearing away of its stem to have fallen over to one side. Not uncommonly large masses have split off from the sides, and south of the entrance is one which has broken into two across the middle. The portion of the platform, which should have been covered by these split-off masses, is not, unless the fall is very recent, visible and must have been washed or dissolved away. Every stage from a well-formed islet to a small patch 2—3 fathoms deep can be seen; no shoals appear to exist, which cannot be ascribed to the solution of these islets. Within the lagoon no corals and no reef-forming nullipores live, the bottom consisting of a fine, soft, white, sandy mud, the deposit of which on any coral would quickly prove fatal; solution is going on continually and there is no balancing growth. The final result should be a perfect atoll-reef just awash, the land being dissolved away while the living reef round it continues to grow. (Plate.)

The whole island is surrounded by a fringing reef, off Quoin Hill very narrow, but to the south-west half-a-mile broad with a well-marked channel, 2 fathoms deep; to the north-east it

extends between the islands, except at the ship's passage, and is very broad. At low tide a few boulders can be seen on the outer edge, especially off the north of the island; all are small, and could have been thrown up, as they doubtlessly were, by strong gales. The 100 fathom line is extremely close to the reef, following the contour of the land; on the north and south it is noticeably nearer than to the east and west, being off Quoin Hill almost contiguous with the outer edge of the reef.

About 5 miles to the east of Fulanga lie the reefs surrounding the two islands, Ongea Levu to the north and Ongea Ndriki to the south. Both in the nature of their rock are of a similar character to Fulanga and resemble it further in each having a small lagoon. Ongea Levu is 4 miles long by 2 broad, lying almost north and south; it is surrounded by a ridge with precipitous overhanging cliffs on the outward side, which are not generally as high as those of Fulanga. Southward this range is broken, where the water of the lagoonlet joins with that of the lagoon of the barrier reef; a number of islets, however, in two lines join it across, and make the ring complete. These latter range up to about 80 feet, while the ridge is about 200 feet high with a hill northwards of 270 feet. These rocks across the entrance of the lagoonlet present the same characters as those within the lagoon at Fulanga; one shows at low tide an arch in the centre, where the middle has been washed away. Near these the lagoonlet is about 5 fathoms deep, but northwards it gradually shoals, uncovering at low tide a reef about 200 yards across; this merges into a sand flat, from which the land rises to the ridge, as at Fulanga.

Ongea Ndriki resembles the last island, except that it is smaller, more circular in shape and rather higher; the ridge is open northwards, opposite the break in that of Ongea Levu. These breaks in the ridges of the two islands are so strongly joined by isolated rocks, and the ridges themselves bend so markedly towards them, that the two islands seem always to have been completely separated.

South of Ongea Ndriki the reef fringes, while it contains on the other sides, as a barrier round the two islands, distant 1—2 miles; it has four passages, one to the north and three to the west opposite the channel between the two islands. The depth of the lagoon is from 4 fathoms between the islands to 13 fathoms outside them, with a bottom of fine muddy sand and few shoals; the depth of the largest western passage is 13 fathoms and the strength of the tide through it is 2—3 knots. The water was very distinctly milky at the ebb tide, forming a well-defined streak in the sea outside. North-east of the barrier is a small reef, Thakau Teteika, a few hundred yards away, with a channel between of over 100 fathoms.

Namuka, 14 miles north-by-west of Fulanga, is about 4 miles long by 1½ broad, its long axis lying east and west. A ridge with cliffs to the sea surrounds it; the heights are, as registered, 260, 240 and 210 feet; otherwise it is very even, but its plateau seems to be much narrower than at Fulanga. To the south is a small lagoonlet, 1 mile broad with a sand flat at its head; the ridge, as in the last islands, is continued across its entrance by isolated rocks. The land at the ends of the island within the circle of the ridge is from 50—100 feet above the sea-level; it has no appearance of a deep valley, but the ridge rises from it with more precipitous and higher cliffs than at Fulanga. The rock is the same kind of limestone, by the lagoonlet being remarkably hard and crystalline with numerous pockets of the red sandy rock. Numerous large caves are found on the outer face of the island, and seemed to be situated in horizontal lines along the cliffs. The reef is barrier about 1 mile distant from the shore, but to the east fringes, and off the north-west consists of isolated patches, covered with reef-building nullipores and corals; the lagoon is 7—14 fathoms deep.

The Yangasa Group lie south-east of Namuka and consist of four islands in the lagoon of a barrier reef, 22 miles in circumference, having a passage on the north-west side. They are stated in the survey to be of coral, and this is corroborated by the appearance of their slopes and vegetation. The natives also have the same name for the rock, and state that it overhangs on all sides, making landing very difficult. Yangasa Levu, the largest island, seemed from the ship to have two flat summits; on the sides of these, running across the island, are two terraces, the slope between being about 70°. The other islands of the group are 210, 240 and 270 feet in height.

About 14 miles east of Namuka, Wangava, a long, narrow island lying north-east to south-west, 3¼ by 1 mile broad, is surrounded on all sides by steep cliffs to the sea and has an extreme height of 350 feet. The ridge round the island with the exception of this hill is almost of a uniform height of 260 feet with few undulations. To the centre its slopes are extremely abrupt, and in it in reply to careful inquiries the natives stated that there is a lake in which the tide regularly ebbs and flows[1]. There is no doubt that its rock is all of the same limestone, the fracture showing clearly in one place white and its cliffs overhanging very conspicuously. Round the island is a narrow fringing reef, which north-west only consists of isolated heads and patches.

North-west of Wangava at a distance of about 35 miles lies

[1] I am indebted to the Hon. L. Allardyce, Native Commissioner, for these and numerous other inquiries.

Vanua Vatu, a saucer-shaped island $1\frac{1}{2}$ miles in diameter with steep cliffs all round; its height is 310 feet, and it appears regularly table-topped from the sea[1]. Its reef fringes except to the north-west, where it is of the barrier class.

Naiau, Tuvutha, Mango, Vatu Vara and Naitamba have all table-tops and the characteristic appearance of limestone islands; they are respectively 580, 800, 607, 1030 and 610 feet high. Vatu Vara has merely one flat-topped peak, but the others are typically round with central depressions about 200 feet below the ridge. I was informed by Capt. Wilson and Mr Chalmers, a former magistrate of Lau, that all are formed entirely of limestone; this is confirmed by the survey[2] and very fully by their appearance from the sea. In the interior of Mango nodules of manganese are stated to be common, and, to drain it, a stream has cut itself a passage to the outer side through the ridge. The other islands are all said to have water holes in the centre, in some of which the water is salt, though not below sea-level. From the sea all appear to have very steeply sloping or precipitous cliffs, and I could see no trace anywhere of volcanic soil or rock; terraces are nowhere visible. The reefs usually are fringing or very slightly barrier; off Mango the reef has a few low islets. Vatu Vara alone has a considerable stretch of low lying land by the shore, rising gradually for 250—300 feet above the sea and then precipitously for the last 700 feet; southward its hill ends in a cliff over 900 feet high.

Thikombia-i-ra is, to judge from the descriptions I have received and its appearance, certainly of limestone. Capt. Wilson also told me that the rock is precisely the same as that of Fulanga, and that at one of the chief landing-places a ladder is let down through a hole in the overhanging cliff to ascend by. Its greatest height is 680 feet, but it has several summits and no ridge; its hills at a distance appear to be very distinctly terraced. There is no living reef of any sort except to the east-south-east, where a barrier extends out for about 2 miles. The 100 fathom line lies, however, about 13 miles beyond the reef, and there is a large plateau 45 fathoms deep; the edges in places are considerably less deep, and it is in all probability an upgrowing barrier reef.

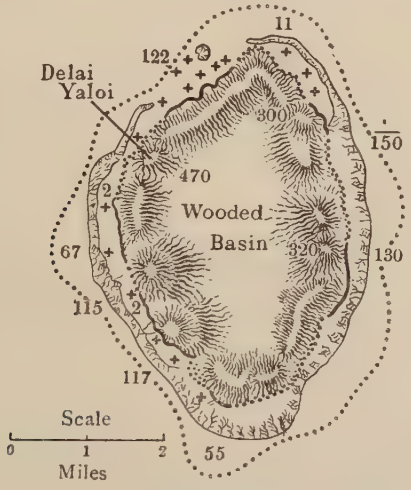
Of the smaller islands in Lau, most are of the same hard limestone, but a few low lying ones seemed to be formed of a soft limestone, in all respects similar to the ordinary reef-rock. Vatoa, I was informed, has both kinds of rock; its reef is both barrier and fringing, the former with in places a deep lagoon.

[1] Captain Wilson informed me that, near the landing place, the rock is much indurated with large clam shells (*Tridacna?*)

[2] *Vide* Sailing Directions for the Fiji Islands, 1882.

2. Limestone and Volcanic Islands.

Chief in this class for interest is Kambara (Fig. 7), an oval-shaped island, close to Wangava, lying north and south, with a length of 4½ and a breadth of 3 miles. The island is bounded by precipitous cliffs on all sides except to the west, where there is a hill, 470 feet high, known as Delai Yaloi. Into the middle of this hill the cliffs surrounding the whole island run. Towards the sea its slope is very steep, up to 45°, but it is nowhere precipitous; it slopes too similarly along the sides of the cliffs and towards the centre of the island. The cliffs around the island average 250 feet in height, but to the east there are two eminences of 320 and 300 feet. By Delai Yaloi the elevations are somewhat lower; and



KAMBARA

Fig. 7.

the cliffs, north and south, are separated from the sea by a sand flat, otherwise they continue evenly round the whole island. This hill is thus 150 feet higher than any other point on the island and about 230 feet above the cliffs near it. Its rock is a dense basalt, which has apparently welled out from a fissure near the base of the limestone cliffs, and gradually increased in height, so as finally to overflow them and form a high hill. Near it, and especially where its rock overlies, the limestone is dense and white with a less tendency to form chasms, pits and caves than is usual; it is also not nearly so hard and crystalline, and preserves traces of its organic origin. Elsewhere the limestone is similar to that of Fulanga, but perhaps a little denser. On the surface of Delai

Yaloi large limestone blocks are found strewn about, as if carried up by the eruption which formed it. On the ascents there are no sea shells, nor sea deposit of any sort; indeed, this hill must have been formed, not only subsequent to the formation of the limestone, but also to its upheaval. It could not have been formed simultaneously with the upheaval, or the precipitous limestone cliffs, cut off by its lower slopes from the sea, would have had no existence.

From the cliffs the land slopes in everywhere gradually towards the centre of the island, which forms a broad plateau about 100 feet above the sea. The slope is, however, sometimes uneven, forming precipices 20—30 feet high. On the plateau itself are many small hills, rising nearly to the level of the rim, with very steep walls, and some springing out almost like buttresses from the rim itself. The natives affirm that in this plateau there are a number of deep holes, in which salt water is found and tidal influences are felt. The rim is somewhat broken as if by dykes 40—100 yards across; its breadth varies greatly, but generally the ridge is at least 100 yards across.

Round the whole island the reef is fringing and narrow with a channel, 1—2 fathoms deep, for some distance to the south-west of Delai Yaloi; north-west of this hill and off it, it is merely represented by a few sunken patches growing up[1].

Vanua Mbalavu, the largest island of the Exploring Group (Fig. 8), is of mixed origin. Within the barrier reef, enclosing the group, islands of all the three kinds are met with; of these Thikombia-i-lau, Sovu, Avea and Munia stand quite apart, while Susia and Malatta really continue the main island to the south and east, the three being surrounded by the same inner fringing reef. Thikombia-i-lau standing 550 feet, Sovu 230 feet, Avea 600 feet, and Malatta 420 feet high, are all of the same dense limestone as Fulanga, while Munia, 950 feet high, consists entirely of erupted rock. Susui, 430 feet high, to the east of Malatta, has its west end of limestone much undermined by the sea with precipitous cliffs; to the east this passes gradually into volcanic rock with more gentle slopes.

Opposite Avea on Vanua Mbulavu is a hill, Koro Mbasanga, 930 feet high. From this the island runs west for 6 miles and south for 8, its average breadth being about 2 miles. North-west at the base of Koro Mbasanga is a deep creek, the rock of which on the west side is limestone, precipitous and much undermined, while on the other side the shore is formed by the slopes of this hill, the rock of which is volcanic. The two rocks join here with no sharp line of demarkation except in the vegetation, the lime-

[1] The reef in Fig. 7, which is taken from the chart, is not quite accurately represented.

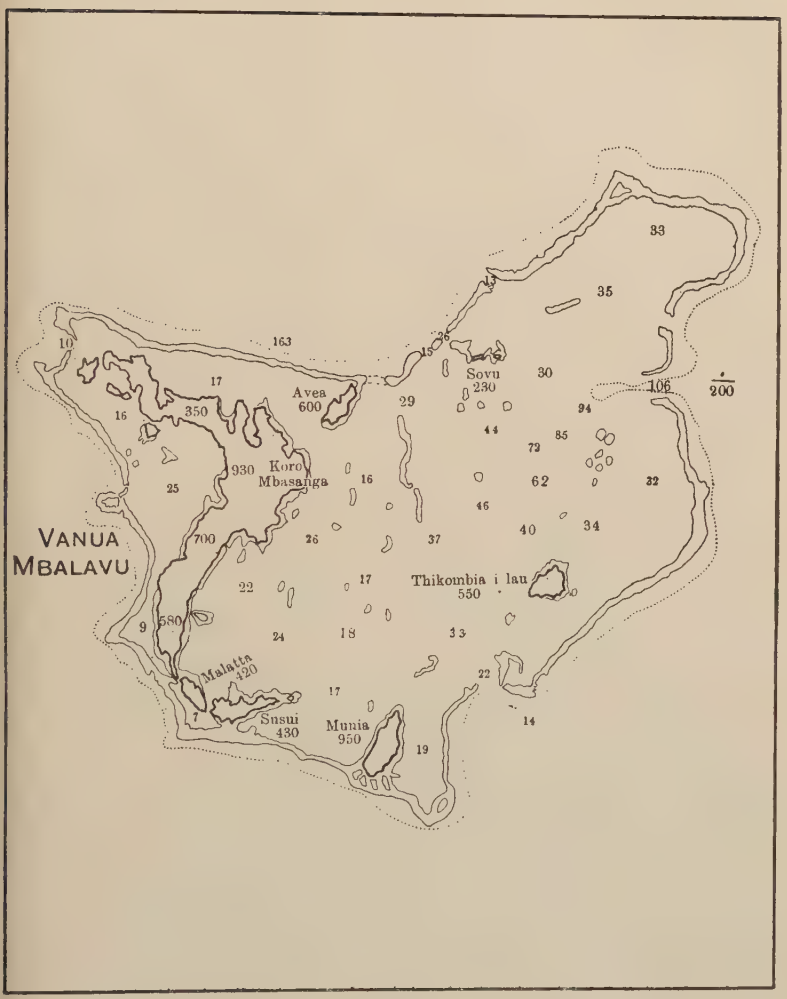


FIG. 8.

EXPLORING GROUP.

stone running straight into the volcanic rock, and being covered by it in many places. In fact it would seem as if Koro Mbasanga had been formed subsequent to the elevation of the limestone in precisely the same way as the volcanic hill at Kambara. To the west the island is all limestone, much cut into by creeks and broken up into isolated hills, forming an irregular range in the centre, with peaks and buttresses on either side, ending in cliffs seawards with heights up to 600 feet. To the south the land is undulating, having down its centre a ridge, steep but not precipitous; to the east is an extensive sand flat by the shore. This volcanic rock is a friable and very crystalline conglomerate, breaking down into a red coloured earth. About a mile from the south end limestone again recurs; it is here exceedingly crystalline and much coloured by oxide of iron. At the juncture of the two rocks are some hot springs, in one of which the water is nearly boiling. Here again the limestone appears to have been formed before the eruption of the volcanic rock. The cliffs, however, are not especially precipitous, nor is the amount of water action so great as to lead to the conclusion that so much time had elapsed between the elevation of the limestone and the eruption of the volcanic rock as at Kambara.

The barrier reef of the whole Exploring Group (Fig. 8) is about 80 miles round, lying far distant to the east, but to some extent fringing Munia, Susui and Malatta. Within it round Vanua Mbalavu is a fringing reef, small and inconspicuous off the limestone part, projecting indeed little beyond the distance to which this rock overhangs. The general depth of the lagoon is about 35 fathoms, with a pool in the centre about 75 fathoms deep; shoals and patches abound in the shallower parts. The passages for such a large lagoon are exceedingly few and small, but all have depths of over 10 fathoms, and around the eastern one the 100 fathom line runs for three miles into the lagoon; the other passages are one to the south, one north-west, and two north, near the island of Sovu.

Lakemba has flat limestone hills, rising abruptly to a height of 60—70 feet on the west coast, the foundation being the volcanic conglomerate, which forms the rest of the island. I was informed by the Rev. C. Heighway, resident missionary, that these are frequent round the whole island, and that inland there is generally a small flat at their base; they should then represent an upheaved barrier reef. Inside these the island trends gradually upwards to two peaks in the centre about 700 feet high; there are good springs of water and deep water-courses. The reef is fringing off the west side, but a distant barrier off the east.

One to the extreme south, Capt. Wilson stated, has some of the islets coral and some volcanic. He considered the three larger islands in the centre to be the remains of an old crater.

3. Volcanic Islands.

The rock of these islands is usually a dense, coarse, and heavy volcanic conglomerate with no vesicular lava and no remains of craters. Most of them have well-defined water-courses, broad fringing reefs and often distant barrier reefs as well. There are no signs of recent activity or movement.

Mothe, to the south, rises to a single peak of 590 feet, and has a considerable barrier reef round it with a long extension southwards. In the same lagoon is a small island, Karoni, 120 feet high, the formation of which is doubtful; as seen from the ship it appeared to be of limestone.

Thithia, 540 feet high, is a larger and rounder island with several peaks of about the same height, fringed by a broad reef. Kanathea has several peaks up to 330 feet, with an extensive barrier reef. Yathata has a single conical hill of 840 feet, with a broad fringing reef.

SECTION III.

Summary and Conclusions.

Considering the peculiar forms that these islands of limestone and of mixed rock in the Lau Group take, it is impossible to conceive that they can have had any origin different to that of many of the atolls, barrier and fringing reefs of the present day. They differ however in the great hardness and dense nature of their limestone, and that it shows little or no signs of organic structure. I have already given my reasons for considering that the soft limestone of the eastern half of Viti Levu had its origin in coral reefs, which, although some may have been inner reefs, cannot have differed essentially from the reefs, which formed the Lau Group. The surface of these soft limestone reefs where it is exposed to the atmosphere is no doubt harder than the interior, but metamorphosis has only taken place in the small fissures, where the carbonate of lime has obviously been dissolved by the carbonic acid gas in solution in the rain water and been redeposited.

The raised reefs of Tonga[1], the Solomon Islands[2], and Cuba[3], seem to be formed of absolutely similar limestone. From the character of these reefs, together with those of Fiji, I can scarcely

[1] The Geology of the Tonga Islands, by J. J. Lister. *Quar. Jour. Geo. Soc.* 1891.
[2] The Geology of the Solomon Islands, by H. B. Guppy.
[3] On the elevated Coral Reefs of Cuba, by W. O. Crosby. *Proc. Boston Soc. of Nat. Hist.* xxii. 1882—3.

believe that the metamorphosis from the heap of organically deposited material, like the reef of the present day, can have been subaqueous. Neither do I believe that the change has been to any great extent effected by the interfiltration from the sea of carbonate of lime in solution. The deposition of finely divided carbonate of lime would, of course, greatly tend to fill up the pores of any coral, but this is not an interfiltration. The cavities in the hard limestone can be accounted for by the shrinkage that would naturally occur in its metamorphosis; I have also already shown, in my description of the reef at Funafuti, that in the growth of a living reef spaces are often enclosed. The pockets of oxide of iron are due to the solution and abrasion of much of the rock by the rain during its metamorphosis, when it would of course be left undissolved; the higher lying rock is much freer from it than the lower, as should in such circumstances be the case. The rock at Kambara, covered by the lava, is almost intermediate between the soft limestone of Suva and this hard limestone. The beach sand rock of Rotuma, a white, loosely compacted limestone, is, as I have pointed out, used by the natives for gravestones, becoming on exposure exceedingly hard and compact, the various constituents being cemented together by calcite. I believe that in these small slabs a metamorphosis is now taking place precisely similar to that which has formed the dense highly crystalline limestone of the Lau Group out of the loosely aggregated masses, such as form a coral reef at the present day.

I am greatly indebted to Dr Pollard, of the Caius College Laboratory, for having kindly analysed two specimens of this hard limestone. Both came from the inner side of the ridge of Namuka, the one being a pure white rock without any large crystals, and the other a very fine grained reddish rock. The white rock (two determinations) gave 78·6 per cent. carbonate of lime and 21·5 carbonate of magnesium. The red rock gave 85·2 per cent. carbonate of lime, 6·7 carbonate of magnesium, 4·2 oxides of iron and aluminium, and 3·5 silica[1].

These analyses show that dolomitization has gone on to a considerable extent in this hard limestone. Dana states[2] that 95—98 per cent. of the common reef-corals is formed of carbonate of lime, the remainder being made up of $1\frac{1}{2}$—4 per cent. of organic matter with about 1 per cent. of earthy ingredients. Dana also notes[3] that coral sand and mud contain little or no magnesia generally; he gives however an instance where 1·38 per cent. was obtained. These analyses, so far as they go, afford no probability

[1] Analyses of two very white specimens from Kambara and Fulanga gave me 19·8 and 10·7 per cent. respectively of carbonate of magnesium.

[2] *Coral and Coral Islands.* J. D. Dana, 1890, p. 99.

[3] *Loc. cit.* p. 394.

of the dolomitization having taken place under water with the reef *in situ*. Abrasion from climatic causes has certainly gone on to a very large extent. The rain water takes up a certain amount of carbonic acid gas, when exposed to the air, and by means of this acts on the limestone, dissolving its carbonates of calcium and magnesium. There can be no doubt that this is due to a chemical compound being formed, and I would suggest that a certain amount of pure calcium magnesium carbonate $\{MgCa(CO_3)_2\}$ is always formed at the same time and deposited on the evaporation of the water. One litre of water saturated with carbonic acid gas under atmospheric pressure dissolves ·31 gram of calcium magnesium carbonate at 18° C., ·88 gram of calcium carbonate at 10° C., and 1·31 gram of magnesium carbonate at 10° C[1]. Pure dolomite hence is far less soluble than either calcium or magnesium carbonates, and thus its percentage in the rock should slowly be increased, especially to the detriment of the carbonate of lime, which in the first place formed the greater part of the rock. The rock should then practically be a mixture of pure calcium magnesium carbonate with pure carbonate of lime and a small quantity of earthy ingredients. It should vary greatly in the relative amounts of the carbonates of calcium and magnesium, and the analyses, before mentioned, indicate that it does so; Dana also from similar rock from the island of Metia noted 38·07 per cent. and 5·29 per cent. of carbonate of magnesium[2].

An additional factor in the formation of the dolomite might be the large amount of magnesium salts raised in the sea-water of the lagoons and pools in the reef at the time that these islands were elevated. Dittmar in the *Challenger Reports* has shown that magnesium chloride forms 10·878 per cent. of the salts in sea water, and that other magnesium salts form nearly 5 per cent., while lime salts do not form quite 4 per cent. On the external cliffs the spray from the sea might have some effect with a slow elevation, but the specimens analysed from Namuka were both taken from the inner side of the ridge; a series of analyses of specimens taken at intervals from a short horizontal core obtained from one of the outer cliffs would, however, at once settle the extent to which this has been a factor of importance.

There is no method of estimating directly the amount of time which would be required for this metamorphosis to a hard crystalline state. If it has taken place in the Lau Group as I suppose, it should in course of time take place also in the raised reefs of Viti Levu. It may be argued that traces of such a metamorphosis are not seen in other parts of the world, and that at any rate they should be seen in the exposed limestone cliffs of Viti Levu. They

[1] *Handbuch der Anorganischen Chemie.* Dammer II. 2.
[2] *Loc. cit.* p. 393.

do show traces of a hardening on the surface, but the raised coral reefs in Viti Levu, closely examined by me, are all overlain by soapstone, a rock of such density and of such a nature that it is very doubtful if it would allow water to percolate through it at all. Considering the large size of the delta of the Rewa, it is obvious that the eastern half of Viti Levu has been upheaved for a considerable period of time, and hence it is only reasonable to suppose that the upheavals which formed the Lau Group must have been extremely ancient.

In Lau it has been seen that there are many perfect specimens of raised atoll-reefs, of the same form as existing atolls, save only that the slope is not gradual below their summits for some distance and then precipitous, but at once an almost perpendicular cliff. Naiau and Tuvutha are such raised atolls with perfect rims, and lagoons apparently seldom more than 30 fathoms deep; Mango has the rim broken and is deeper, probably from denudation. Kambara, once a typical atoll, has a broad flat rim, remains of numerous shoals in a flat even lagoon, and many passages in the original reef. Round Tuvutha are cliffs of over 100 fathoms, and these are approached or exceeded by those of many of the other islands. The cliffs are limestone; in no place except Lakemba, where the raised reef is of the barrier type, is the foundation visible. Considering the known sections of atoll-reefs, the precipitous cliffs to the sea can only be accounted for by the splitting off of great masses of the rock, owing to undermining, and their subsequent solution (or the formation on them of the fringing reef). The reef sections of Funafuti[1] and of Sydney and Canton atolls in the Phœnix Group[2] show that it is only at a distance of 150—200 yards from the edge of the reef that the steep commences. Yet there is no trace of any distance approaching this from the summits to the cliffs in such islands as Fulanga, Ongea, Namuka and Kambara; indeed the edge of the cliff is often the actual summit of the island. Hence the action of the sea undermining the cliffs must have been extremely extensive. As the breadth of the fringing reef, supposing it to have been largely built up on the fallen fragments, is in many places not comparable to such an amount of undermining, it necessarily follows that the solution under water of the fallen fragments must for some time at any rate have kept pace with the breaking away of the land. The higher elevations probably represent the highest points of the reef, parts awash or low islands on the rim of the atoll; when first upheaved, such would naturally be the most consolidated, and hence least liable to solution.

Denudation owing to climatic causes cannot, I consider, have

[1] *Nature*, vol. LV. 1897, p. 375.

[2] *Vide* Admiralty Charts.

been of sufficient importance to have greatly affected the position of the summits, or it should have left traces of itself in water channels, etc. in the cliffs. It no doubt has been considerable, but I think that the heights of the highest points are not far, not more perhaps than 40—50 feet, below their original elevations, when first upheaved. Crosby, in the case of El Yunque in Cuba, has come to the conclusion that "erosion must have diminished the height by as much as 200 feet[1]," but such an amount is impossible for the Lau Group and seems to me very extreme for that mountain.

That solution in the sea is taking place very rapidly admits of little doubt from the presence of small lagoons within many of the raised atolls. The beginning is seen in the pools in the centres of Tuvutha, Naiau and other islands; in Mango these have further dissolved themselves out a channel to the coast. Wangava is said to have a tidal lake in the centre, while in Namuka and the two Ongea, the water has broken through the reefs of the former atolls, and opened lagoonlets to the sea; in their channels the water is deep and within shoals gradually, denoting the extent to which solution has taken place. Fulanga is an example of a later stage, where a perfect lagoon, 10 fathoms deep, has been formed by this means. The islands of its rim are high and protected from all rapid solution oceanwards by the living reef, which now surrounds them. The same action is, however, still going on within the lagoon, the rocks of which, the shoals of the former atoll, are seen to be disappearing one by one, and its ring of land to be lessening. Only on the seaward face is any increase of the land visible, and there is due merely to the growth of sand flats at the base of the cliffs; such flats, however, I only saw on this one limestone island in the Lau Group. Should the Yangasa Group, as I am led to believe from the appearance of their hills, be the remains of a raised atoll, they would form another stage, only a few isolated peaks being left. Within the barrier Navatura and Yavutha are connected by a reef awash, while the terraces on Yangasa Levu I regard as caused by denudation in a channel through the original reef; if they represent progressive steps in elevation, they should rather have been separated by perpendicular cliffs. Finally, some of the neighbouring atoll-reefs may possibly represent the last stage, where all the raised reef has gone, and only the living reef is left. In support of this I may here note that there are, in addition, a number of large barrier reefs in the Lau Group, surrounding very small islands, or rocks of a similar limestone to that of these raised atolls.

The thickest cliffs, which I had the chance of closely examining,

[1] On the Elevated Coral Reefs of Cuba. *Proc. Boston Soc. of Nat. Hist.* Vol. xxii. 1882–3.

are all under 350 feet, but those seen on Tuvutha and Vatu Vara are over 800, and those of Naiau, Mango, Vanua Vatu and Vanua Mbalavu are about 600. The absence of terraces is remarkable, as it is extremely characteristic of Tonga, the Solomon Islands, and Cuba. One is driven from this to the conclusion that elevation must have taken place for each island in one act to its present level, or the undermining and wearing away must have been still greater than I am prepared to admit: Thikombia-i-ra is, I believe, an exception, as its terraces looked very distinct, and it is in no way atoll-shaped; it stands also quite apart from all the other islands.

Necessarily each island need not have been elevated to the same height, but two areas of elevation are clearly defined to the north and south of Lakemba, the elevation of which may have been a separate act, as its limestone is nowhere thicker than 100 feet, and represents a barrier reef. Northwards the elevation was about 700 feet, but southwards from Vanua Vatu to Ongea about 400. The latter is very regular in neighbouring islands, but yet great variety at the present day exists in the nature of their living reefs. By Kambara and Wangava they are narrow and fringing, while off Fulanga the reef is broader with a boat channel. Namuka and Vanua Vatu have reefs both barrier and fringing, while round the two Ongea the reef is typically barrier, but slightly fringes the southern island. Round the Yangasa Group the reef is a widely separated barrier with a lagoon 20 fathoms deep.

None of the pure limestone islands show their foundations, but at Kambara the volcanic rock has welled out over the coral. This island is otherwise typical of what a raised atoll might be supposed to be, and here the nature of its foundation is exposed; it is also shown in the same way in three places on Vanua Mbalavu. The volcanic nature of the rock on Ono, Vatoa, Mothe, Lakemba, Thithia, Kanathea, and Yathata proves that there must have existed here a broad range of volcanic hills. The whole group of Fiji stands on a plateau of 1500 fathoms, so that their general height to bring them within the limits of coral growth, assuming it at 50 fathoms, must have been over 8700 feet. Between the islands and reefs of this range the depths are not usually marked on the charts: in the Nanuka passage, and between Vatoa and Ono there are a few soundings of considerably less than 1000 fathoms. There thus seems to be here a special ridge at 1000 fathoms, which possibly may be continuous southward with that on which the Minerva reefs are situated. If so, it joins the line of volcanic activity from New Zealand to the Kermadec Islands, which then branches at the Minerva reefs through Tonga to Samoa, and through the Lau Group of Fiji[1].

[1] *Vide* Lister, *loc. cit.* p. 595.

I have already pointed out that several of the islands have a greater thickness of limestone than 600 feet; Tuvutha further is 800 feet high, and Vatu Vara 1030, with a cliff over 900 feet. It is important to note that the latter is the greatest vertical thickness of coral-limestone yet recorded. Prof. Crosby estimates the thickness of the reef on El Yunque, Cuba, at 1000 feet but, as he allows 200 feet for denudation, and the base of the mountain consists of "eruptive rocks with some slates" and its summit is "three-quarters of a mile long and a third as broad," it is probable that his estimate is far too high, much of the thickness perhaps being due to the talus on the slope of the original reef.

I see nothing inconsistent, with any of the theories of the formation of atolls and barrier reefs, in the shallow reef, together with the talus slope, advancing outwards in a long period of rest and so getting a spurious thickness. Darwin and Dana, however, do not seem to have considered the possibility of such a period in the slow subsidence they require. The real depth of the limestone of Vatu Vara to the rock on which the island is founded might have to be somewhat reduced owing to such a cause, but in contradistinction to El Yunque the elevated reef of Vatu Vara is small at the base and precipitous above, with a flat top not more than 100 yards across in any direction. The same remarks may apply also to a limited extend to Funafuti, where a thickness of 693 feet has lately been obtained, and to Vavau, Tonga, where the cliffs are 500 feet high. Of other Pacific Islands Dana gives a thickness of 250 feet at Metia, Society Group, and of 300 at Mangaia, Hervey Group; the sailing directions, however, for the Eastern Pacific give the height of the latter island as 650 feet[1].

In contradistinction to these Guppy asserts that the raised reefs of the Solomon Islands are nowhere more than 150 feet thick.

The mud, coating the bottom of the lagoon at Fulanga, has apparently been formed by the wearing action of the sea on the raised limestone cliffs. It has no trace, except in a few small pieces of shell, of any organic origin, and is quite different from what is known as coral mud. As Dana's description[2] of the chalk of Oahu, Sandwich Islands, applies equally well to this mud, I would suggest that it was formed in a similar way, and on upheaval solidified. Oahu has the same kind of hard limestone, and the recurrent eruptions, which have taken place on that island, might well account for all traces of its origin being lost.

[1] *Pacific Islands*, Vol. III. 1885, p. 14.
[2] *Loc. cit.* p. 395.

PART V.

Special Features in the Natural History of Corals.

Section 1. *The Structure of a Reef.*

I have already described fully the external appearances of the coral reefs of Funafuti and Rotuma. In Fiji the reefs, which are freely exposed to the ocean, have all a well-defined rim and reef-flat; some also have a boulder zone, which is naturally absent from the reef at Funafuti owing to the elevation of that atoll. Few have such a luxuriant growth of corals as are found on the reef-flat off the windward end of Rotuma, but otherwise their differences are merely differences in detail.

The boulder zone shows, I think, the distance from the edge of a reef, at which boulders can be piled up on the reef-flat to form, subsequently, land. From the *Penguin* the outer reefs on the windward side of Nukulailai, an atoll about 65 miles south of Funafuti, could be seen to be as narrow as in the similar position at that atoll; on the other hand the same reefs off the leeward island are as broad as on the leeward side of Funafuti, but with no rough zone. In this position at Nukulailai there is a rim much broken up by fissures, followed by a reef-flat upwards of 200 yards broad, hollowed out for 2—3 feet with, in places, pools of water 5—6 feet deep; this extends to the island itself, which is not more than 2—3 feet above the high-water level. The island is formed principally of small boulders and sand, and rests on the reef-flat all round, so that the latter has a total breadth of over 600 yards. This atoll differs greatly from Funafuti, as its lagoon is almost completely encircled by the reef and has everywhere a sandy bottom. Land too encloses the greater part, the broadest gaps in it being to the north-west and south-west. There are a few isolated blocks of rock on the reef, but these are not regular in position and seem rather to be negroheads than pinnacles; they are more abundant to leeward.

From the appearance of the atoll of Nukulailai I should not judge that any upheaval is necessary to explain its characters; there may, however, have been one of 2—3 feet. The reef to the south-west, where there is no land, bears a close resemblance to that on the west side of Wakaya, Fiji. It consists of pools of water 1—10 feet deep, fringed with corals, nodulose, and laminated nullipores; *Millepora* is everywhere very common, as also is *Heliopora*, a genus only found at 35 fathoms off Funafuti. The

islands to windward seem to have been formed by an aggregation of boulders, and their subsequent consolidation underneath by the deposition of carbonate of lime, caused by the evaporation of the spray and salt water at low tide. The large island to leeward appears to be a large sand cay on the reef, formed partially by fragments of reef-rock, but principally by sand washed up from the lagoon and formed into a bank by the assistance of the wind. The lagoon has a depth of about 10 fathoms, and is stated by the natives to be of a basin-like form, rather than a flat with precipitous walls. As the tide falls, the water rushes out of the lagoon by a few narrow passages, which, however, would seem to be insufficient to keep its water at the same level as that of the external ocean; hence it is probable that (as the natives stated) there are tunnels from the lagoon, which assist in carrying away the water. It is interesting to note that on the western island of Nukulailai there is a spring of pure fresh water, which, according to the natives, is continually running. This probably indicates that the foundation of this atoll is situated at no great depth. I was informed by Capt. Wilson that Arecifos, or Providence Island, to the east of the Marshall Group, has also a spring of clear fresh water in the coral-rock, and that this and Nukulailai are the only coral islands in the Central Pacific, where such are found.

The appearance outside a reef is that of a number of more or less overhanging and anastomosing masses of rock with pools between, having sandy bottoms. The same appearance is retained on protected reefs, until they become awash, when by the growth of corals and nullipores the masses fuse, and a smooth platform is formed. The inner part of the Great Barrier Reef of Fiji consists, where it is single, of deep pools, separated from one another at the surface but joined below. Inside the more solid outer part of the reef here mushroom-shaped heads of coral rock grow up, anastomosing with one another at the surface to form an almost solid platform, resting on so many large pillars. I have obtained, in pools on the inner parts of the Great Barrier Reef and of the reef to the west of Wakaya, 5—6 fathoms of water, and at the same time noticed that the tide sets strongly through them, indicating a free connection with the ocean beneath the surface. Outside reefs, where they are exposed to the ocean, there seems to be the same process as within the Great Barrier Reef, but the joining organisms are almost entirely nullipores; in the account of the outer reef of Funafuti I have shown how it is going on at the surface, and it certainly seems to be taking place outside the rim. At greater depths the same formation also, I believe, occurs; the corals and nullipores cannot grow upwards with such vigour as inside the reef on account of the strong backwash of the waves which break on the reef, and so grow relatively

more outwards, and anastomose more frequently with one another. The whole structure then, provided that there are no movements of elevation or subsidence, should be an open spongework, full of chambers and passages, some of which are partially, or completely, blocked by sand and boulders.

The consolidation of sand and fragments by the infiltration of sea water is not, I believe, of any importance. That molecules of a salt can out of a saturated solution of that salt attract more molecules admits of no doubt, but there is no evidence to show that this is going on with regard to lime salts to any considerable extent in the sea; the pores of the dead corals in reef-flats and in the raised reefs of Viti Levu do not show any marked filling up, which can be ascribed to such a cause. Sand is constantly to be found in small holes in the reef completely walled in by the growths of nullipores and corals; it exists also in greater or less quantity in nearly all the pools on the reef, in all boat channels and on all inner reefs, but I could not succeed in finding it consolidated together in any way or to the rock below it under low tide level. The sand in the soft limestone quarry at Suva, although it has evidently been upheaved for a considerable time, is still badly consolidated, and the thin layer between this rock and the soapstone is almost loose. The *Report on the Coral Reef at Funafuti* by Prof. Sollas mentions that loose sand and boulders were met with as deep as the bore went. At a depth of 72 feet in the bore, on the outer sides to windward of the atoll "the bottom of the hole was surrounded by quicksand, containing boulders of coral, and as fast as the sand was got out, so fast it flowed in and faster." The consolidation of the fragments here has certainly not been effected, and I am inclined to doubt whether infiltration does not have an entirely opposite effect by means of the carbonic acid gas in the water.

The hard parts of a reef, above low tide, are made up of fragments of coral, sand and nullipores, the consolidation of which seems to be due to the deposition of carbonate of lime on the evaporation of the water at low tide. Loose fragments after a time get a bed of fine mud, which, as more fragments are piled up, gradually surrounds and buries them. On the reef-flat there is less coral and less sand, and much of the rock is formed by a structureless and very compact limestone, which also forms the whole rim. Where corals are found in this rock, they are not as a rule as much eaten into or worn, as they are in the parts above the low tide level; they do not seem like fragments at all, but as skeletons of corals in the position of growth. This appearance leads me to believe that the rock here has grown up to the surface exactly in the position in which it is found, and is not made up of masses, thrown up from outside the reef by the seas and cemented together.

Of the structure of the surface of a reef Dana gives the following account:—"The reef rock, when broken, shows commonly its detritus origin. Parts are of compact, homogeneous texture, a solid white limestone without a piece of coral distinguishable and scarcely an imbedded shell. But generally the rock is a breccia, or conglomerate, made up of corals cemented into a compact mass, and the fragments of which it consists are sometimes many cubic feet in size[1]."

The last sentence certainly will not apply to the reefs of Fiji. The parts of "compact homogeneous texture" are very numerous, and are formed, I believe, mainly of the carbonate of lime secreted by incrusting nullipores. The importance of the incrusting nullipores, in the formation of the reefs of the Central Pacific, cannot be overestimated. The greater part of the rim of the reef and its outer slopes is covered by these algæ. A coral dying is at once grown over by them; sand grains and even loose boulders are enclosed, and fissures as well as small pits are bridged. These incrusting nullipores grow close to the surface they are on, and spread gradually outwards, fixing themselves as they grow, so that they form a very strong bulwark against the sea. At the bottom of the sea, outside the reef in 7—15 fathoms, they often form on the tops of the ridges large nodular masses round fragments of coral and rock; the smaller nodules (which alone could be loosened by the use of Priestman's grab) showed thicknesses of upwards of 4 inches of compact white homogeneous limestone secreted by them, and I have no doubt that by their agency the greater part of these large masses was formed.

The incrusting nullipores of the reef belong to the genus *Lithothamnion*, which Walther has shown covers and has apparently formed the greater part of certain shoals in the Bay of Naples[2].

Walther afterwards investigated the tertiary "Nulliporenkalk" of Syracuse, and found all stages from well-preserved specimens of *Lithothamnion* to a structureless limestone. Of the lias of "Todten Gebirges" it is remarked that "the strata consist of coral rock, detrital calcareous deposits, and associated with these, masses of limestone, in which microscopic examination fails to detect either vegetable or animal structure. These structureless beds are considered to have been *Lithothamnion* banks from which percolating water has removed all traces of algal cells[3]."

The outer reefs of Fiji, Rotuma, and Funafuti seem as if they

[1] *Loc. cit.* p. 175.

[2] Die Gesteinbildenden Kalkalgen des Golfes von Neapel, etc. *Zeitsch. deutsch. Geol. Ges.* XXXVII. 1885, p. 329.

[3] Algæ as Rock-building Organisms. A. C. Seward. *Sci. Progress*, Vol. II. 1894, p. 19.

form structures, which, if upheaved, would give a rock precisely similar to this.

In the lagoons, it must be remarked, the shoals are formed much more of corals and have everywhere precipitous sides. Nodulose, mamillated and laminated species of *Lithothamnion* grow, but, while they are often very abundant, they cannot have the same binding effect as the incrusting forms. Algæ of the genus *Halimeda*, Mollusca and *Sarcophytum* abound, and the whole rock seems a somewhat loose conglomerate of the remains of all these, held together mainly by coral growth.

SECTION II.

The Bathymetrical Limits of Coral Growth.

Dana on this subject sums up the evidence to 1890 in these words[1]:—"There is hence little room to doubt that 25 fathoms or 150 feet, may be received as the limit in depth of flourishing banks of reef corals." Although I cannot admit some of the arguments that he urges in favour of this view, there is really no evidence which proves that "flourishing banks of reef corals" grow at a greater depth; indeed it would be equally correct, if Dana placed the depth at 15 fathoms, which is about the limit of the visible zone.

Yet I think that, considering the few dredgings that have been taken outside coral reefs and in spite of the few species of incrusting and massive corals, which have been brought up, there is now strong presumptive evidence that corals live down to depths of 40—50 fathoms in great luxuriance. Dr Bassett Smith's dredgings on the Macclesfield Bank showed that 18 genera of corals with 40 species were living at depths from 20—44 fathoms[2]. "The following 10 genera were found at a greater depth than 30 fathoms:—*Stylophora, Astraea, Pavonia, Cycloseris, Leptoseris, Stephanaria, Psammocora, Montipora, Alveopora* and *Rhodaroea*, besides seven small scarcely reef-building genera." The total number of species collected was 142, so that nearly one-third were represented in depths over 20 fathoms.

The character of the reef off Rotuma is certainly such as to indicate a growing coral bed to depths up to 40 fathoms, and the soundings and dredgings off Funafuti give for this atoll a similar depth. The dredge and the swabs when placed down in depths of 20—40 fathoms were constantly caught up, and their whole behaviour indicated such a very rough and uneven bottom, that

[1] *Loc. cit.* p. 118.

[2] *Nature*, Vol. XL. p. 223.

I had no doubt that it must have consisted of a growing coral bed, of which Dana remarks[1]:—"The character of a growing coral bed is so strongly marked in its uneven surface, its deep holes and many entangling stems, to the vexation of the surveyor, that in general the danger of mistake is small."

At Funafuti, eight dredgings were made on the steep slope of the reef between 40 and 140 fathoms, none of which gave any evidence of reef-building corals at these depths. There were a few patches of solitary corals, but even these were rare on the stones brought up. At a depth of 35—45 fathoms off the south end I took a haul of the swab and obtained four species of *Madrepora*, three of which I have identified with *M. reticularis*, *M. angulata*, well-known deep living species, and *M. scabrosa*. No other true corals were obtained, but there were two small pieces of *Heliopora*. At a greater depth than 30 fathoms, also off the south end, I obtained in two hauls three species each of *Pocillopora* and *Madrepora*, and one each of *Stylophora* and *Porites*; there were also several small pieces of *Millepora*. Off the north of the island, I could not keep in these depths as the slope of the reef was too abrupt, but one long haul, with the shallowest depth 21 fathoms, resulted in three species of *Madrepora*, one each of *Heliastræa* and *Pocillopora*, and also several pieces of *Millepora*. In all, from depths of over 20 fathoms were obtained 7 species of *Madrepora*, 4 of *Pocillopora*, and one each of *Heliastræ*, *Porites* and *Stylophora*, making in all 13 species in four hauls of the swabs. Outside the reef, five hauls of the swabs at depths up to 7 fathoms gave in all 11 species, represented by 5 *Madrepora*, 2 *Pocillopora* and one each of *Seriatopora*, *Montipora*, *Porites* (apparently *P. arenosa*, a very common species on the shoals of the lagoon) and *Stylophora*. *Millepora* was, however, very common in all the hauls up to 7 fathoms, and evidently grows extremely luxuriantly at about this depth.

SECTION III.

The Food of Corals.

The conclusion of Murray[2], "that the chief food of the corals consists of the abundant pelagic life of the tropical regions" does not seem to rest upon many direct observations, though the same statement is repeated, or inferred, by nearly all writers on the theory of the formation of these reefs. A further conclusion is

[1] *Loc. cit.* p. 117.
[2] *Coral Reefs.* By Chas. Darwin, 1889. App. II. p. 286.

then drawn by Murray that "when coral plantations build up from submarine banks, they assume an atoll form owing to the more abundant supply of food to the outer margins and the removal of dead coral rock from the interior portions by currents and the action of the carbonic acid dissolved in the water." With the latter statement we are not at present concerned, but that the more abundant supply of food to the outer margins causes these to grow up quicker, I am inclined to doubt. I have already stated that I do not consider that the Madreporaria, together with *Heliopora* and *Millepora*, are the chief factors in the formation of coral reefs within shallow depths, as Murray seems to infer, but at the same time I do not deny to them a very considerable influence.

At Funafuti I did not have many opportunities of examining the "auftrieb" outside the reef, and none in the early morning, when it is always most abundant near the surface. On the few occasions that I used my nets outside the atoll I found that the deeper I sank them into the water, up to 50 fathoms, the greater were the quantity and variety of pelagic animals obtained. Near the surface and down to 25 fathoms the nets were useless, and on no occasion did I find, even at 50 fathoms, a luxuriant pelagic fauna. This proves nothing, but it indicates that, at any rate during the daytime, the fauna must sink down to very considerable depths; unfortunately I had neither the time nor the opportunity to investigate depths below 50 fathoms. In the lagoon, the surface fauna, before daybreak, is generally very rich in the pelagic larvæ of Crustacea (especially the *Phyllosoma*), pelagic Crustacea and *Sagitta*. There are usually found in addition veliger and trochosphere larvæ of the Mollusca, and larvæ of Tunicata, Chætopoda, fishes and Echinodermata. Pteropoda, especially of the genus *Creseis*, are occasionally very abundant, while the remainder of the 'auftrieb" is almost entirely made up of bits of green seaweed, dirt, etc. Before dawn, nets at 10, 15, and 20 fathoms in the lagoon give only a few *Sagitta*, but a large amount of sand, with bits of weed, etc. in suspension. After dawn, the fauna for an hour can be followed down, as it sinks from the surface, and is obtained progressively in the deeper nets. At midday, even the deepest nets in the lagoon almost failed to obtain any animals; the whole fauna seemed to sink until it lies in the layer of water which immediately covers the bottom, as it was found, when a net was accidentally scraped along the bottom, that with the dirt brought up there was a decidedly rich fauna. Generally outside the atoll the fauna was the same, but unicellular algæ and Radiolaria are at times very common. At Rotuma I fixed my "auftrieb" nets in the passages of the reef to find out what animals were driven over the reef to form the food of the corals on the reef-flat and in the boat

channel. This fauna differed little from that inside the lagoon at Funafuti; but Radiolaria were more abundant, and often the pelagic eggs of different species of fishes were extremely numerous. Bits of green weed and pelagic algæ were very common, but there was little sand, or dirt of any sort. When the tide was ebbing the nets gave few animals; there was generally, however, a large accumulation of sand with bits of weed in the bottle.

In the "auftrieb" collected in the daytime at Rotuma, animals were rarely to be found, while the fauna at night was very rich. In agreement with this it was noticeable that only at night time were the corals of the family Astræidæ fully expanded; these corals expand somewhat on the tide rising, but I never saw any with the exception of *Euphyllia*, *Symphyllia* and *Mussa* fully out in the daytime, while at night parts of the reef looked quite different by reason of the expanded polyps.

The examination of the food, found in the cœlenteron of different species of *Prionastræa*, *Favia* and other genera, killed in the early morning, was unsatisfactory. In most I could only find pieces of green weed with sometimes a few small, oval, unicellular algæ; animal remains were rare, and could seldom be identified. I next tried to feed different species of corals, placing them in bottles, into which a certain amount of the "auftrieb" from the passages in the reef was poured, and then sinking them in a pool off Solkopi; I could get no results with any genera, except *Euphyllia* and *Mussa*. Other genera with large calices, such as *Prionastræa* and *Favia*, could not be obtained in small enough pieces without damaging some of their polyps, and, after removal from the mass, were rarely found expanded either by night or day; these two genera further were never seen extended in the daytime, while most of the pelagic animals in the bottles, although they were sunk in 4 feet of water under an overhanging ledge of rock, never lived more than a few hours. The different polyps of the species *Euphyllia* and *Mussa*, on which I experimented, were not in physiological continuity; increasing by fissiparity, their calcinal centres soon become very distinct, the corallites separating, and never remaining in long series. As the living tissues only cover the top $\frac{1}{2}$—$\frac{3}{4}$ of an inch of the corallum, the branches, where they are broken off, are dead and these corals, when placed in the bottles, easily expanded. The animals in the "auftrieb" gradually became less active, until they moved but slowly over the bottom, so that the conditions should have been eminently favourable for the polyps. The tide would not allow the polyps to be kept in the position I chose for more than four hours, so that in about this time the bottles were lifted and the corals killed by being dropped into a saturated solution of corrosive sublimate.

I examined eight calices, thus fed, on five branches of *Euphyllia gaimardi*; they all showed an abundance of unicellular algæ and were mostly stuffed with pieces of green seaweed. The eggs and larvæ of Mollusca and Echinodermata were common in all, and in some also fish eggs; there were a fair number of Radiolaria (9) identified and also the remains of some chætopod larvæ. In addition, one Nauplius larva, one Appendicularia larva, two *Sagitta* and two *Atlanta* were observed. In seven calices of the *Mussa*, experimented on, unicellular algæ and pieces of seaweed were very common. There were also identified two *Ceratia*, one Radiolaria, one crustacean (apparently *Nebalia* or a larva allied to it), some Protozoa like *Noctiluca* and the remains of a few Medusæ. All the "auftrieb" in Rotuma was very rich in crustacean larvæ, and *Sagitta*, but these, although they had nearly all sunk, when the bottles were lifted, did not seem to have been consumed by the polyps.

In accordance with a suggestion of Prof. Hickson, I took especial note of the colours of the different species of corals. On our arrival at Funafuti the whole party was struck with the colours of the *Pocillopora* living on the outer reef; some were a brilliant pink, others an equally brilliant green, and others almost colourless. Closer examination showed that the latter had their calices coloured light green also, and I found that the green coloured forms had the polyps expanded and the colourless contracted, and so I concluded that, when contracted, the green forms would be almost colourless. There was one species, living in the fissures of the reef, which showed all three appearances, pink, green, and colourless; I brought back more than a dozen pieces of it, to see if they were different species, but all can be identified with *Pocillopora grandis* (Dana). I fixed two bottles firmly on the top of two branches of one of the green forms with tow round their necks and two also on branches of one of the colourless forms. Three days afterwards I found at the top of each bottle on the green forms a few cubic centimetres of gas, and about one centimetre at the top of the other two bottles. The gases from the first two bottles added together seemed to make the spark at the end of a match glow and so to indicate the presence of oxygen; the result from others was unsatisfactory. I soon after had to leave Funafuti and unfortunately at Rotuma this species was only once obtained. Other *Pocillopora*, most of which are green, gave for a long time unsatisfactory results, as it was hard to remove this coral from the reef without damaging it, and the species were too small and too exposed to fix bottles over them. Finally I placed in bottles six pieces belonging to one or more of the following species of *Pocillopora*, between which at that time I did not properly distinguish:—*P. verrucosa, P. lacera, P. lobifera,*

and *P. brevicornis*. These bottles I fixed upside down with open mouths in a pool off Solkopi, where there was a free circulation of water, but no sand or dirt; at low tide they were covered by 4 feet of water and their position was one in which they would be fully exposed to the light. At the end of a week five bottles were happily unbroken, and of these, four had a small amount of gas in each. The gas was then all collected together into a tube, which was next held in the water over a small piece of metallic sodium in a perforated tin box, until twice the quantity of hydrogen gas was added to the other gas. The tube was then removed and, on being turned up, a lighted match was placed at its end and, the thumb being removed, a distinct explosion resulted. From the four pieces of *Pocillopora*, only about 8 cms. of gas were obtained in a week; the same experiment, however, carried out for a like time with four pieces of *Prionastræa* (the species being taken from the *rim* of the reef, but not distinguished specifically) gave upwards of 15 cubic centimetres of gas, which exploded in a like manner. From *Euphyllia gaimardi*, over the polyps of which I fixed the bottles without breaking their stems, I obtained an even greater amount of gas, into which I placed a glowing match, which at once burst into a flame. *Madrepora*, if broken, gave very little gas, but over some of the branches of a species which grows very abundantly on the reef-flat, and in the pools off Solkopi, I fixed bottles; the gas given off, though not as great as from the *Pocillopora*, also gave an explosion. I furthermore proved that gas was given off from a species of *Fungia*, and I had traces from several *Favia* and one *Stylophora*; my experiments, however, on these genera were unfortunately shortened by a strong gale in the end of November, 1896, which broke all my bottles.

Pavonia divaricata and *Pavonia cristata*, which were also very abundant in the pool by Solkopi and over which I fixed bottles, gave negative results, as also did some experiments on *Porites* and *Astræopora* in the boat channel. *Heliopora* and *Millepora* abounded in the boat channel, but no gas from them was obtained, although they were especially carefully managed.

On the whole the experiments were not altogether satisfactory, as the amount of gas, given off by different pieces of the same species, varied much more than the sizes of the pieces did. All the reef species of *Prionastræa* and *Pocillopora* were, I believe, experimented on and never failed to give a certain amount of the gas, but the different species of *Favia* were uncertain. It would have been interesting to discover how much of the gas given off was oxygen, by exploding it by means of a battery, but experiments to this end were unsatisfactory owing to the want of proper apparatus.

The spirit material I brought from Rotuma is unsatisfactory, so far as I have examined it. Some sections, however, of *Euphyllia* and *Prionastræa* polyps show apparently the presence of unicellular algae in the endoderm and structureless lamella.

Prionastræa and *Pocillopora*, the two genera which gave by far the best results, both live on the rim of the reef and are uncovered, though never dry, at low tide. Should the giving off of oxygen, as my experiments seem to indicate, be confined to certain genera it would be of interest to note their distribution on the reef, and in depth. That oxygen is given off, I do not believe, to be due in all cases to parasitic algæ; in the genus *Madrepora* I have completely failed, so far, to find any such, and while the species of this genus only gave off small quantities of gas, all the species yet gave off a certain amount. Experiments at Wakaya, Fiji, on species of *Pocillopora* and *Prionastræa*, fully confirmed my results in Rotuma, though the quantity of gas seemed relatively less. I do not know how far the gas in some cases may not have been due to the algæ in the cœlenteron, but I do not think this factor is of any importance. As the species of *Madrepora* and *Pocillopora*, obtained from depths off Funafuti up to 35 fathoms, all had their corallites distinct to the eye, like pins' heads, when fresh, and of a dark greenish colour, it seems to me probable this assimilation of carbon from carbonic acid gas is a very important factor in the growth of corals and hence in the formation of coral reefs.

SECTION IV.

Conditions affecting the Growth of Coral Reefs.

Most of what I have now to draw attention to under this heading has been already anticipated in my descriptions of the different reefs. I have, in connection with many of the reefs, referred to the effects of currents, of which two kinds must be distinguished, namely, tidal and ocean; the first are usually shallow currents, but the latter are both surface and deep, and do not, I think, vary much in the central and western parts of the Pacific with moderate winds. The tidal currents show their influence mainly in the passages of reefs and inside reefs, while the ocean currents affect rather the growth of the whole reef. In fringing reefs the passages are generally free from corals, which nevertheless sometimes grow very luxuriantly, where the tidal current commences to spread out over the whole boat channel. It is in this position, where the current slackens, that sediment would

be deposited, rather than in the passages themselves in which the absence of corals is more probably to be accounted for by the inability of the larvæ to fix themselves. The ship's entrances at Fulanga and Lakemba in the Lau Group of Fiji, where tidal currents of 4—5 knots an hour are met with, are very free from corals, which yet live in great luxuriance off their entrances as deep down as the bottom can be seen. At Funafuti, in the ship's entrances, the genus *Millepora* is extremely abundant, but true corals are rare.

With regard to ocean currents, which, in some groups of islands, are very powerful in certain positions, I think that they probably modify considerably the form of the whole reef, by preventing the proper fixation of the larvæ both of corals and nullipores, and by the distribution of the talus on the reef slopes. The form of the channel, between Wakaya and Ovalau, is probably due to this, the slope being very sharp down to 40 fathoms off each island, where the steep is reached, the reefs on either side running also very evenly, and with few projecting points. Such currents as cannot pass this way must pass to the south of Viti Levu, and all along the coast here the reef lies close to the land. The current between Taviuni and Vanua Levu is not altogether tidal, but probably has had a still greater effect on the reefs. I cannot entirely agree with Semper[1], when he states his opinion that currents are one of the principal causes which have given the reefs their very remarkable forms. Given a bank at a suitable depth, I think that it would require a very powerful current, a current such as is only found in narrow straits between islands, to prevent a reef from forming on it in suitable latitudes.

Where currents are not too strong, they assist no doubt very materially in the formation of the reefs in some way. Most atolls have their land to windward and their passages to leeward; Charybdis reef to the north of Viti Levu, a case in point, is well protected against the wind, but its reef is, apparently owing to the current, especially well formed to the east but quite open to the west. In the Gilbert Group the current and wind both come from the east, on which side also the land generally lies. In the Marshall Group, the current sets east and the prevailing wind is east-north-east, the two being thus opposed to one another; the atolls have therefore land both on their windward and leeward sides.

At Rotuma corals are found in greatest abundance in the boat channel at points intermediate to passages in the reef. Before I observed this, however, I removed specimens of a number of different species down the coast for about half a mile to a position almost opposite to a big passage—from an area very rich in corals

[1] *Animal Life.* By Carl Semper, 1881, p. 229.

to one proportionately poor—so as to observe them constantly and have them handy to kill, when they were properly expanded. I fixed them between stones on the sandy bottom, but they soon became silted up with sand and were killed. I then placed a number of pieces on the hollowed out top of a species of *Astræopora*, but debris soon collected round them, and, when I last visited them, I found only the clean coralla without polyps. Generally I could not, in the boat channel, get any true coral to grow if placed on the bottom owing to the sand and mud deposited on them, but all would grow if not too near the passages, on the tops of the stone heaps, which are built up by the natives for fish trapping. In the few fringing reefs, with boat channels, which I examined, in Fiji it was unusual to find any corals growing inside the reef, and at Rotuma the presence of corals is largely due no doubt to the presence of these stone heaps, on which the larvae fix themselves above the level of the scour of the tide.

When I saw that the corals on the *Astræopora* were being killed, I tried transferring all the commoner species out of their own peculiar zones into other zones. I also wounded a certain number of corals over a definite area to see how soon their wounds would heal and be grown over. Of course in the limited space of three months no decisive experiments could be carried out, the results are yet not without interest. Broken-off arms of branching species and small pieces of massive species all died. Branching forms removed by the root and large blocks of massive species, if kept in their own proper zone, continued to live. Corals could be removed from the rim to the reef-flat; from the nature of the reef it was impossible, however, to try the opposite experiment. Some species from the reef of *Pocillopora* and *Prionastræa*, fixed on the top of a stone heap near one of the passages, seemed to flourish, but species of *Madrepora* and most other genera died. In no case was there any sign of the coral attempting to fix itself. The polyps near the damaged part of a colony died, and there was no attempt in the time to heal over the scars, which simply gave an easy entrance to boring animals. The experiments with *Millepora* and *Heliopora* were inconclusive; their lamellae fixed on the hollowed out top of the *Astræopora* certainly seemed alive after three months.

I never saw the corals over any considerable area, or individual corals, killed by the heat of the sun at low tide. The corals on the rim, at even the lowest tides, are constantly wetted by spray, and on the reef-flat the water remains constant in height. In the boat channel at Rotuma, and on certain shoals at Funafuti, I noticed species of *Montipora*, *Porites*, *Favia* and many other genera exposed for upwards of three hours at low water without any effect on them. Nearly, if not all, had some part in the water, and doubtlessly

water can be passed on by diffusion, or in some other way, from polyp to polyp. At spring tides at Rotuma, parts of the *Heliopora* and *Millepora* clumps in the boat channel were exposed for four hours. I examined them when they were thus exposed, and also on my return from the reef after the tide had turned, when they were just covered, and found that all the polyps of the *Heliopora* were expanded fully and those of the *Millepora* partially so. I found, too, that true corals also tended to partially expand on being immersed again.

The effect of rivers is due more to the sediment which they carry down in suspension, than to the freshness of their water. Yet at the chief mouth of the River Rewa, the bottom is hard from the edge of the reef for some distance up the river, and corals are absent from the whole of the inner part of this area. The streams on the east of Taviuni vary enormously in size and very quickly run down; the reef outside, where it exists, is well formed and covered entirely above the low water level by *Lithothamnion*. The fresh water, being lighter, must spread out in a sheet, when it meets the ocean, and yet the reefs, in the neighbourhood of the Rewa, Navua, Penang and Lambasa Rivers, are all well developed and awash at low tide.

SECTION V.

The Rate of Growth of Corals.

I had an opportunity of assisting in cleaning the chain of a buoy, which had been down in Levuka Harbour, Fiji, for one year and ten months. The first, or upper, fathom of chain had no corals, but was covered completely with Tunicata, bivalve Mollusca and sponges. In the next two fathoms I obtained six different coral growths which with their weights were as follows:—

1.	*Turbinaria*	45 grams.
2.	*Pocillopora*	41 and 13 grams.
1.	*Heliastraea*	34 grams.
1.	*Madrepora*	20 grams.
1.	*Galaxea*	Two corallites weighing nearly 1 gram.

A rusty iron chain is not likely to be a good foundation for the corals to fix themselves on, so that these growths in this space of time are most remarkable.

PART VI.

The Formation of Coral Reefs.

The generally accepted theory of the formation of atolls, before Darwin brought forward his well-known theory of subsidence, was that they were situated on the rims of submarine volcanoes. It is possible to explain the formation of atolls quite satisfactorily on this theory, if such craters really can and do exist at the requisite depths. Prof. Sollas, without expressing an opinion, has drawn attention to the striking similarity of the slopes of Funafuti to those of a volcano[1]. "The conical mountain below the 140 fathoms line, with its parabolic slope is suggestively similar to a volcano; but if so its crater must have been immense, 10 miles across at least." Many arguments can be advanced against the possibility of there being such large craters and such a large number, as would be requisite to account for the foundations of the atolls of the Indian and Pacific Oceans. No volcanoes with craters even as large as the medium sized atolls and atoll-reefs exist at the present day; and there are no indications, except in the Andes, of such large chains of volcanoes as would be requisite to form the Ellice, Gilbert and Marshall Groups, which form a line as if of one mountain chain. Another great objection to this theory lies in the fact that it leaves the formation of barrier reefs completely out of account, and anyone, who has examined both atoll and barrier reefs, cannot fail to have been struck by their striking similarity to one another. Again, there is found every intermediate variety from the narrow fringing reef to the atoll with perhaps the greater part of its rim converted into land.

It is, however, very doubtful whether a submarine crater could be formed as such; lava meeting the water would usually break up into loose vesicular cinders and ash. Falcon Island, Tonga, is said[2] to be "just a bare brown heap of ashes." Sir A. Geikie mentions[3] Graham's Island, Sicily, erupted 1831, Sabrina Island, Azores, 1811, a new island in the basin of Santorin, 1866, and an island formed in 1783, 30 miles west of Cape Reykjanaes, Iceland, all of ash, and either completely worn away or being rapidly destroyed. The only permanent cone, known to have been thrown up by a submarine eruption, is the island of Johanna Bogoslawa,

[1] *Loc. cit.*, p. 375.

[2] "The Newly Emerged Falcon Island." By J. J. Lister. *Proc. R. G. Soc.*, March, 1890.

[3] *Text-Book of Geology*, 1893.

Alaska, 1796; it is 3,000 feet high. Captain Dawson, H.M.S. *Waterwitch*, informed me that in the eruption of Ambryn, New Hebrides, in 1895, the lava where it met the water in its flow, burst into fine cinders and ash; of course under the ash the lava might possibly still flow. Further, so far as I am aware, no deep banks of crater-form have yet been discovered in the sea. The foregoing is sufficient evidence to make it very doubtful whether craters can be formed under the sea, and if the foundation of any atoll should be on the top of a crater; I consider it can only have been thus formed owing to the subsidence of the crater. The island of Totoya, about 1° due west of Kambara, Fiji, is the remains of an extinct crater, 3 miles across and 1184 feet high, with the south side blown out; it now has a depth of 30 fathoms of water in the original crater. If it subsided 1300 feet, an almost perfect ring of coral would grow up, but it would take a very long time for the crater to fill up to 30 fathoms, the usual depth of an atoll; and whether it could ever do so, on account of its opening to the south and the consequent scour inside, is doubtful. I can find no atoll or horse-shoe reefs with depths in any way approaching this, and I conclude that it is absolutely necessary to reject this theory in its entirety, as applied to any atolls.

Darwin's theory has, however, a great advantage over all others which have before or since its publication been put forward, in that it explains all the different forms of reefs on the same hypothesis. There can be from the nature of the case little direct evidence brought forward of subsidence. "There is," Dana writes[1], "not merely probable but positive evidence of subsidence in the deep coast indentations of the high islands within the great barriers. The long points and deep fiord-like bays are such as exist only where a land, having been deeply gouged by erosion, has become half submerged." Such evidence as applied to volcanic islands is, I submit, of very doubtful value. Viti Levu and Vanua Levu, according to Dana's views, must have subsided at least 1000 feet, and, as there are practically no atoll or barrier reefs with land on their rims in the Fiji Group, it must be still subsiding slowly; yet, although the coast is by no means an even one, a subsidence of even 100 feet would make a very great difference in the direction that Dana has suggested. The coast, too, of Australia inside the Great Barrier Reef certainly does not show the characters that Dana propounded.

Saville Kent has brought forward an argument for subsidence from the distribution of the animals, now peculiar to Australia and the neighbouring islands. He draws attention especially to the Darling Downs deposit in which the bird remains find their

[1] *Loc. cit.*, p. 273.

nearest allies in New Zealand, of which one species is classed directly with the genus Dinornis[1]. The deposit is referred to an epoch not later than the early Pliocene. The time at which the Great Barrier Reef of Australia originated is placed "somewhere about the period of the later tertiaries." The Rev. A. H. Cooke, in speaking of the distribution of the land Mollusca of the Melanesian Province, including the New Hebrides, New Caledonia, the Loyalty and Fiji Islands, remarks that "these islands are very probably the remains of what was once a much wider extent of land[2]." The arguments to be deduced in favour of subsidence from the distribution of animals are worthy of every attention. If the views of Saville Kent and Cooke are correct the origin of coral reefs must be, comparatively speaking, either quite recent, or the subsidences must have been very rapid and the traces of all the former reefs have been obliterated, or, finally, the origin of coral reefs must be put down to some other cause than subsidence.

Again, the subsidence, required by Darwin's theory, must be slow and long-continued to form the atolls and barrier reefs of the Pacific. All the evidence of movement in this ocean such as it is, decidedly points, however, to the movement as being rapid and local. There are many local upheavals, and conversely, if local upheavals occur, there should also be taking place in a similar manner local subsidences. The evidence does not show that these elevations have been slow, but rather in sudden steps. I have already given my reasons for thinking that the upheavals, which formed the several limestone islands of the Lau Group of Fiji, were rapid ones. The characters of the raised reefs of the Solomon and Tongan Islands as described by Guppy and Lister also go to show that the upheavals, which formed these, were probably rapid. The terrace formation of Christmas Island[3], in the eastern part of the Indian Ocean and of Cuba, is not what one would expect to find if the elevation had been slow; there would rather be required a series of sudden upheavals with, between these upheavals, periods of rest. Conversely there should also have been sudden local subsidences with long periods of rest. I have examined all the charts of the islands in the Pacific Ocean, and of no island can I find any indication of such subsidences in the soundings off them. Professor Bayley Balfour, in his description of the island of Rodriguez[4], mentions the existence of what he believes to be an old reef at a depth of about 90 fathoms; it must be remarked in connection with this that he considers that the present reef rests on it, but whether directly, or by an outward

[1] The Great Barrier Reef of Australia, p. 136 *et seq.*
[2] Mollusca in the *Cambridge Nat. Hist.*, p. 323.
[3] *Dana, loc. cit.*, p. 274.
[4] *Phil. Trans. R. Soc.*, CLXVIII., 1879.

extension of its talus slope he does not say. The sections of the sea bottom off Masamarhu Island[1] in the Red Sea can only, I hold, be taken as indicating sudden and rapid subsidences.

The limestones 4,000 to 6,000 feet thick mentioned by Dana[2], with similar fossils from top to bottom, afford no evidence to show that there have been to form them "slowly progressing subsidences" of 4,000 to 6,000 feet. I do not contend that these have not been formed by subsidence, or that the subsidence was not a slowly progressing one, but I do protest against the too rapid conclusions, which fix at once the nature of the bottom, and the depth at which the organisms lived, without allowing them any power to adapt themselves to changed surroundings and conditions of life. An absolutely vertical thickness of coral limestone of more than the 1000 feet of Vatu Vara has, as far as I know, not yet been proved; the real thickness to the volcanic rock indeed may be considerably less. Dana and others do not show of what constituents the raised limestone reefs are formed. Hence it is purely an assumption to suppose that the whole thickness of any of these raised reefs has been formed mainly of coral in the way which the subsidence theory requires; there is no more evidence that their lower parts are formed in this way than, for instance, that they are formed by the fragments of a talus slope, which have been consolidated together.

Dana explains the absence of fringing reefs and shoals inside barrier reefs on the supposition that the conditions are much more favourable for the latter than the former. "If the barrier increases one foot in height in a century, the inner reef, according to this supposition, would increase but half a foot; and any rate of subsidence between the two mentioned, would sink the inner reefs more rapidly than they could grow and cause them to disappear[3]." According to Dana's view, as already mentioned, the Fiji Group forms an area, where subsidence is still going on. It must be remembered, however, that in Fiji the land is fringed with a reef inside the lagoons of barrier reefs, where the island has a broad stretch of water between the barrier and itself. The great luxuriance of the reef east of Wakaya and the volcanic part of Vanua Mbalavu is very noticeable, and yet on the subsidence continuing both of these are to disappear. Coral grows in the localities I have visited far more luxuriantly inside the lagoon than where the reef is exposed to the full force of the ocean. Bourne remarked on the rich growth in the lagoon of Diego Garcia, in positions where sand is not deposited, and at Funafuti it was apparent to all the members of the first expedition

[1] *Nature*, xxxvi., p. 413.
[2] *Loc. cit.*, p. 278.
[3] *Dana, loc. cit.*, p. 265.

how much more luxuriant the growth of coral was in the lagoon than on the outer reef. Study of the charts of such islands of the Pacific Ocean as have been surveyed and other information, collected by me, has shown me that shoals in lagoons are generally nearly awash, and thus a conclusive proof is afforded, if we accept the theory of subsidence, that at present the whole Pacific Ocean must be in a period of rest. When the barrier reef is far distant to windward, the sea may have great force within the lagoon, the inner reef be well formed and somewhat strewn with boulders. Yet the building-up power of the coral is so small here, or the subsidence has been so rapid, that nowhere are there to be found secondary barrier reefs, as it were, inside the primary reefs. There must be other causes inside the lagoon which prevent the formation of these, or, as I fully believe, the constituents of a barrier reef are radically different from those of an inner reef, which has grown up as such.

In my opinion, the effect of the waves on loose coral and shell materials, of which Darwin supposes the reef to be mainly formed should be noted. Dana states precisely his opinion on this point[1]. It serves to "widen the reef," the windward reefs being the widest, as more fragments would be heaped up by the heavier seas, the growth also being probably greatest to windward. Yet the windward reef, including the raised islands of Funafuti, is narrower in many places than the leeward reef, and the same is often the case in Fiji. All the windward reefs in Fiji are of about the same breadth, and indeed through the whole Pacific barrier and atoll reefs to windward show a striking uniformity among themselves in width. The material then is "to steepen, as a consequence of the widening, the upper part of the submarine slopes." The sections off the barrier of Tahiti, mentioned by Murray[2], and the sections run in 1889 by the *Egeria* off Sydney and Canton islands in the Phoenix Group, and by the *Penguin* in 1896 off Funafuti[3] all show a fairly even slope to about 40 fathoms at about 250 yards from the edge of the reef, and then a sudden precipitous fall of about 75°. My own observations off the Fijian and Rotuman reefs show the same characteristic slope to about 40 fathoms and then a sudden fall. This steep would seem to be a feature of all reefs, rather than one due to particular reefs being in a condition of rest in a general area of subsidence. Where the deposits from corals and shells do not exceed greatly the loss by subsidence and current waste "the atoll reef should lose in irregularity of outline and thus approximate towards an annular form." The effect of this should be seen in the so-called "sunken"

[1] *Loc. cit.*, p. 270.
[2] *Dana, loc. cit.*, p. 282.
[3] *Nature*, Vol. LV, 1897, p. 375.

atolls, such as the Coombe, Waterwitch and Alexa banks and in the atoll-reefs in the Lau Group of Fiji. Some atolls too might have been supposed to have passed through this condition, nevertheless the study of the charts of the atoll-reefs, in comparison with the barrier reefs of the Pacific, has completely failed to show in them any increased regularity of outline, or any approach to an annular form. The smaller atolls inside the larger atolls of the Maldive Group are of a similar character to the atoll shaped shoals inside the Great Barrier Reef of Fiji; here the conditions, which Dana suggests, would naturally be expected to exist with a like effect to follow. However, I do not think that it can be maintained that these are the effects of the debris of the outer reefs at the present day, or have been in the formation of the atolls of the Pacific in the past.

The reef debris would next fill the lagoon, or, failing sufficient contributions, the atoll bank would, according to Dana, lose its channels into the lagoon and "become at last a small bank of reef rock with a half obliterated lagoon basin." The observations of Admiral Wharton are summed up on this question, when he remarks[1], "I have no hesitation in saying that a flat floor is an invariable characteristic of a large atoll and I cannot find his (*i.e.* Darwin's) 'deeply concave surface' in any large atoll. On the contrary, a flat surface is found in all these, whether the rim be above, or below the surface." The study of the charts of the different islands and barrier reefs of the Fiji Group show that, while there may be abundant shoals, the bottom between is fairly even. The fall from the reef, and then from the shoals to the bottom of the lagoon, is cliff-like. The depth towards the side of the lagoon gradually lessens, and in no place which I examined, except where the sand bank runs off the main island of Funafuti, was this cliff absent. The bottom beyond the cliff is usually visible and generally covered by sand. By the reef it does not show masses of debris, nor is debris being piled up and gradually broadening the reefs, which are awash. There are plenty of banks known in the Pacific in all conditions of submergence and all seem to be of true atoll-form, none occurring with "half obliterated lagoon-basins." It is possible that such constructive agencies inside atoll-reefs may be in excess of the destructive, but it is more probable that the widening of the reef within the lagoon takes place by the growth upwards within it of shoals, formed by means of coral and other lime-secreting organisms, and their gradual fusion with the reef itself; this is what appeared absolutely to be going on, as previously stated, inside the Great Barrier Reef of Fiji and the lagoon of Funafuti.

[1] Foundations of Coral Atolls. By W. J. L. Wharton. *Nature*, Vol. LV, 1897, p. 390.

Where land is formed on the reefs of lagoons, other agencies, such as the deposition of sand, drift, etc. come into play, and the whole lagoon may be finally converted into land. The Ellice Islands show the stages of this conversion, which depended largely on the size of the atolls, and the number and depth of the passages through their rims. Funafuti is the largest, and has six deep passages, one of 24 fathoms, and a general depth of 25 fathoms in the lagoon, with only a deposition of sand in one place off the large island to windward. Nukufetau has only one passage, the depth of its lagoon is about 22 fathoms, with a sandy bottom in most parts. Nukulailai has more land, broader reefs, no passages, and a depth of 10 fathoms with sandy bottom. Oaitupu is nearly all land, there being only a small lagoon with a depth of 1—2 fathoms and a narrow channel to the sea at its north end; this island is completely covered by sand, except along the coast. Niutau, another sandy island, has in its centre a small mangrove swamp, which is dry at low tide. Gente Hermosa, one of the Union Islands, is of interest, having in its centre a lagoon a few hundred yards across with precipitous sides, a depth of 5—6 fathoms and a small island in its centre; there is also, slightly to the north, another smaller lagoon with a similar depth; unlike the other islands which are covered with sand, the solid rock is here continued from the reef to the lagoons, and it is probable that the atoll owes its land to a different mode of formation[1]. There is no evidence that the channels of the atoll and barrier reefs are filling up in Fiji, but if they are doing so, it is certainly not by the accumulation of debris in them, but rather by the growth upwards and inwards of corals and nullipores on their bottoms and sides.

Lastly, the waves "accumulate on the reef material for beaches and dry land." Certainly in the boulder zone at Rotuma an accumulation of this kind is going on; it is also especially noticeable on the reefs to the south-east of Viti Levu and on the Mumbolitha reef to the south of Ngau. Most of this debris comes, according to Dana, from the rim and outer portion of the reef, but even narrow fringing reefs have these parts, and yet the land they surround does not show much debris piled up, or any extension outwards by its means. Such an accumulation may take place but, in my opinion, both Darwin and Dana have greatly overestimated the amount of the debris thrown up by the waves.

Semper, Murray, and other observers, lay great stress on the presence in the same group off different sides of the same islands of fringing and barrier reefs. Semper[2], dealing with the Pelew

[1] I am indebted for this information and much of that about the Ellice Group to Capt. Wilson.

[2] *Animal Life.* By Carl Semper. 1881. Chap. VIII.

Islands in particular, considered the fact of great importance, and discussed very fully its bearing on the theory of subsidence joined with the complicated system of up and down movements, that would have been required under this theory to explain their formation. For the Fiji Group Moore[1] and Murray[2] have already dealt with Kandavu. Wakaya has to the west off its highest summit (590 feet) a narrow fringing reef, but elsewhere a broad barrier reef; the land has cliffs in accordance with this to the west, but toward the east near the sea there is a gradual slope of about 10°. Assuming the slope to be only 5°, Dana's more moderate estimate, the thickness of this reef would be 1840 feet, and the cliff to the west, if we accept his theory of subsidence, must have been 2345 feet high. Mbatiki, Ngau, Nairai, Makongai, Moala and Matuku, all of which are volcanic islands, slope to the sea fairly evenly on all sides, and show every variety of reef from a narrow fringing to a well-formed barrier. Lakemba slopes evenly from its two central peaks; its reef to the west fringes, and to the east is a barrier, inclosing a lagoon 5 miles broad. The greater part of the coast of Taviuni has no reef at all or only a narrow fringing one, yet a barrier reef lies off Vuna to the south, and to the north-east, including also the islands of Ngamia and Lauthala; northwards there is a regular line of shoals enclosing an area with a depth of 40—50 fathoms. The slope from the hills of Taviuni is in all directions very uniform, and it is inconceivable that a subsidence of this kind of 300 feet (calculating it only from the depth of the water inside the reef to the north), has left no trace of any reef along the greater part of its coast. There is no difference between the appearance of the hills north and south of Vanua Levu, or in their rock; there is, however, a far distant barrier reef to the north and along the greater part of the south coast, a narrow fringing reef.

I submit that the above observations afford strong reasons against Darwin and Dana's theories of the structure and formation of coral reefs in general and of those of Fiji in particular. As Dana spent three months in the Fiji Group, and had a very unique experience of coral islands, I have given his account more weight than that of Darwin, which has the priority. Although it is very unlikely that subsidence has been one of the determining features in the formation of the reefs of Fiji, or of most of the other reefs of the Pacific, I consider that, if a slow subsidence as is suggested, or a series of rapid subsidences, as would be more probable, took place, barrier and atoll-reefs would undoubtedly result, not, however, of the internal structure as suggested by Dana. My opinion is that the growth of the debris and sand is not so great, as Dana has suggested, and

[1] *Nature*, XL., 1889, p. 203. [2] *Nature*, XL., 1889, p. 222.

accepting the theory of subsidence for the moment, the wonder would be that the atoll and barrier reefs have not greater depths than even that of 70 fathoms found on the Saya de Malha bank, north-east of Madagascar.

I have already, in treating of the Lau Group, referred to the great destruction of the limestone islands, which is continually taking place by means of the wearing and dissolving actions of the sea water. The fact of such a solution occurring here cannot be doubted; even in the most protected parts of the salt lake of Fulanga, which is most completely inclosed, the rocks everywhere show the undermining action of the sea. The narrow edge of reef under the overhanging limestone does not extend beyond the edge of the limestone, and is evidently also being dissolved. There are many fallen masses of rock, and islets have slipped over completely, while there are no growing shoals, so that it seems as if they were being dissolved away, until they became level with the general flat of the lagoon. Corals and nullipores do not grow in the lagoon, and there is nothing to make up for the continual waste from solution. Murray and Irvine[1] have shown that one part of calcspar is soluble at 10° C. in something over 136,000 parts of sea water; three corals, taken at the same temperature, belonging to the genera *Goniastraea*, *Melobesia* and *Oculina* were soluble one part in from 12,000 to 43,000 parts of sea water. The soft limestone of an atoll is hence probably about three times as soluble as the hard limestone of the Lau Group, so that the formation by solution in the middle of Fulanga of a lake 10 fathoms deep with only one narrow tidal passage, furnishes a strong argument that solution must be of immense importance in the formation of atoll and barrier reefs.

The experiments of Reid, quoted by Murray and Irvine[2], show that, under pressure, carbonate of lime is more soluble in sea water containing carbonic acid gas. He took about 1 gram of the solid nearly pure carbonate of lime, forming the skeletons of different organisms, and tested how much of it would be dissolved by 1 litre of sea water, containing carbonic acid gas. His results are of great importance, and his table may be briefly summarised:

	Experiments.	Pressure.	Average Amount dissolved.
Globigerina Ooze	4	4 tons	·1166 grams
„	1	2 tons	·0999 „
„	2	Atmosphere	·057 „
Coral Sand	2	4 tons	·1205 „
„	3	Atmosphere	·0418 „
Crystalline Limestone	3	4 tons	·0339 „
„	5	Atmosphere	·0067 „

[1] On Coral Reefs and other Carbonate of Lime Foundations in Modern Seas. *Proc. R. Soc., Edin.*, 1889.

[2] *Loc. cit.*, Table xvi.

The extent to which solution has been going on is very great in the Lau Group of Fiji; and Reid has thus shown how much greater the action is under pressure. Given a bank at a suitable depth, nothing further is required to demonstrate how its lagoon has been formed; the edges being covered with living organisms would be protected while the depth of the lagoon would be regulated by the amount of decay of animal matter giving off carbonic acid gas, by the amount of carbonate of lime deposited on its floor, and by the extent to which the currents passing in and out of the lagoon renew its bottom layers of water. At the same time, in large atoll and barrier reefs, I would emphasize the importance also of these currents in removing sand and silt. Both effects of the current depend largely on the number, size, and depth of the passages; of these the depth is of most importance. Considering how rarely a depth of over 50 fathoms is obtained within barrier reefs, and how common depths up to that are, there is probably to be found a mean, below which increase is greater than solution, and above which solution has the upper hand. The lagoon of Exploring Group of Fiji has depths up to 100 fathoms, with a passage equally deep, to which in this particular case the depth is no doubt due; this case, however, should be considered in connection with the raised limestone islands of Lau, the reefs of which will be treated of later.

Considering that, in Fiji, coasts can be found with no reef, or with a broad fringing reef, or with a far distant barrier reef, and indeed with every intermediate condition between these, and that there are many atoll-reefs such as the Charybdis, the Horse-shoe and the Mumbolitha, I can only reiterate Murray's conclusions, summed up by Professor Bonney in these words[1]: "That when coral plantations build up from submarine banks they assume an atoll form owing to the more abundant supply of food to the outer margins and the removal of the dead coral-rock from the interior portions by currents and the action of the carbonic acid (gas) dissolved in the water."

"That barrier reefs have built out from the shore on a foundation of volcanic debris, or on a talus of coral blocks, coral sediment and pelagic shells, and the lagoon channel is formed in the same way as a lagoon."

As mentioned before, my opinion is that the reef is formed rather by the growth of calcareous algæ of the genus *Lithothamnion* than by either the direct building action of corals, or the consolidation of their fragments. Hence the formation of a rim would be due rather to the constant replacement of the sea water outside the reef by fresh sea water renewing the supply

[1] *Coral Reefs.* By Charles Darwin, p. 286, 1889.

of carbonic acid gas, than to the more abundant food supply. Undoubtedly the better food supply would be advantageous to the corals so that these would flourish also, but my experiments have shown that possibly the fresh supplies of carbonic acid gas are here too the more important factor.

My reasons have already been given for thinking that reefs are growing outwards at about the low tide level, and the deep dredgings off Funafuti showed that stones and fragments of coral rest on the steep slope, while there is an area of about 250 yards in width outside the reef before the steep commences. The uniformity in width of this gives evidence, I think, of the depth to which the greatest effective wave action extends, and also gives approximately the depth at which nullipores, the chief reef-builders, flourish. I found at Rotuma that there is over this area, when the tide commences to rise as well as when it is falling, a distinct undertow from the edge of the reef. I found that if I threw a fishing line with a heavy weight attached from the edge of the reef it was not swept back again to me. Making a further experiment by substituting a large round pebble for the lead, the whole line of 60 fathoms was quickly drawn off its stick by the strain; it was necessary, however, to throw the pebble a distance of about 30 yards out, as otherwise it was simply washed in at my feet. I never repeated the experiment, but the absence of pieces of reef debris in my dredgings from this area leads me to believe that there is really such a backwash, which sweeps the slope clean, and piles up the talus to form the steep.

The precipitous character of the reef towards the lagoon, and the mushroom-shaped heads of coral are, no doubt, due to solution. This theory implies that the reef, as fast as it is formed outside, is dissolved away inside as these barrier and atoll-reefs are fairly uniform in breadth. The solution forms the cliff, and undermines the shoals; the sand at the base of the cliff represents the debris thrown over the reef, and is in this position reformed as quickly as it is dissolved.

The rate of growth of the reef outwards depends on many considerations, foremost amongst which must be placed the depth of the ocean floor. The reef, as it grows, has to build up its foundation, and, until it has built up this to the requisite height, it cannot grow outwards; hence naturally the reef should extend out fastest where the slope is smallest. Again, deep currents running along the shore would naturally clean the steep of its talus, where it projects, and so the growth outwards should be slowest here. Thus currents would act in such a case in opposition to the slope, and the result would depend upon which was the stronger. The reef also should grow faster on the side on which the current runs, as it would constantly change the surrounding water. The waves on the

windward side would do the same, and, where the wind and current are in opposition, the reef would be growing equally vigorously on all sides, as is found to be the case in the Marshall Islands.

It is not, of course, necessary that all barrier reefs should have gone through the fringing stage to the barrier condition. They might equally well grow up on any bank off an island, which is within the requisite depth for the reef-building organisms. I believe that the Great Barrier Reef of Fiji and the reef, which bounds the deep pool between Viti Levu and Vanua Levi, has been thus formed. The reef round this pool is by no means complete, and if it grew out from a fringing reef, there is no reason why it should not be moderately so. There are, also, off the north of Vanua Levu a large number of islands, and the line of mountains, of which the Yasawas are the emerged peaks, appears as if it extended round nearly in the line of the Great Barrier Reef. Should such be the case, the double nature of this reef would be at once explained. Indeed incrusting nullipores seemed to cover most of the surface of the inner part. These only grow where the water is constantly in motion, and to their protection outside from solution the reef, I think, owes its existence. This talus theory serves also to explain why terraces due to sudden subsidences are not found outside the reef off islands, in the same way as terraces are found in the raised reefs of the Solomon Islands, Cuba and other lands.

It is inconceivable that with the elevations which formed the Lau Group of Fiji there have not been other upheavals, which have brought areas within the depths, required by the reef-building organisms, but many of its atoll-reefs are probably due to different causes. After the reefs were first upheaved, it would take a long time, owing to the loose nature of the rock on the outside, for the organisms to get a firm hold. The result has been that solution has gone on outside to such an extent, that the whole thickness of the reef almost from its highest point to the sea level has been exposed as a cliff in many of the islands. After the reef was formed, the base of the island would be protected by it and its solution would then be very small. As the reef, however, spreads outwards on its own talus a lagoon would be formed and the solution of the limestone would at once recommence with renewed activity. The final result would be that the whole island would be washed away and an atoll-reef left. It is quite probable that the atoll-reefs of the Lau Group, particularly the larger ones, owe their origin to this. The Mbukatatanoa atoll-reef, east of Lakemba, with a depth of 35 fathoms has probably been formed in this way, and every intermediate stage from such an atoll-reef to high coral islands with narrow fringing reefs can be found in the Lau

Group. The following list of the coral islands with their heights and depths will make this still clearer:

Name of the Island.	No. of Islands.	Greatest Height.	Reef greatest distance from land in miles.	Depth lagoon in fathoms.	Situation of islands in lagoon.
Vatu Vara	1	1030	—	—	—
Naiau	1	580	—	—	—
Wangava	1	350	—	—	—
Tavunisithi	1	200	—	—	—
Mango	1	670	$\frac{1}{4}$	2	C (centre)
Tuvutha	1	800	$\frac{1}{2}$	9	C
Vanua Vatu	1	310	1	2	C
Fulanga	1	260	$\frac{1}{4}$	5	rim
Namuka	1	260	$\frac{3}{4}$	13	C
Ongea	2	300	1	12	C
Naiambo	1	40	$\frac{1}{2}$	3	C
Malima	2	130	$1\frac{1}{2}$	11	C
Komo	1	270	2	17	SE
Kimbombo	3	190	1	13	C
Yangasa	4	390	3	19	C
Vekai	1	30	$1\frac{1}{2}$	17	E on reef
Aiwa	2	210	2	23	S
Vanua Masi	2	80	5	20	C
Reid Reef	3	60	4	20	C

The lagoons inside all the barrier reefs in the above list are singularly free from shoals; this, probably, is largely due to the fine mud, caused by the solution of the limestone, killing the corals by which such shoals are mainly formed, and, as it coats the bottom, allowing few firm foundations for the larvæ to fix themselves upon.

The elevation of Thikombia-i-ra, which was probably the last upheaved of the limestone islands to the east of Fiji, raised with the reef a long bank to the east. On this side there is a fringing reef about 3 miles broad, while round the greater part of the island no reef has as yet been formed. On the bank a number of isolated reefs are now growing up and it appears as if a barrier reef is being formed. A long time, however, would be required for this to reach the surface, and so in the meantime a fringing reef has been formed, and, the depths being favourable, is pushing its growth rapidly over the whole bank, perhaps to form ultimately a barrier reef in quite a different manner.

According to Dana[1] "an atoll-reef without green islets, or with

[1] *Loc. cit.*, p. 356.

but few small spots of verdure" is "probable evidence of subsidence now in progress," and the existence of barrier reefs, atolls or submerged atolls, is an evidence of subsidence; altogether it is clear that Dana, with the present charts before him, would consider that the Fiji Group is, or has been up till a very recent period, subsiding. Support, however, is scarcely lent to this by the broad fringing reefs especially to the south-east of Viti Levu. At this part the Rewa river, after a tortuous course of nearly 130 miles, opens into the sea. I estimated the area of its mud deposits and its delta at about 80 square miles, all of which would be completely submerged by a subsidence of about 8 feet. A small river, the Navua, has a delta of about 20 square miles, and like the Rewa, is also largely fed by a region of soapstone; accordingly they might be equally supposed perhaps to form their deltas somewhat rapidly. But the Ba river in Vitu Levu and the Ndre Keti and Lambasa rivers in Vanua Levu draining only areas of volcanic rock would require a vast period of time to form their large deltas. The sand flats at Wakaya, Taviuni, and Fulanga, as noticed before, are very considerable, while by Nateva Bay and to north of Viti Levu they are in places 1—1½ miles broad; these would only be formed in a period of rest, and their formation can be but slow, so that they also point to the probability that movements of elevation and subsidence have not occurred throughout the whole group for a considerable period.

In the next place it is necessary to consider what are the foundations of atolls. Here I can only emphasize still further Admiral Wharton's views, when he says[1]:—"I would venture to suggest the cutting down of volcanic islands by the action of the sea and that this action has a far greater share in furnishing coral foundations than has been generally admitted." While agreeing mainly with these views, I do not know that the sea would have in all these cases merely a bank of cinders and ashes to act upon; lava might well rise to a certain height and then burst its way out above or below the water. The lava, when it first met the water, would probably burst up into cinders and ashes, but beneath these a crust would form and the lava might continue to flow[2].

At the same time I consider that mounds can be, and are being, built up to the requisite height for the reef-building organisms by the skeletons of marine animals. But what is the requisite height? The chief building organism is *Lithothamnion*, the bathymetrical zone of which must be limited to a large degree by the extent to which light can penetrate sea water. On this

[1] *Loc. cit.*, p. 392.
[2] *Geology.* By A. H. Green. Part I., 1876, p. 224.

question Prof. Hartley says[1]:—"The penetration of light to the bottom of the deep sea is a possible, if not a probable, phenomenon. It is true that in the clear waters of Lake Leman photographic plates are affected at mid-day at no depth beyond 120 fathoms; but Messrs Fol and Sarasin[2] are led to suppose that the greater transparency of sea water will admit of the extreme limit of luminous rays reaching a still lower level. We have accordingly no grounds for the conclusion that because chlorophyll has been found at a depth of 553 fathoms, even if the pigment be produced *in situ*, therefore light has not penetrated so far and has not operated in its formation." I do not wish to contend that the bathymetrical limit of nullipore life is to be placed at so great depth, but that these algæ grow down to 75 fathoms was shown by one of the dredgings made by the *Penguin* to that depth. The rate of growth of all algæ depends largely on the strength of the light, for on it their power of absorbing carbonic acid gas, and giving off oxygen directly depends. Light also is probably an extremely important factor to many reef-building genera of corals.

I can add nothing fresh to Murray's account of the methods by which mounds can be built up to the requisite depth, whatever that may be, but I would call attention to the fact that the growth of corals especially contribute towards this. The evidence, so far as it goes, shows that animals in the sea have a tendency to collect especially on any mounds; and I do not suppose that corals are any exception to this rule. In his report[3] on Madreporaria obtained during the expeditions of H.M. ships *Porcupine* and *Lightning*, Martin Duncan noted "that the growth of deep sea corals is very vigorous at great depths and at low temperatures," and "that the present deep sea faunas contain species and varieties of species of the Pliocene, Miocene and Cretaceous deep sea faunas and offshoots of the Palaeozoic fauna." Only 30 species were obtained, but four colonial genera were represented; of one, *Lophohelia*, Sir C. Wyville Thomson remarked[4] that it formed "stony copses which cover the bottom of the sea for many miles." Of *Dendrophyllia ramea*, obtained in the cruise of the yacht *Norna*, Saville Kent stated[5] that its growths "were massive and weighty as any of the most solid branching *Madreporæ* of the Great Barrier Reef (of Australia)."

L. F. de Pourtalès obtained[6] six colonial genera of corals from

[1] On Chlorophyll from the Deep Sea. *Proc. R. S. Edin.*, 1884—5.
[2] Sur la Pénétration de la Lumière du Jour dans les Eaux du Lac de Genève. *Compt. Rend.* XCIX., p. 783.
[3] *Trans. Zool. Soc. Lond.* VIII., pt. v.
[4] *The Depths of the Sea.*
[5] *Loc. cit.*, p. 72.
[6] *Cat. Mus.*, Harvard Coll., 1871.

the plateau 50—250 fathoms in depth, which bears his name. Of this plateau L. Agassiz says[1] that it "is built up entirely by animals now living upon its surface and constantly increasing the thickness of the bed by their accumulation. Large fragments of the rock were brought up by the dredge; so that its structure and characteristic remains of animals could be studied at leisure. I do not know that there is on record a more direct illustration of the manner in which mountain masses of calcareous deposits have been accumulated on the bottom of the ocean." This rock it must be observed Agassiz compared to the coral rag in the Jurassic. Agassiz speaks also of the abundant fauna of this plateau, one remark, "the variety of marine plants does not keep pace with the variety of marine animals," being of interest as indicating that some plants at any rate were found all over this plateau.

I have shown sufficiently by these few quotations, how banks may grow up to the requisite height for the reef-building nullipores. In the tropics corals would grow more vigorously and at greater depths on account of the abundance of food and probably the greater intensity of the light than in the temperate zone, where the pelagic life is not so rich. That reefs are not formed in the temperate zones is due, in the first place to solution, in the next to the inability of the reef-building nullipores to live and flourish in these temperatures; these algæ require moreover a strong and direct light, and such light can only be obtained at considerable depths in the tropical regions.

These few remarks, which I have made, are intended to apply more particularly to the coral reefs of Fiji, as well as generally to those of the Pacific and Indian Oceans. I would not wish to apply them to the Florida Reefs and other reefs in the West Indies, where I think the conditions of formation were radically different; the formation of the Great Barrier Reef of Australia may probably be explained in a similar way to that by which L. Agassiz has explained the Florida Reefs. Each step in the formation of atoll and barrier reefs, as I have tried to sketch it, cannot, I fear, be applicable to all atoll and barrier reefs. Nevertheless the main outlines will, I believe, be found to be correct for most of the so-called coral reefs, but each group of islands and each reef should be considered by itself in relation to the known meteorological and biological conditions of its region.

[1] Report on Deep Sea Dredgings in the Gulf Stream. *Mus. Comp. Zool.*, 1870.

(2) *Methods for the Demonstration of "Connecting Threads" in the Cell Wall.* By WALTER GARDINER, M.A., Clare College.

(Received 7 March, 1898.)

I HAVE already published in the *Proceedings* of the Royal Society[1] a brief account of a new method for demonstrating the "connecting threads" which traverse the walls of plant cells.

In the present paper I propose to make some further remarks upon that method, which may be called the "Kolossow-Safranin method," and also to describe a second means of investigation, which may be briefly described as the "Iodine-Acid-Violet method."

A. *The Kolossow-Safranin Method.*

1. *Killing and swelling.* In this method the two processes are accomplished simultaneously either with watery Picric acid, Picro-sulphuric acid or Picro-acetic acid, and in certain cases with Sulphuric acid. With refractory tissue, treatment with Picric acid may be supplemented by that of Picro-sulphuric acid. Picro-acetic acid was used with success in the investigation of the tissue of ferns.

2. *Fixing*, is effected with Kolossow's mixture of Uranium nitrate and Osmic acid, either of normal strength or diluted with once or twice its own volume of water[2]. Hermann's mixture also gives good results, but Flemming's mixture and fixatives containing chromic acid were found to be unsuitable.

3. *Preservation.* After fixing, the material may be preserved (apparently indefinitely) in a solution of Thymol water containing ·5 grms. Thymol to 1 litre of water, and sections may be taken as required.

4. *Staining*, is accomplished either with a saturated aqueous solution of Safranin or a similar solution containing 2 °/₀ alcohol and 1 °/₀ aniline. The sections after washing in water are treated for some time with a 2 °/₀ solution of orange G., which dissolves the dye from the cell wall and leaves the protoplasm and connecting threads stained. The threads may now be further stained by substitution. In this direction Gentian violet gives generally the most satisfactory results, especially if Gram's method of intensification be employed. Staining with Gentian violet (as with Safranin) affects the whole tissue, but the dye may again be removed from the cell wall by the action of a 5 °/₀ solution of acid fuchsin. In certain cases, e.g. the endosperm of *Hordeum*

[1] Gardiner. *Roy. Soc. Proc.*, Vol. XXII. 1897.
[2] Kolossow. *Zeit. f. Wiss. Mik.*, v. I. 1888.

vulgare, substitution of the Safranin by Eosin is advantageous, and for many ferns Poirier's orange may be similarly employed. In some instances I have used Cyanin with favourable results.

5. *Mounting.* The sections are placed in dilute glycerine and then transferred for permanent mounting into glycerine jelly.

Some observations on the rationale of the method may now follow.

As regards the Picric acid no more need perhaps be said than that the lethal action of this quickly penetrating reagent upon the protoplasm is a well-known matter of experience, and that living tissue exposed to its poisonous influence rapidly succumbs. Besides killing the tissue, it swells the walls. This swelling of the cell wall seems to me to be essentially a phenomenon of hydration, and it is possible that the protoplasm is similarly affected. The action of Sulphuric acid is not unlike Picric acid itself, and possibly Acetic acid also belongs to the same category. Certain of my experiments appear to indicate that both Picric acid and Acetic acid function as mordants both for Safranin and Gentian violet, though in view of the great solubility of both reagents and the comparative readiness with which they may be removed from tissues by washing, the real explanation may be that cytoplasm or cellulose after having been submitted to these actions has a much greater affinity for the two dyes than before. At the same time it must be remembered that Picric acid gives a precipitate both with Safranin and Gentian violet.

Kolossow's mixture is a far more deadly poison than Picric acid; is exceedingly rapid in operation; and fixes the tissue with little or no change. It also hardens the structures and makes them very favourable for cutting and subsequent manipulation.

Unlike Picric acid there seems to be no question of hydration here, and the action of both the Osmium and Uranium salt appears to consist in forming a fixed and insoluble compound with the tissues exposed to their influence.

From the point of view of staining it is the Uranium which plays the more important part in that it and not the Osmic acid acts as a mordant to the Safranin. In the case of Hermann's mixture it is the Platinum salt which performs a similar function.

The Uranium on the other hand does not mordant Gentian violet, and no satisfactory results can be obtained by staining with Gentian violet direct. Thus the staining has always to be effected through the intervention of Safranin, which with this particular method must be taken as the starting point of all further treatment.

It must not however be imagined that this fact constitutes an objection to the use of Safranin. Safranin is on the contrary almost an ideal dye for such an investigation as the present,

and any tissue which can be stained with it, admits of very wide treatment on account of the numerous and important substitutions and combinations which can be subsequently effected.

In connection with the Safranin method it is important to note that it is the protoplasm and connecting threads which appear to be mordanted by the Uranium, and that it is in these structures that the precipitate of Uranium and Safranin which cannot be dissolved by the Orange G. is formed.

B. *The Iodine-Acid-Violet Method.*

As the Kolossow-Safranin method depends upon the fact that the Uranium salt in combination with certain elements of the dead cytoplasm acts as a mordant to the Safranin, so in the Iodine-Acid-Violet method, the Iodine is the mordant and an Aniline violet the dye.

We owe to Meyer[1] the latest exposition of the relations between Iodine and certain of the Aniline violets, as well as the explicit account of an excellent method, for the demonstration of "connecting threads."

Meyer brings clearly into prominence the part played by Iodine as a mordant, and to him belongs the credit of stating with precision the great importance of this fact and dwelling upon the significance of it in connection with his method.

His method is as follows:

Sections of fresh tissue are treated with a solution made by adding ·5 grms. Iodine and ·5 grms. Potassic Iodide to 100 c.c. of water. They are transferred to a slide together with a small quantity (a half drop) of the Iodine solution. One or two drops of a 30 °/₀ solution of sulphuric acid saturated with Iodine are added and the cover-slip is put on.

Then a half drop of a 3 °/₀ solution of Pyoktanin is run under the edge of the cover-slip. The Pyoktanin is at once dissolved by the sulphuric acid, forming a brownish yellow or greenish yellow solution which, together with the Iodine present, stains the protoplasm and the "connecting threads" black. The various details of staining may be observed under the microscope. After some three minutes have elapsed the slide and its contents are placed in a vessel containing about 400 c.c. of water. In successful preparations the cell wall is stained light blue and the protoplasm and "connecting threads" blue black. The fine precipitate formed by the union of the Iodine and Pyoktanin, which is frequently deposited upon the section, may be removed by brushing.

The author made a series of observations upon the aniline

[1] Meyer. *Besichte der Deutsch. Bot. Gesell.* Bd. xv. Heft. 3. 1897.

violets suitable for the above reaction, and has found that the methyl violet manufactured by Merck and known as Pyoktanin, (a chlorhydrate of pentamethyl and hexamethyl pararosanilin) gave the best results.

I may add that Gentian violet gives almost (if not quite) as good results as Meyer's Pyoktanin.

Meyer's method has lately been used with success by Kohl[1] in demonstrating the "connecting threads" in the epidermal cells of *Viscum album* and in the parenchyma cells of the "leaf" of *Mnium undulatum* and other mosses.

On repeating the experiments of Meyer it is easy to see that the method is an admirable one and is a great advance upon any of the analogous methods which have preceded it. Moreover Pyoktanin has excellent staining powers and constitutes a reagent of the first rank for investigations of this kind.

Meyer's method has however certain drawbacks, which become apparent when actual experiments are made, and they may be summarised as follows.

1. Want of delicacy and control.
2. Want of certainty and uniformity of results.
3. The formation of the Iodine-Methyl-Violet precipitate.
4. The use of fresh tissue.

One finds in practice that it is difficult to arrange with certainty that the union of the Pyoktanin and Sulphuric acid shall take place in the neighbourhood of the section, and even when this is accomplished, the varying proportions of the acid and the dye may end in the formation of either a propitious or unpropitious mixture, which in consequence may or may not stain in the proper manner. The somewhat voluminous blue-black precipitate which accompanies the union of the two liquids is an impediment to observation, and too frequently a group of granules appears at the identical place one wishes to examine. The results produced by the final treatment with water are by no means uniform, and one has frequently to perform the whole process over again. Finally the use of fresh material involves the possibility of structural changes during the cutting and manipulation, as well as inability to be independent of times and seasons.

The chief drawbacks to which I have alluded are not unconnected with the fact that the various operations of killing, fixing, swelling and staining are carried on simultaneously. In the modification I am now about to describe these operations are conducted apart, so that the whole process involves several separate stages, and each stage can command its own special

[1] Kohl. *Bot. Central.* Bd. LXXII. No. 8. 1897.

treatment. By this means also the rationale of the whole process is considerably elucidated.

(1) *Killing and fixing* are in all cases brought about by means of Iodine. Small pieces of tissue are treated either with Iodine water or more usually with a solution of Iodine dissolved in Iodide of Potassium. For delicate tissues of small bulk a strength of ·1 or ·2 °/₀ Iodine in ·15 or ·25 °/₀ Potassic Iodide in water is sufficient, but for the average tissues a solution containing ·5 °/₀ Iodine in ·75 °/₀ Potassic Iodide in water is necessary for rapid penetration. The tissues may either remain in the Iodine solution as required for cutting or may be preserved in a solution of Thymol water.

(2) *Swelling.* Sections having been cut in water are swollen in sulphuric acid. The strength of the acid employed is essentially dependent upon the character of the tissue; 1—5 °/₀ being sufficient for such tissue as that of *Chara* or the "leaf" of *Mnium undulatum*, 5 °/₀ for *Viscum album* and 30 °/₀ for *Aucuba Japonica*. Solutions containing Picric or Acetic acid do not give satisfactory results. My experiments appear to show that sulphuric acid is for this particular method the swelling agent *par excellence*.

(3) *Staining.* The sections are first mordanted with Iodine. To this end they are treated for a short time with an Iodine solution in 5 °/₀ sulphuric acid. The Iodine solution may contain either ·5 °/₀ Iodine in ·75 °/₀ Potassic Iodide, or 1 °/₀ Iodine in 1·25 °/₀ Potassic Iodide. Meanwhile the staining mixture is prepared, and must be freshly prepared for each occasion. Two stock solutions are made up. The first is a 10 °/₀ solution of Sulphuric acid and the second is a ·5 °/₀ or 1 °/₀ solution of Pyoktanin or Gentian violet in water. The staining solution is prepared by mixing equal parts (say 5 c.c.) of each, so that what is produced is a ·25 or ·5 °/₀ solution of the Violet in 5 °/₀ Sulphuric acid. A sufficient quantity having been poured out into a watch-glass, the sections are taken out of the acid Iodine solution, quickly passed through a solution containing ·1 °/₀ Iodine and ·15 °/₀ Potassic Iodide in 5 °/₀ Sulphuric acid, and then placed in the solution of acid Violet. After some ten minutes have elapsed they are taken out, and washed freely in water and may be forthwith examined.

(4) *Intensifying.* If the swelling and staining have been properly conducted it will be found that the protoplasm and "connecting threads" are stained blue, and that the cell wall remains almost uncoloured. It is however a great advantage, and frequently a necessity, to make the staining much more pronounced, so that the threads may be the more easily observed.

This may be done either by means of Iodine alone, or by repeating the operation of mordanting and staining one or more times. Intensification with Iodine is accomplished by simply treating a stained section either with a solution containing ·1 °/₀ Iodine and ·15 °/₀ Potassic Iodide in 5 °/₀ Sulphuric acid, or with a solution of 5 °/₀ Sulphuric acid saturated with Iodine and then examining the section mounted in one of these reagents. The Iodine forming a black precipitate with the Aniline violet causes the protoplasm and threads to stand out much more clearly than before. The blue tint assumed by the cell wall in consequence of slight staining also disappears, since the Iodine in virtue of its intense affinity for the Violet combines with every trace of the dye.

When intensification by staining is desired the tissue is mordanted with an Iodine solution, which should be more dilute than on the first occasion of staining. It is then passed through a 5 °/₀ solution of Sulphuric acid and again treated with the Sulphuric acid solution of Gentian violet or Pyoktanin. If again treated with and examined in dilute acid Iodine, the protoplasm and the threads stand out intensely black and peculiarly clearly. One should mention that when secondary staining is resorted to, a certain amount of precipitation upon the section may occur which may necessitate the use of a brush before examination.

(5) *Mounting.* In order to render the violet stain "fast," I mount in glycerine containing a small percentage of Zinc chloride, and still better results may sometimes be obtained when a trace of Iodine is present. The Zinc chloride acts in virtue of its property of forming a dense blue precipitate with Gentian violet and Pyoktanin. The mounting solution may be made as follows: 30 c.c. Glycerine, 60 c.c. of water and 10 c.c. of a 20 °/₀ solution of Zinc chloride are mixed together. To this is added a flake or two of Iodine and the whole is slightly warmed over a warm bath. Sections may be transferred from the above solution into Glycerine jelly.

The demonstration of "connecting threads" in such structures as the "leaves" of mosses and the like, where the whole organ is implicated and the cutting of sections is either undesirable or difficult, involves considerable modification of the above treatment and is not unattended with many manipulative troubles, by reason of the fact that it is exceedingly difficult to limit the staining to the protoplasm and threads, and to prevent it extending to the walls.

Successful results appear to depend upon the use of low percentages of Iodine for the killing and still lower for the mordanting. For the swelling I find that a 5 °/₀ solution of sulphuric acid is sufficient. As the result of an elaborate series

of experiments I also find that if the swelling is followed by treatment with a dilute solution of sulphuric acid containing a trace of Potassic permanganate the walls are bleached, and at the same time so altered in character that after mordanting they no longer stain with the acid Violet, and thus the threads are brought very clearly into view. The threads may be readily demonstrated at the edges of the preparation by means of the ordinary technique when due regard is paid to the special requirements already mentioned. The complete absence of threads from the external walls is very striking, and still more striking is the enormous number of the threads present.

It now remains for me to make a few concluding observations upon the rationale of the Iodine-Acid-Violet method.

The great merit of Meyer's modification is, as I have already remarked, that it places in the foreground the important *rôle* of the Iodine and makes definite provision for mordanting the protoplasm and threads before the application of the dye. This was not done by earlier observers, although I quite agree with Meyer in thinking that what success attended the somewhat analogous methods of Russow[1], Terletzki[2], Kienitz-Gerloff[3], and (I may add) Poirault[4], was not unconnected with the fact that at the time of staining some traces of Iodine were present in the tissue.

The same is possibly true of the Iodine-Chlor-Zinc-Iod-Aniline-Blue methods of Russow[5] and myself[6].

The mordanting properties of Iodine in combination with Aniline dyes has of course been long known, having been used in conjunction with Gentian violet by Gram[7] in 1884 and Bizzozero[8] and Nissen[9] in 1886; and I see from my notes that I have myself used it since 1895 in connection with my Kolossow-Safranin method. These facts however in no way detract from the excellence of Meyer's own researches.

The mordanting properties of Iodine appear to result from the remarkable and phenomenal manner with which it is taken up by the protoplasmic structures of the cell.

It is difficult to frame any adequate explanation of this interesting fact, and the very expressions one uses in connection with it must, I fear, be but vague and unsatisfactory. No

[1] Russow. *Sitzber. der Dorpat. Naturf. Gesell.* 1883.
[2] Terletzki. *Jahrb. f. Wiss. Bot.* Bd. xv. 1884.
[3] Kienitz-Gerloff. *Bot. Zeit.* 1891.
[4] Poirault. *Annales de Sci. Nat. Bot.* 1893.
[5] Russow. *Sitzber. der Dorpat. Naturf. Gesell.* 1882.
[6] Gardiner. *Phil. Trans. Roy. Soc.* 1883.
[7] Gram. *Fortschr. d. Medecin.* II. No. 6. 1884
[8] Bizzozero. *Zeit. f. Wiss. Mik.* III. 1. 1886.
[9] Nissen. *Arch. f. mik. Anat.* 1886.

doubt the phenomenon is partly one of simple diffusion, and results from the circumstance that diffusion into the protoplasm is easier than into the refractory and markedly mucilaginous cell wall. But in addition to this simple diffusion there also appears to be a definite selective affinity between the Iodine and the dead protoplasm, which stated more philosophically is in reality a chemical affinity, as is also in a smaller degree diffusion itself.

But be this as it may, the extraordinary rapidity with which mere traces of Iodine are absorbed by the protoplasm and the threads is almost beyond belief.

The precise manner in which the Iodine travels from cell to cell is not quite clear, but in certain instances I am led to believe that it is *viâ* the threads rather than *viâ* the walls.

Not less important than the relations between the protoplasm and its mordant are the relations between the mordant and its dye, but all that one need say on the present occasion is that Iodine and many of the Aniline violets have an intense affinity the one for the other, and that even in the most dilute solutions where only traces are present the characteristic precipitate which plays so important a part in the Iodine-Acid-Violet method is at once formed.

A second and almost equally important factor in the success of the method is the part played by the sulphuric acid as the solvent of the Aniline violets; and in this connection two points especially deserve notice, viz. (1) that the dye when first dissolved in the acid exists in a very perfect state of solution, and (2) that upon solution the tinctorial properties of the dye become profoundly modified.

As to the first point it will be found that watery solutions of the Violets penetrate into tissues with some difficulty, but that alcoholic solutions, and especially solutions in acids, are much more readily diffusible.

The second point is however much the more important, and it is this; that Gentian violet or Pyoktanin when dissolved in sulphuric acid stains the protoplasmic structures only and ceases to colour the cell wall. That this is so may be easily demonstrated by the following experiments, A and B.

A. (1) Sections of normal tissue having been treated with an aqueous solution of the Violet—the cell wall and protoplasm are both deeply stained.

(2) Sections of tissue having been swollen with sulphuric acid and washed and treated with an aqueous solution of the Violet—the results are practically the same as before.

B. (1) Sections of normal tissue treated with a ·5 °/₀ solution of the Violet in 5 °/₀ solution of Sulphuric acid—the protoplasm is stained but the cell wall hardly at all.

(2) Sections of swollen tissue treated as in B. (1) give a result practically the same as B. (1).

It is not the alteration in the cellulose produced by the acid which determines the reaction, since experiment shows that a swollen wall can be subsequently stained by an aqueous solution of the dye; neither is it necessarily all acid solutions which produce the result since when the dye is dissolved in acetic acid a sensible staining of the wall occurs.

As regards the relations between the dye and its solvent, the following points deserve notice.

A freshly made solution of Pyoktanin or Gentian violet containing ·5 °/₀ of the dye dissolved in a 5 °/₀ aqueous solution of sulphuric acid is at first of a vivid green colour with a yellowish tinge. The colour however soon commences to fade, and after some time entirely disappears. For a short period this liquid would appear to contain a solution of the dye as such, but decomposition soon commences and ends (I presume) with the formation of a colourless sulphate of the base. Whether the sulphuric acid may at any time be regarded as containing unaltered Pyoktanin or Gentian violet, or whether at the moment of admixture a chemical change occurs must for the present be left an open question, but one salient fact becomes obvious, viz. that the staining powers are greatly diminished and bear little comparison to those exhibited by the dye when in aqueous solution. Further, such staining as does take place is fairly limited to the protoplasm.

The selective characteristics of the Acid violet are naturally accentuated by the use of Iodine, since the Iodine itself has analogous affinities for the cytoplasm and threads, and has in addition an intense affinity for the dye.

That the mordanting action of the Iodine is mainly limited to the protoplasm may also be proved by the following experiment:

A section is swollen with acid and washed, and is then treated with Iodine and stained with an aqueous solution of one of the Violets, when it will be found that both the protoplasm and the cell walls are deeply stained. If now placed in a 5 °/₀ solution of sulphuric acid the dye is dissolved from the walls but not from the protoplasm, so that both the protoplasm and the threads come into view.

As this paper is concerned with methods only, I do not propose to give an account of my more recent results, but my work in this direction is proceeding and will be published in due course. I also intend to deal with other modifications of the two methods I have now described.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Monday, May 2, 1898.

MR F. DARWIN, PRESIDENT, IN THE CHAIR.

At a Meeting of the Society held at the Cavendish Laboratory the following were elected Fellows:

J. Stanley Gardiner, M.A., Gonville and Caius College.
W. F. Sedgwick, M.A., Trinity College.

The following Communications were made to the Society:

(1) *On the theory of Order.* By E. T. DIXON, M.A., Trinity College.

All the theorems of non-metrical (projective) geometry depend solely on the conception of "order" so defined as to be independent of the idea of "before and after," which belongs only to Time. It follows from this definition that no "order" can be ascribed to less than four units in any uniform group; and this is why less than four points have no "projective relation" or anharmonic ratio. The paper further discusses the way in which numbers (or coordinates) may be assigned to the units of a group for purposes of analysis, with or without a system of "unique" lines having already been determined.

(2) *On a certain system of differential equations defining periodic functions*.* By H. F. BAKER, M.A., St John's College.

(1) Suppose we have a consistent system of differential equations

$$\frac{\partial^4 w}{\partial u_i \partial u_j \partial u_k \partial u_l} = R_{ijkl}\left(\frac{\partial^2 w}{\partial u_r \partial u_s}\right),$$

* [21 September. The contents and the title of this note have been modified since it was presented to the Society.]

expressing that partial differential coefficients, of the dependent variable w, of the fourth order, are rational functions of the partial differential coefficients of the second order, neither the partial differential coefficients of other orders, nor the independent variables $u_1, u_2, \ldots$, entering explicitly. For definiteness suppose there are three independent variables; then the greatest possible number of equations is fifteen, and the rational functions contain six partial differential coefficients of the second order. If we differentiate the equations once, and eliminate the resulting differential coefficients of the fifth order between pairs of the equations, we shall obtain at most twenty-four equations, each linear in the ten partial differential coefficients of the third order, the coefficients of these being rational in the differential coefficients of the second order. The elimination of the differential coefficients of the third order, which we suppose to be possible, will then lead to rational relations connecting the differential coefficients of the second order. We suppose that these rational relations are algebraically equivalent to as many as there are independent variables, namely three, which we may denote by

$$f_1(x, y, z, \xi, \eta, \zeta) = 0,\ f_2(x, y, z, \xi, \eta, \zeta) = 0,\ f_3(x, y, z, \xi, \eta, \zeta) = 0,$$

where $x, y, z, \xi, \eta, \zeta$ denote the partial differential coefficients of the second order,

$$x = \frac{\partial^2 w}{\partial u_3^2},\ y = \frac{\partial^2 w}{\partial u_3 \partial u_2},\ z = \frac{\partial^2 w}{\partial u_3 \partial u_1},\ \xi = \frac{\partial^2 w}{\partial u_2^2},\ \eta = \frac{\partial^2 w}{\partial u_2 \partial u_1},\ \zeta = \frac{\partial^2 w}{\partial u_1^2}.$$

In the course of deducing these three rational equations we shall obtain such equations as

$$\frac{1}{P}\frac{\partial^3 w}{\partial u_3^3} = \frac{1}{Q}\frac{\partial^3 w}{\partial u_3^2 \partial u_2} = \frac{1}{R}\frac{\partial^3 w}{\partial u_3^2 \partial u_1}$$

$$= \frac{1}{P_1}\frac{\partial^3 w}{\partial u_3 \partial u_2^2} = \frac{1}{Q_1}\frac{\partial^3 w}{\partial u_3 \partial u_2 \partial u_1} = \frac{1}{R_1}\frac{\partial^3 w}{\partial u_3 \partial u_1^2} = \ldots,$$

where $P, Q, \ldots, R_1, \ldots$, are definite rational functions of $x, y, z, \xi, \eta, \zeta$. Putting each of the fractions equal to μ we have

$$\frac{\partial^4 w}{\partial u_3^4} = P\frac{\partial \mu}{\partial u_3} + \mu^2\left(\frac{\partial P}{\partial x}P + \frac{\partial P}{\partial y}Q + \frac{\partial P}{\partial z}R + \frac{\partial P}{\partial \xi}P_1 + \frac{\partial P}{\partial \eta}Q_1 + \frac{\partial P}{\partial \zeta}R_1\right),$$

$$\frac{\partial^4 w}{\partial u_3^3 \partial u_2} = Q\frac{\partial \mu}{\partial u_3} + \mu^2\left(\frac{\partial Q}{\partial x}P + \frac{\partial Q}{\partial y}Q + \frac{\partial Q}{\partial z}R + \frac{\partial Q}{\partial \xi}P_1 + \frac{\partial Q}{\partial \eta}Q_1 + \frac{\partial Q}{\partial \zeta}R_1\right),$$

hence, replacing the left sides by their rational expressions $R_{3333}(x, y, z, \xi, \eta, \zeta)$, $R_{3332}(x, y, z, \xi, \eta, \zeta)$, and eliminating $\frac{\partial \mu}{\partial u_3}$, we obtain μ^2 as a rational function of $x, y, z, \xi, \eta, \zeta$—and can hence

express the partial differential coefficients of the third order in terms of $x, y, z, \xi, \eta, \zeta$, in the forms

$$\frac{\partial^3 w}{\partial u_3^3} = \mu P, \text{ etc.},$$

where P and μ^2 are rational in $x, y, z, \xi, \eta, \zeta$. We obtain several different forms for μ^2 in a similar way. It is supposed that the differential equations are such that these are algebraically equivalent in virtue of the equations $f_1 = 0$, $f_2 = 0$, $f_3 = 0$.

Now we have

$$dx = \frac{\partial^3 w}{\partial u_3^3} du_3 + \frac{\partial^3 w}{\partial u_3^2 du_2} du_2 + \frac{\partial^3 w}{\partial u_3^2 du_1} du_1 = \mu (P du_3 + Q du_2 + R du_1),$$

and similarly,

$$dy = \mu (Q du_3 + P_1 du_2 + Q_1 du_1), \quad dz = \mu (R du_3 + Q_1 du_2 + R_1 du_1);$$

thus we have

$$u_3 = \int \frac{1}{\mu} (A\,dx + H dy + G dz),$$

$$u_2 = \int \frac{1}{\mu} (H dx + B dy + F dz),$$

$$u_1 = \int \frac{1}{\mu} (G dx + F dy + C dz),$$

where A, B, C, F, G, H, μ^2 are rational functions of $x, y, z, \xi, \eta, \zeta$, these latter being connected by the rational equations $f_1 = 0$, $f_2 = 0$, $f_3 = 0$.

Therefore, if the original differential equations have a solution of which the second partial differential coefficients are single-valued functions, *these integrals will be capable of inversion, and define $x, y, z, \xi, \eta, \zeta$ as single-valued, in general simultaneously periodic, functions of u_3, u_2, u_1.* Whence the solution of the differential equations can be completed.

In what follows we obtain systems of differential equations of the form just considered, the system with p independent variables being satisfied by all hyperelliptic theta functions of p variables, independently of the characteristic of the theta function (the dependent variable w being the negative logarithm of the theta function), and we verify in detail for the cases of two and three variables that the necessary conditions for the application of the method of integration sketched above are satisfied. The method would appear to be of importance as an elementary introduction to the theory of theta functions, but it has a wider application, namely to the general theory of functions with simultaneous period-systems.

(2) Let

$$\sigma(u) = \sum_{n_1 \ldots n_p} e^{au^2 + 2\pi iv(n+r) + i\pi\tau(n+r)^2 + 2\pi iq(n+r)},$$

be, as usual, the general hyperelliptic theta function of p variables with any characteristic, the quantities a_{ij} occurring in the quadratic form au^2 satisfying the equation

$$2\Sigma a_{ij} u_i^{x,a} u_j^{z,c} = \Pi_{z,c}^{x,a} - \int_a^x \int_c^z \frac{dx\,dz}{ys} \frac{2ys + F(x,z)}{4(x-z)^2},$$

where* $F(z,z) = 2f(z) = 2s^2$, $\left[\frac{\partial}{\partial x} F(x,z)\right]_{x=z} = \frac{d}{dz} f(z)$;

further let $\wp_{ij}(u) = -\frac{\partial^2}{\partial u_i \partial u_j} \log \sigma(u)$, $\wp_{ijkl}(u) = \frac{\partial^2}{\partial u_k \partial u_l} \wp_{ij}(u)$,

and let e_1, e_2, e_3, e_4 be any quantities whatever.

Then we have the relation

$$\begin{aligned}
8(e_2-e_3)(e_3-e_1)(e_1-e_2)(e_4-e_1)(e_4-e_2)(e_4-e_3) \sum_{\lambda\mu\nu\rho}\sum \wp_{\lambda\mu\nu\rho}(u) \cdot e_1^{\lambda-1} e_2^{\mu-1} e_3^{\nu-1} e_4^{\rho-1} \\
= (e_2-e_3)(e_4-e_1)[F(e_2,e_3) - 4(e_2-e_3)^2 \sum_{\lambda\mu}\sum \wp_{\lambda\mu}(u)\, e_2^{\lambda-1} e_3^{\mu-1}] \\
\qquad [F(e_4,e_1) - 4(e_4-e_1)^2 \sum_{\lambda\mu}\sum \wp_{\lambda\mu}(u)\, e_4^{\lambda-1} e_1^{\mu-1}] \\
+ (e_3-e_1)(e_4-e_2)[F(e_3,e_1) - 4(e_3-e_1)^2 \sum_{\lambda\mu}\sum \wp_{\lambda\mu}(u)\, e_3^{\lambda-1} e_1^{\mu-1}] \\
\qquad [F(e_4,e_2) - 4(e_4-e_2)^2 \sum_{\lambda\mu}\sum \wp_{\lambda\mu}(u)\, e_4^{\lambda-1} e_2^{\mu-1}] \\
+ (e_1-e_2)(e_4-e_3)[F(e_1,e_2) - 4(e_1-e_2)^2 \sum_{\lambda\mu}\sum \wp_{\lambda\mu}(u)\, e_1^{\lambda-1} e_2^{\mu-1}] \\
\qquad [F(e_4,e_3) - 4(e_4-e_3)^2 \sum_{\lambda\mu}\sum \wp_{\lambda\mu}(u)\, e_4^{\lambda-1} e_3^{\mu-1}]
\end{aligned}$$

wherein

$$u_i^{x,a} = \int_a^x \frac{x^{i-1}dx}{y}, \quad y^2 = f(x) = (x,1)_{2p+2}, \quad u_i = \sum_{r=1}^{p} u_i^{x_r, a_r},$$

and $F(x,z)$ may have its most general form. The summation for each of the symbols λ, μ, ν, ρ is from 1 to p.

The deduction of the explicit forms of the differential equations, by equation of like powers of e_1, e_2, e_3, e_4 on the two sides, is a question in the calculus of symmetric functions.

(3) For the case of 2 variables we put down the differential equations in two forms, (α) the explicit form, in accordance with § 1, (β) a symbolical form, itself of some interest.

* Compare, e.g., *Math. Annalen*, L. (1898), p. 471.

(α) Putting

$$f(x) = \lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + \lambda_4 x^4 + \lambda_5 x^5 + \lambda_6 x^6,$$

$$F(x, z) = \sum_{i=0}^{3} x^i z^i \left[2\lambda_{2i} + \lambda_{2i+1}(x + z)\right], \text{ with } \lambda_7 = 0,$$

and

$$\wp_{22} = x, \ \wp_{21} = y, \ \wp_{11} = z,$$

the equations are

$$\begin{aligned}
-2(\wp_{2222} - 6\wp\wp_{22}^2) &= \lambda_2\lambda_6 - \tfrac{1}{4}\lambda_3\lambda_5 - 2\lambda_4 x - 2\lambda_5 y + 6\lambda_6 z \\
-2(\wp_{2221} - 6\wp\wp_{22}\wp_{21}) &= \tfrac{1}{2}\lambda_1\lambda_6 \qquad\qquad - 2\lambda_4 y + \lambda_5 z \\
-2(\wp_{2211} - 2\wp\wp_{22}\wp_{11} - 4\wp\wp_{21}^2) &= \lambda_0\lambda_6 \qquad\qquad - \lambda_3 y \\
-2(\wp_{2111} - 6\wp\wp_{21}\wp_{11}) &= \tfrac{1}{2}\lambda_0\lambda_5 \qquad + \lambda_1 x - 2\lambda_2 y \\
-2(\wp_{1111} - 6\wp\wp_{11}^2) &= \lambda_0\lambda_4 - \tfrac{1}{4}\lambda_1\lambda_3 + 6\lambda_0 x - 2\lambda_1 y - 2\lambda_2 z,
\end{aligned}$$

nich take a still simpler form when we put $\lambda_6 = 0$, $\lambda_5 = 4$, as we ay do without loss of generality. By differentiating the first ıation in regard to u_1, and the second in regard to u_2, and sub- cting, and acting similarly for the other pairs, we obtain four ations, of which, for instance, one is

$$-2y\wp_{222} + 2x\wp_{221} + \tfrac{1}{2}\lambda_5\wp_{211} - \lambda_6\wp_{111} = 0,$$

thence by elimination we obtain

$$\begin{vmatrix}
-2y & 2x & \tfrac{1}{2}\lambda_5 & -\lambda_6 \\
2z & 2y + \tfrac{1}{2}\lambda_3 & -(4x + \tfrac{1}{2}\lambda_4) & \tfrac{1}{2}\lambda_5 \\
\tfrac{1}{2}\lambda_1 & -(4z + \lambda_2) & 2y + \tfrac{1}{2}\lambda_3 & 2x \\
-\lambda_0 & \tfrac{1}{2}\lambda_1 & 2z & -2y
\end{vmatrix} = 0,$$

v geometrically is a transformation of the equation of the s ıı-nodal quartic surface first found by Göpel and considered b mmer. The nature of the transformation is best described g trically—suppose $\lambda_6 = 0$, $\lambda_5 = 4$, and use the abbreviations

I $+ bx - b^2$, $P_{bc} = z + (b + c)\,y + bcx - e_{b,c}$, $e_{b,c} = \dfrac{1}{4}\dfrac{F(b, c)}{(b - c)^2}$,

th c_1, a_1, c_2, a_2, c are the five roots of the equation $f(x) = 0$, an 'enotes the plane at infinity, the sixteen nodal points are :

The infinite end of the axis of z, through which pass the six ılar tangent planes

$$P_{c_1},\ P_{a_1},\ P_{c_2},\ P_{a_2},\ P_c,\ P;$$

II. The five points lying on the plane at infinity given by

$$x = -\frac{y}{b} = \frac{z}{b^2} = \infty,$$

where b is any one of c_1, a_1, c_2, a_2, c, the singular tangent planes which pass through this point being

$$P,\ P_b,\ P_{bb_1},\ P_{bb_2},\ P_{bb_3},\ P_{bb_4},$$

where $b_1, \ldots, b_4$ are the four of the set c_1, a_1, c_2, a_2, c other than b:

III. The ten points given by

$$x = b_1 + b_2,\ y = -b_1 b_2,\ z = e_{b_1, b_2},$$

where b_1, b_2 are any two of c_1, a_1, c_2, a_2, c, the singular tangent planes which pass through this point being

$$P_{b_1},\ P_{b_2},\ P_{b_1 b_2},\ P_{b_3 b_4},\ P_{b_4 b_5},\ P_{b_3 b_5},$$

where b_3, b_4, b_5 are the three of the set c_1, a_1, c_2, a_2, c other than b_1, b_2.

Still supposing $\lambda_6 = 0$, $\lambda_5 = 4$, the expressions for the partial differential coefficients of the third order in terms of x, y, z referred to in § 1, can be taken so that the squares and the products of twos of $\wp_{222}$, $\wp_{221}$, $\wp_{211}$, $\wp_{111}$ are rational integral cubic polynomials in x, y, z. I have obtained the ten equations, of which for instance three are

$$\wp_{222}^2 = \lambda_2 + \lambda_3 x + \lambda_4 x^2 + 4x^3 + 4xy + 4z,$$

$$\wp_{221}^2 = \lambda_0 + \lambda_4 y^2 + 4xy^2 - 4yz,$$

$$2\wp_{222}\wp_{221} = \lambda_1 + \lambda_3 y + 2\lambda_4 xy + 8x^2 y + 4y^2 - 4xz.$$

The first of these equations shews that all the functions $\wp_{111}$, $\wp_{221}$, $\wp_{211}$, $\wp_{111}$ can be expressed rationally in terms of the three $x = \wp_{22}$, $y = \wp_{21}$, $\zeta = \wp_{222}$, (a remark easily generalisable to higher values of p). It is known that there exist cubic surfaces touching the sixteen-nodal quartic surface along sextic curves; denoting the right-hand sides of these equations momentarily by X, Y, Z, the equation of the quartic can be put into the form

$$4XY = Z^2,$$

which shews that each of the cubic surfaces X, Y touches the quartic along a sextic curve lying on $Z = 0$. There are also irrational forms for the equation of the quartic; for instance one arises from the verifiable identity $\wp_{211} = \wp_{21}\wp_{222} - \wp_{22}\wp_{221}$. Further in the form $t^2 = \chi(x, y)$, to which the equation is reducible birationally, $\chi(x, y)$ is the product of the five expressions P_b.

The expressions for u_2, u_1 as linear integrals can now be obtained as in § 1. The result can be brought into a familiar form

$$u_2 = \int \frac{L\,dx + M\,dy}{\dfrac{\partial \Psi}{\partial \zeta}}, \qquad u_1 = \int \frac{M\,dx + N\,dy}{\dfrac{\partial \Psi}{\partial \zeta}},$$

where $\Psi = 0$ is what the equation of the sixteen-nodal quartic becomes when we substitute for z in terms of x, y and ζ, where $\zeta = \wp_{222}$, and L, M, N are rational integral polynomials in x, y, ζ. From the point of view of the theory of functions it is the surface $\Psi = 0$, rather than the sixteen-nodal quartic, which is to be regarded as fundamental.

(β) Coming next to a certain symbolical form of which the differential equations are capable, we notice that we have

$$\wp_{22} = \frac{\sigma_2^2 - \sigma\sigma_{22}}{\sigma^2} = -\frac{1}{2\sigma^2}\Delta_2^2\sigma\sigma', \quad \wp_{21} = -\frac{1}{2\sigma^2}\Delta_2\Delta_1\sigma\sigma', \quad \wp_{11} = -\frac{1}{2\sigma^2}\Delta_1^2\sigma\sigma',$$

and similarly

$$\wp_{2222} - 6\wp_{22}^2 = -\frac{\sigma\sigma_{2222} - 4\sigma_2\sigma_{222} + 3\sigma_{22}^2}{\sigma^2} = -\frac{1}{2\sigma^2}\Delta_2^4\sigma\sigma', \text{ etc.},$$

where

$$\Delta_2^2\sigma\sigma' = \left(\frac{\partial}{\partial u_2} - \frac{\partial}{\partial u_2'}\right)^2 \sigma(u_1, u_2)\,\sigma(u_1', u_2'), \text{ etc.},$$

the variables u_1', u_2' being replaced after differentiation by u_1, u_2. Now let k_2, k_1, λ, μ, λ', μ' be arbitrary constants, let us take Klein's invariantive form for the polynomial $F(x, z)$, and

$$v_2^{x,a} = \int_a^x \frac{(\lambda x + \mu)\,dx}{\sqrt{\phi_x^6}}, \quad v_1^{x,a} = \int_a^x \frac{(\lambda' x + \mu')\,dx}{\sqrt{\phi_x^6}}, \quad \phi_k = \phi_1 k_2 + \phi_2 k_1, \ \phi_k^6 = \psi_k^6;$$

then the differential equations are summed up by

$$\frac{1}{3}\left[\left(k_2\frac{\partial}{\partial v_2} + k_1\frac{\partial}{\partial v_1}\right) - \left(k_2\frac{\partial}{\partial v_2'} + k_1\frac{\partial}{\partial v_1'}\right)\right]^4 \sigma\sigma'$$

$$= (\lambda\mu' - \lambda'\mu)^4 \cdot (\phi, \psi)^4 \phi_k^2 \psi_k^2 \cdot \sigma\sigma'$$

$$-(\lambda\mu' - \lambda'\mu)^2 \cdot \phi_k^4 \left[\left(\phi_2\frac{\partial}{\partial v_2} - \phi_1\frac{\partial}{\partial v_1}\right) - \left(\phi_2\frac{\partial}{\partial v_2'} - \phi_1\frac{\partial}{\partial v_1'}\right)\right]^2 \sigma\sigma',$$

where σ is regarded as a function of v_1, v_2, and, after differentiation, v_2', v_1' are to be replaced by v_2, v_1.

This form shews, by equating terms of the same dimension on the two sides, that the equations can be used to obtain the expansion of the σ-functions in integral powers of the arguments, and gives an immediate proof of the known theorem as to the invariant and integral character (in the coefficients of ϕ_x^6) of the

terms; as a practical example I have obtained the expansion of the function $\sigma_{12}(u_1, u_2)$ up to terms of the ninth order, assuming only the terms of the first order to be u_1. It is probably better however that the *linear* partial differential equations, which for any specified function are more convenient than these, should first be deduced from them*.

From either point of view it is clear that the differential equations, regarded as given, are sufficient entirely to define the theta functions.

(4) The case of three variables may be dealt with more cursorily. Putting

$$x = \wp_{33},\ y = \wp_{32},\ z = \wp_{31},\ \xi = \wp_{22},\ \eta = \wp_{21},\ \zeta = \wp_{11},$$

$$\Delta = \wp_{32}\wp_{21} - \wp_{31}\wp_{22} + \wp_{31}^2 - \wp_{33}\wp_{11},$$

$$f(x) = \lambda_0 + \lambda_1 x + \ldots + \lambda_6 x^6 + 4x^7,$$

$$F(x, z) = \sum_{i=0}^{i=3} x^i z^i \left[2\lambda_{2i} + \lambda_{2i+1}(x + z)\right],$$

the fifteen differential equations are

(1) $\wp_{3333} - 6\wp_{33}^2 = \frac{1}{2}\lambda_5 + \lambda_6 x + 4y$, (2) $\wp_{3332} - 6\wp_{33}\wp_{32} = \lambda_6 y - 2\xi + 6z$,

(3) $\wp_{3331} - 6\wp_{33}\wp_{31} = \lambda_6 z - 2\eta$,

(4) $\wp_{3322} - 4\wp_{32}^2 - 2\wp_{33}\wp_{22} = \frac{1}{2}\lambda_5 y + \lambda_6 z - 2\eta$,

(5) $\wp_{3321} - 4\wp_{32}\wp_{31} - 2\wp_{33}\wp_{21} = \frac{1}{2}\lambda_5 z$, (6) $\wp_{3311} - 4\wp_{31}^2 - 2\wp_{33}\wp_{11} = 2\Delta$,

(7) $\wp_{3222} - 6\wp_{32}\wp_{22} = -\lambda_2 - \frac{1}{2}\lambda_3 x + \lambda_4 y + \lambda_5 z - 6\zeta$,

(8) $\wp_{3221} - 4\wp_{32}\wp_{21} - 2\wp_{31}\wp_{22} = -\frac{1}{2}\lambda_1 + \lambda_4 z - 2\Delta$,

(9) $\wp_{3211} - 4\wp_{31}\wp_{21} - 2\wp_{32}\wp_{11} = \lambda_0 + \frac{1}{2}\lambda_3 z$,

(10) $\wp_{3111} - 6\wp_{31}\wp_{11} = \lambda_0 x - \frac{1}{2}\lambda_1 y + \lambda_2 z$,

(11) $\wp_{2222} - 6\wp_{22}^2 = -\frac{1}{2}\lambda_2\lambda_6 + \frac{1}{8}\lambda_3\lambda_5 - 3\lambda_2 x + \lambda_3 y + \lambda_4\xi + \lambda_5\eta - 3\lambda_6\zeta + 12\Delta$,

(12) $\wp_{2221} - 6\wp_{21}\wp_{22} = -2\lambda_0 - \frac{1}{4}\lambda_1\lambda_6 - \frac{3}{2}\lambda_1 x + \lambda_3 z + \lambda_4\eta - \frac{1}{2}\lambda_5\zeta$,

(13) $\wp_{2211} - 4\wp_{21}^2 - 2\wp_{22}\wp_{11} = -\frac{1}{2}\lambda_0\lambda_6 - 2\lambda_0 x - \frac{1}{2}\lambda_1 y + \lambda_2 z + \frac{1}{2}\lambda_3\eta$,

(14) $\wp_{2111} - 6\wp_{21}\wp_{11} = -\frac{1}{4}\lambda_0\lambda_5 - 2\lambda_0 y + \frac{3}{2}\lambda_1 z - \frac{1}{2}\lambda_1\xi + \lambda_2\eta$,

(15) $\wp_{1111} - 6\wp_{11}^2 = \frac{1}{8}\lambda_3\lambda_1 - \frac{1}{2}\lambda_0\lambda_4 + 4\lambda_0 z - 3\lambda_0\xi + \lambda_1\eta + \lambda_2\zeta$.

From the first twelve of them we find the three independent equations

$$S = R^2/P,\ T = QR/P,\ U = Q^2/P,$$

* For the case of $p = 1$, see Weierstrass, *Werke*, I. p. 7.

where $P = 2x^3 + \frac{1}{2}\lambda_6 x^2 + 2xy + \frac{1}{2}\lambda_5 x - 2z + 2\xi + \frac{1}{2}\lambda_4,$

$Q = 2x^2 y + \frac{1}{2}\lambda_6 xy + 2xz - x\xi + y^2 + \frac{1}{4}\lambda_5 y + \eta + \frac{1}{4}\lambda_3,$

$R = 2x^2 z + \frac{1}{2}\lambda_6 xz + yz - x\eta + \frac{1}{4}\lambda_5 z - \zeta,$

$S = 2xz^2 + \frac{1}{2}\lambda_6 z^2 - 2z\eta + \frac{1}{2}\lambda_0,$

$T = 2xyz + \frac{1}{2}\lambda_6 yz + 2z^2 - y\eta - z\xi + \frac{1}{4}\lambda_1,$

$U = 2xy^2 + \frac{1}{2}\lambda_6 y + 4yz - 2y\xi + 2\zeta + \frac{1}{2}\lambda_2.$

Each of the three relations is, when reduced, of the fourth degree in the aggregate. We find other relations, also of the fourth degree, which are deducible from the three above. Of these two have forms which deserve to be recorded :—

$$\begin{vmatrix} \eta, & \xi - 2z, & y + \frac{1}{4}\lambda_5, & 2x + \frac{1}{2}\lambda_6 \\ \zeta, & -(\eta + \frac{1}{4}\lambda_3), & 2\xi - 2z + \frac{1}{2}\lambda_4, & y + \frac{1}{4}\lambda_5 \\ \frac{1}{4}\lambda_1, & 2\zeta + \frac{1}{2}\lambda_2, & -(\eta + \frac{1}{4}\lambda_3), & \xi - 2z \\ \frac{1}{2}\lambda_0, & \frac{1}{4}\lambda_1, & \zeta, & \eta \end{vmatrix} = 0,$$

$$\begin{vmatrix} z, & y, & -x, & 1, \\ \zeta, & -(\eta + \frac{1}{4}\lambda_3), & 2\xi - 2z + \frac{1}{2}\lambda_4, & y + \frac{1}{4}\lambda_5 \\ \frac{1}{4}\lambda_1, & 2\zeta + \frac{1}{2}\lambda_2, & -(\eta + \frac{1}{4}\lambda_3), & \xi - 2z \\ \frac{1}{2}\lambda_0, & \frac{1}{4}\lambda_1, & \zeta, & \eta \end{vmatrix} = 0;$$

if in the first of these determinants we omit x, y, z and replace ξ, η, ζ by x, y, z, it becomes identical with the Kummer nodal quartic obtained in § 3; an independent set of three of these equations is indeed to be regarded as the generalisation of the Göpel relation to the hyperelliptic case $p = 3$. For any value of p the generalisation should consist similarly of a set of $\frac{1}{2}p(p-1)$ equations in $\frac{1}{2}p(p+1)$ Abelian functions. There is a generalisation in another direction in which there enter $2^p - 1$ Abelian functions—holding however for the general case of p variables.

(3) *On the Total Eclipse of the Sun* 1898 *Jan.* 22. By H. F. Newall, M.A., Trinity College.

A general account was given of the observations made during the recent eclipse, and photographs were exhibited, shewing (i) the general appearance of the corona, (ii) the spectrum of the sun's limb as photographed with prismatic cameras by Sir Norman Lockyer's party, and by Mr Evershed, (iii) the spectrum of the sun's limb as photographed with a slit-spectroscope by Mr Newall, and a diagram was shewn illustrating the distribution of glowing 'coronium' as observed by Mr Newall with a slitless grating spectroscope.

Capt. E. H. Hills, R.E., exhibited and described the photo-

graphs obtained by him of the spectrum of the corona, and also the two series of photographs of the spectrum of the sun's limb at the beginning and end of totality.

Monday, May 16, 1898.

Mr F. Darwin, President, in the Chair.

At a Meeting of the Society held at the Cavendish Laboratory, B. Cookson, Trinity College, was elected an Associate.

The following Communications were made to the Society:

(1) *On the figures produced on photographic plates by electric discharges.* By J. A. McClelland, M.A., Trinity College.

Several experimenters have lately drawn attention to the figures produced on photographic plates on development, after an electric discharge has taken place on to or near the surface of the plates. The figures are very interesting and well defined, and differ in a marked degree according as the discharge is positive or negative.

The manner of the production of these figures seems to be imperfectly understood, and it has not been shown whether they are due to a direct chemical action of the discharge on the plate, or whether they are simply produced by the light of the discharge. If the figures are produced by a direct chemical action on the plate it would point to a difference in the manner in which the positive and negative discharges spread over the plate; if produced by the action of light on the plate the figures would be explained by the different forms of the discharge in the air close to the plate.

In this paper the effect of the gaseous medium surrounding the discharging point on the form of the figures is investigated, and also the effect of interposing thin plates of different material between the film and the discharge. In this way we vary the form of the visible discharge in the gas close to the plate and can compare it with the figure produced, and we test the transparency of various substances to the action.

It will appear from the figures given that the effect on the films is produced principally by the light of the discharge.

The discharge passes through the air quite close to the film and the sharpness of the lines on the figures is explained by the nearness of the source of light to the plate, only the part of the plate just beneath the lines of discharge being affected.

On this explanation the difference in the positive and negative figures corresponds to the difference in the discharges in air; and the photographic action brings out these differences very clearly.

The figures Nos. 1 to 6 were taken in air at different pressures as given below.

No.	Positive Pressure.	No.	Negative Pressure.
1	1 atmosphere	5	1 atmosphere
2	·75 „		
3	·5 „	6	·5 „
4	·25 „		

The figures were taken with all other conditions the same but the pressure varied. The photographic plate was laid on a piece of ebonite and put under a bell-jar in which the air could be exhausted or compressed; a pointed metal rod just touched the surface of the film and was connected to the outer coating of a Leyden jar set on the table, the inner coating of which was joined to one of the knobs of a Wimshurst machine; the distance between the knobs was the same in all the figures.

The marked change in the figures produced by a change in the pressure of the air shows that the form of the figure depends to a great extent on the form which the discharge takes in the air near the film; even when the discharging point touches the film the condition of the air near the film is the principal factor in determining the nature of the figure.

With the intensity of discharge used the negative figure shows little detail at atmospheric pressure, while the positive shows its characteristic fine tapering lines. As the pressure diminishes both positive and negative figures increase in size and the negative breaks up into branching feathery lines. At the lowest pressure given ($\frac{1}{4}$ atmos.) the lines in both the positive and negative figures have broadened out until nearly all the plate is affected (the negative figure for this pressure is not reproduced but it was very similar in appearance to the positive).

Figures were also taken for pressures greater than atmospheric, and as the pressure increases the figures get smaller and smaller, the positive still showing its fine tapering lines and the negative becoming a circular spot in which no lines can be detected.

Now when we do the experiments in a dark room and watch the discharge carefully the form of the visible discharge is the same, as far as can be judged by the eye, as the form of the figures; in the positive we see the sharp branching lines and in the negative we see less distinctness of outline. As the pressure is lowered the lines broaden out until at the lowest pressures we see only a continuous flash all over the plate round the discharging point. The photographic figures are the same, but showing more detail than we can notice with the eye. In the figures given above the intensity of the discharge is small; it is interesting to increase the intensity and produce larger figures.

Figures 7 and 8 are positive and negative taken by placing the photographic plate on a metal plate connected to the outer coating of a Leyden jar; the outer coating of a second jar was joined to a metal rod which just touched the film. The jars were then charged until a spark passed between their inner coatings.

Here we have the discharge running out in heavy lines, from each of which at various points branch off small discharges similar to those in the smaller figures. In the positive the potential rose high enough to break down the air to the edge of the plate and on to the metal base, and the greater part of the discharge passed this way. Even the heavier lines in the negative are less bent about than in the positive.

To test whether the effects on the film are due to the light of the spark a number of experiments were made with thin plates, some transparent to light and some opaque, placed between the film and the discharging point.

With a thin plate of mica on the film the branching lines of the positive figure are only slightly less distinct than when the film is uncovered.

Figure 9 is taken with a small circular piece of glass $\frac{1}{3}$ mm. thick placed on the centre of the photographic film, and the discharging point touches the glass. The lines of the positive figure beneath the glass are quite distinct but broader than they otherwise would be; we would expect this broadening as the light is further from the film by the thickness of the glass.

For figure 10 the same piece of glass was used but half of it was rendered opaque to light by a thin coating of Brunswick black on the side next the film. Here again the figure comes out distinct through the transparent part but not through the opaque part.

In figure 11 part of the film was covered with a piece of ebonite $\frac{1}{2}$ mm. thick; the opaque ebonite entirely shielded the film although not much thicker than the glass which had little shielding effect.

These figures seem to prove that the light of the discharge is necessary to produce the figure on the plate.

As it is well known very perfect impressions of coins or any metal dies can be obtained by placing them on a photographic plate and passing a discharge to them. Here again the light of the discharge from the coin appears to be sufficient to explain the effect produced. Around the edge of the coin we have a discharge and also minute discharges from the edges of the raised portions of it, where the film is affected. Where the coin touches the film it is not affected.

Figure 12 is taken by weak sparking on a coin laid on a photographic plate; the edge of the coin is brought out by the

Fig. 1.

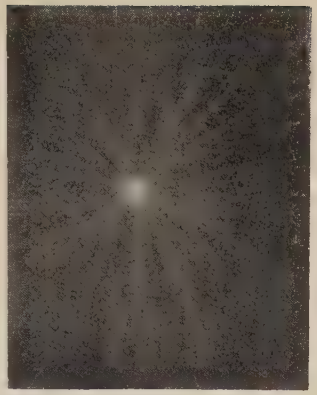


Fig. 2.

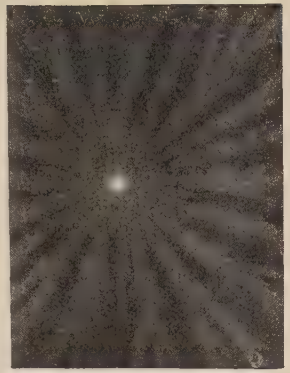


Fig. 3.

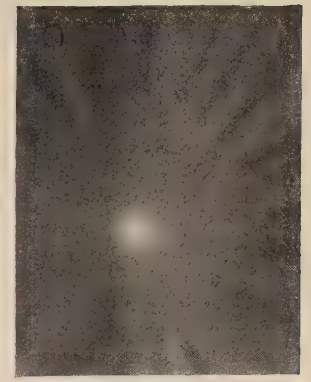


Fig. 4.

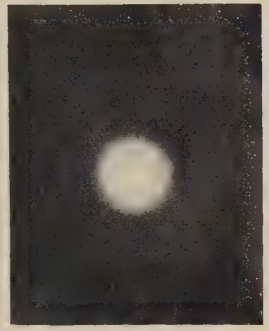


Fig. 5.

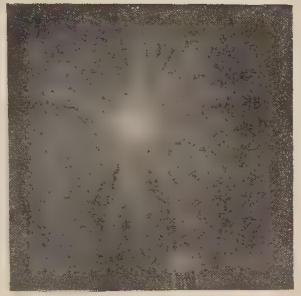


Fig. 6.

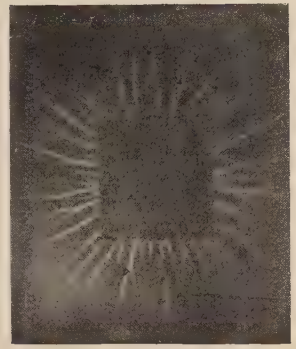


Fig. 11.

Fig. 12.

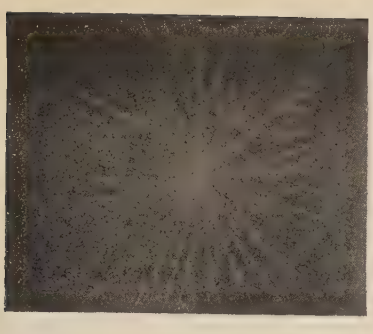


Fig. 9.

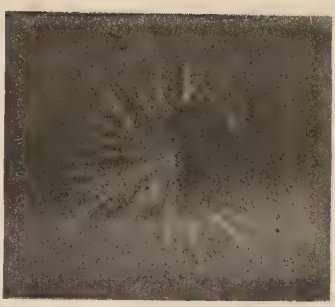


Fig. 10.

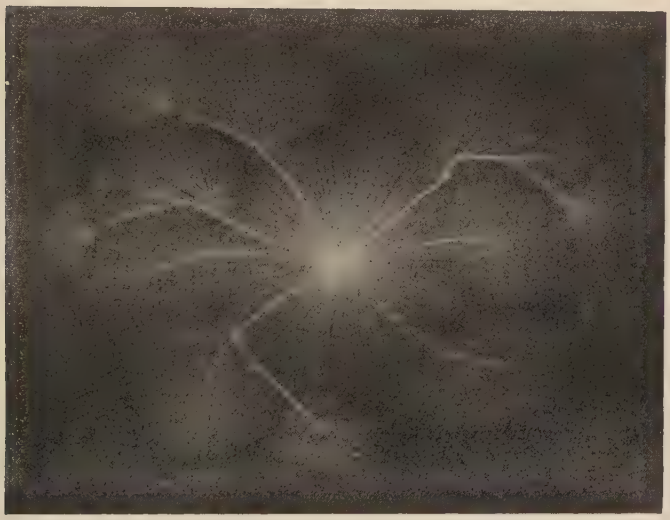


Fig. 8.

Fig. 7.

discharge from it and the letters by the minute discharges from their edges. Under the parts which touch the film there is no action, and under the level parts which did not touch there is weaker action.

We also get a very good impression of the coin if it is laid on the film and covered with an ebonite plate and the discharge allowed to pass to the ebonite; with this arrangement we have an induced discharge around the coin which gives the impression. If the ebonite is between the coin and the film no effect is produced.

All these effects on photographic plates would seem to be due, in great part at least, to the light of the discharge, although the amount of light may be very small indeed.

That the positive and negative figures differ so radically is due to the different forms of discharge in air in the two cases; the positive is more in the form of a brush and the negative of a glow.

Why the nature of the discharge into air from a positive and a negative point should be so different is a property as yet not understood.

It is interesting to compare these photographic figures with the figures produced by sprinkling a mixture of sulphur and minium on an ebonite plate on which a discharge has passed. The negatively electrified sulphur adheres to the plate if the discharge has been positive, the positively electrified minium if the discharge has been negative.

The positive and negative figures thus produced are very similar to the photographic figures, only the latter show more detail. Here again when the pressure of the air surrounding the plate is reduced the effect is similar to that with the photographic plate; the radiating lines in the positive become fainter and broader as the pressure is diminished. This effect of the pressure of the air has been observed by Joly (*Proc. Roy. Soc.* Vol. 47) who concludes that the figures owe their form chiefly to the manner in which the discharge spreads in the surrounding air.

In experiments with these dust figures one often gets a secondary positive in the centre of the primary negative or *vice versa*, but this is simply due to a secondary discharge back from the ebonite to the point used to convey the discharge to the plate. In fact if we touch with an earthed metal rod a plate of ebonite which has been electrified by a discharge having passed to it, we get a figure around the point touched opposite to the sign of the charge on the plate.

In conclusion I wish to thank Prof. Thomson for the assistance afforded by his suggestions.

(2) *On a Method of facilitating the measurement of temperature by means of Platinum Thermometry.* By E. B. H. WADE, B.A., Trinity College.

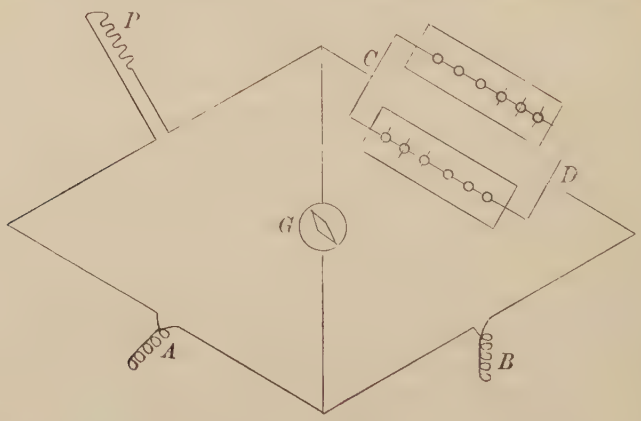
§ 1. *Object of the Paper.*

The method of measuring temperatures by finding the resistance of a platinum wire, claims a high degree of accuracy and is also widely applicable. Probably one reason why it is not more frequently used, is that equal increments of resistance do not represent equal increments of temperature, so that inconvenient calculations are required to deduce the latter. The object of this paper is to describe a means of avoiding these calculations. It is assumed that the formula employed by Callendar and Griffiths is exact, and if the directions contained in the paper are followed every unit of resistance will represent the same interval of temperature, which may be 1° Centigrade or any other. It is hoped that this will lead to the adoption of platinum thermometry by those who have hitherto been deterred by the calculations.

§ 2. *Description of a modification of Wheatstone's Bridge, which gives direct readings with platinum thermometers.*

Fig. 1 represents a form of Wheatstone's Bridge in which P is a platinum thermometer having resistance R_t at temp. t.

A, B, are coils of equal resistance of a galvanometer. If a single adjustable resistance were situated between points C, D,



the whole arrangement would be that in ordinary use. But in the suggested modification this interval is occupied by two adjustable resistances in parallel forming a shunt. And these coils are to be manipulated according to the following rule.

"By whatever amount the resistance of one arm of the shunt is increased, that of the other is to be diminished." Thus the sum of the resistances of the arms of the shunt is constant. Let it equal S. It will be shewn that if S is chosen rightly at the outset, and if the above rule is obeyed, then the condition that the bridge is balanced is that the arms of the shunt are linear functions of the temperature. By suitable choice of constants we may make the resistance in one arm numerically equal to the temperature. The rule is most easily followed by connecting two ordinary resistance boxes in parallel and transferring plugs from one to the other, so that one box gains in resistance what the other loses.

§ 3. *Demonstration of the accuracy of the Method suggested.*

Since the equation connecting resistance and temperature is that of a parabola with axis inclined, any of the following equations may be used in which R_t is the resistance at t, R_0 the resistance at $0°$, ϕ is the change of resistance between $0°$ and $100°$.

$$R_t = C + \alpha t - \beta t^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1),$$

$$R_t = \alpha'(t+z) - \beta'(t+z)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2),$$

$$\frac{R_t - R_0}{\phi} 100 = \delta \left\{ \frac{t^2}{100^2} - \frac{t}{100} \right\} \quad \ldots\ldots\ldots\ldots\ldots (3).$$

The constants in the three equations are connected as follows:

$$C = \alpha' z - \beta' z^2 = R_0,$$

$$\alpha = \alpha' - 2\beta z = \frac{(100+\delta)\phi}{100^2},$$

$$\beta = \beta' = \frac{\delta\phi}{100^2}.$$

Now equation (2) can be written as follows:

$$R_t = \frac{1}{\dfrac{1}{\alpha'(t+z)} + \dfrac{1}{\dfrac{\alpha'^2}{\beta} - \alpha'(t+z)}},$$

which will at once be recognized as the expression for the effective resistance of two conductors in parallel whose individual resistances are $\alpha'(t+z)$ and $\dfrac{\alpha'^2}{\beta} - \alpha'(t+z)$ respectively. Since these two are each linear functions of the temperature and since their sum is

constantly $\frac{\alpha'^2}{\beta}$, the method of § 2 is shewn to be justifiable. The constant $\frac{\alpha'^2}{\beta}$ will in future be called S.

In order to make this result practically serviceable, it must be remembered that at the present day it is usual to find the values of the constants ϕ, δ, R_0, (see § 3 equation (3)), in preference to any three others, and it will be necessary to shew how to obtain z, α', and S from these quantities.

To find z,

$$\alpha = \alpha' - 2\beta z,$$

$$R_0 = \alpha' z - \beta z^2,$$

$$\therefore\ \alpha = \frac{R_0 - \beta z^2}{z};$$

putting $$\alpha = \frac{(100+\delta)\phi}{(100)^2},\quad \beta = \frac{\delta\phi}{100^3},$$

we easily get

$$\frac{2z}{100} = \frac{100+\delta}{\delta} \pm \sqrt{\left(\frac{100+\delta}{\delta}\right)^2 + \frac{4R_0 \times 100}{\phi\delta}}.$$

Again $$\alpha' = \frac{R_0 + \beta z^2}{z}$$

$$= \frac{R_0 + \frac{\phi\delta}{100^3} z^2}{z},$$

$$S = \frac{\alpha'^2}{\beta}$$

$$= \frac{\alpha'^2 100^3}{\phi\delta};$$

as a control of the accuracy of the calculations we may use

$$R_0 = z\,\frac{\alpha + \alpha'}{2}.$$

Messrs Neville and Heycock have kindly furnished me with the constants of several pyrometers and from these I have calculated the values of S, α', z, which would convert them into direct reading instruments.

Example 1.

Pyr. 22. $\phi = 99{\cdot}786$, $z = 245{\cdot}771$,
$\delta = 1{\cdot}490$, $\alpha' = 1{\cdot}085815$,
$R_0 = 257{\cdot}885$, $S = 7929{\cdot}71$,
$[\alpha = 1{\cdot}012728]$.

Test: $z\dfrac{\alpha+\alpha'}{2} = 257{\cdot}885$,

but $R_0 = 257{\cdot}885$.

Example 2.

Pyr. 25. $\phi = 99{\cdot}206$, $z = 246{\cdot}765$,
$\delta = 1{\cdot}5135$, $\alpha' = 1{\cdot}081240$,
$R_0 = 257{\cdot}666$, $S = 7786{\cdot}18$,
$[\alpha = 1{\cdot}0070850]$.

Test: $z\dfrac{\alpha+\alpha'}{2} = 257{\cdot}666$,

but $R_0 = 257{\cdot}666$.

I have calculated what would be the resistance of these pyrometers at the melting point of gold (960°), using first equation (3) of § 3, secondly the rule

$$R = \alpha'(t+z) - \frac{[\alpha'(t+z)]^2}{S}.$$

The exact agreement only proves of course that the arithmetic and algebra has been correctly performed, but this it seems to me is all that need be proved. The conclusion is that by using this form of bridge the same temperature can be arrived at *without calculation* as would have been arrived at by the δ formula. Whether this method of calculating the temperature is itself correct is of course outside the object of this paper.

In concluding this note it may be well to point out a curious analogy between the action of the shunt and the action of the impurities in the wire. It is regarded by many as not improbable that in a wire of perfectly pure platinum the resistance might be a linear function of the temperature, and then no shunt would be required to give direct readings. If so we may regard the action of the impurity in the wire as similar to a shunt in providing an alternative path to the flow of electricity. It seems hardly worth while to push the idea further in a paper whose object is entirely practical.

I am greatly indebted to Mr Griffiths and also to Messrs Peacille and Heycock for the assistance received from them.

(3) *The development of Peripatus novae-britanniae.* By Dr A. Willey (Balfour Student).

The ova are without yolk and the nutrition of the embryo is effected by the development of a large trophic vesicle which occupies the entire dorsum of the embryo and projects far in front of the embryo as a head-fold and behind as a tail-fold. The trophic vesicle is thus a hollow closed cylinder lined internally by endoderm and externally by ectoderm, the cells of the latter being adapted for absorption of nutriment. The trophic folds were compared with the amniotic folds of insects. The trophic cavity becomes the gastral cavity of the adult and in the transformation from one to the other the endoderm undergoes certain changes. It secretes a basal membrane and a cuticular membrane simultaneously with a great increase in thickness; and between the two membranes the endoderm contains numerous small and large yolk-like globules which are probably to be regarded as reserve nutrient matter to tide the embryo over the first few days of its independent life. This late deposition of reserve nutrient matter derived ultimately from the maternal organism, as opposed to foreign ingested matter, is probably of some significance with regard to the question of the lecithality of the ovum. The embryo lies outside on the ventral surface of the trophic vesicle just as an insect embryo lies upon the yolk.

(4) *On the possibility of deducing magneto-optic phenomena from a direct modification of an electro-dynamic energy function.* By Mr J. G. Leathem, M.A., St John's College.

[Printed in the *Transactions*, Vol. xvii. Pt. I.]

The method initiated by Maxwell for the explanation of the Faraday effect depended on the direct insertion of a magneto-optic term in the energy. This method was extended by Fitzgerald and others to the explanation of Kerr's effect, namely the modification introduced in the circumstances of optical reflexion by magnetisation of the reflector. A difficulty occurred however in satisfying all the interfacial conditions, which virtually showed that such a scheme was not formally self-consistent. The origin of the discrepancy has been traced by Mr Larmor (Report on the Action of Magnetism on Light, *Brit. Assoc.* 1893) to omission to secure what may for shortness be called the electromotive incompressibility of the medium: in the ordinary problem of optical reflexion there is no tendency for this to be disturbed, but when Maxwell's magneto-optic energy terms are included the reaction against compression introduces what may be termed an electric pressure, which must appear in the equations

It was necessary to compare the modified scheme thus obtained with experimental knowledge: and the calculations given in this paper show that in fact it does not represent the phenomena. The paper is only a summary of the actual calculations, because since they were completed the author has shown (On the Magneto-optic Phenomena of Iron, Nickel and Cobalt, *Phil. Trans.* 1897) that the other rigorous theory formulated as an alternative by Mr Larmor (*loc. cit.*), which leads to an analytical scheme practically the same as those advanced on various hypotheses by Fitzgerald, Goldhammer, Basset, Drude, and others, is in much more satisfactory agreement with experiment. This brief history of the subject shows the desirability of the examination of the consequences involved in the former method of explanation: the result is however what was to be expected by those who adhere to the more recent formulation (Larmor, A Dynamical Theory of the Electric and Luminiferous Medium, Part III., *Phil. Trans.* 1898) of optical theory which treats a material medium as free æther pervaded by discrete molecules involving in their constitution electrons considered as nuclei of intrinsic æthereal strain. On such a view a continuous energy function is not the starting point, and the influence of these discrete nuclei could hardly be conceived to modify the propagation in the intervening æther in so fundamental a manner as an electromotive pressure would demand.

(5) *On the solutions of the equation* $(\nabla^2 + \kappa^2)\psi = 0$ *in elliptic coordinates and their physical applications.* By R. C. Maclaurin, B.A., St John's College.

[Printed in the *Transactions*, Vol. XVII. Pt. I.]

(6) *On the interpretation of divergent solutions of the hypergeometric equation.* By Mr W. M^{c}F. Orr, Professor at the Royal College of Science, Dublin.

The Harmonic Expression of the Daily Variation of Solar Radiation, and the Annual Variation of its coefficients. By R. HARGREAVES, M.A., formerly Fellow of St John's College, Cambridge.

[*Read* 22 November 1897. *Revised* 1898.]

A harmonic series, applied to express the intermittent daily supply of heat from the Sun, consists of a series of terms with the day, half-day, one-third of a day &c. as periods, and an opening term on which the total supply for the day depends. Each coefficient depends on the latitude of the place and on the Sun's declination, and through the latter is subject to an annual variation.

This annual variation for the opening term was discussed in a previous communication to the Society*. The present paper contains the solution for the general case, and tables are appended giving numerical values for latitudes up to 60°, at intervals of 10°.

A corresponding analysis of the daily variation of temperature, and the yearly variation of its coefficients, will be found in a volume published by the Meteorological Council, under the title 'Harmonic Analysis of hourly observations of Air Temperature and Pressure at British Observatories'; and an account of the method and general results in a paper, bearing the same title, by Lieutenant-General Strachey, published in the *Transactions of the Royal Society* for 1893. I understand that this is practically the only case in which the Harmonic Analyser has been applied to the second stage, *i.e.* to express the *annual* variation of coefficients of the *daily* expansion. The preliminary comparisons which I have made of these results of observation, with the calculations of the present paper, are sufficiently promising to justify a detailed study, should time permit. But the time and labour required are very serious, and the reader who gives a casual glance at the collection of numerical tables appended here, will hardly realise the weeks of laborious calculation involved. These tables, with explanations in sections 1 and 5 are all that is needed for meteorological purposes. Section 3 gives a brief *résumé* of properties of the coefficients, which in the more general form of the mathematical problem, bear a curious resemblance to the Associated Functions of Laplace.

§ 1. With a diathermanous atmosphere, the amount of solar radiation reaching a unit area of the earth's surface, depends on

* 'Distribution of Solar Radiation on the Surface of the Earth, and its dependence on Astronomical Elements,' printed in *Transactions*, 1896.

the distance of the sun and the angle of exposure to the sun's rays. The element of heat-supply may be written $\frac{Hdt}{r^2}\cos I$, in which dt is time-element, r the distance of the sun from the earth, and I the sun's zenith distance. In latitude λ, when the sun's declination is δ, $\cos I = \sin\lambda\sin\delta + \cos\lambda\cos\delta\cos\psi$; ψ the hour-angle varies in the course of the day from $-\psi_1$ to $+\psi_1$, where ψ_1 the hour-angle at sunset is given by

$$\sin\lambda\sin\delta + \cos\lambda\cos\delta\cos\psi_1 = 0.$$

The factor $\cos I$ is positive during the day, and at night is replaced by zero.

In lieu of this discontinuous expression, we propose to write the quasi-continuous expansion in cosines of multiples of ψ, in which ψ varies from $-\pi$ to $+\pi$, embracing the whole twenty-four hours. Fourier's method gives for this

$$\frac{2Hdt}{r^2}\Sigma\chi_s\cos s\psi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

in which

$$\chi_s = \int_0^{\psi_1}(\sin\lambda\sin\delta + \cos\lambda\cos\delta\cos\psi)\cos s\psi\, d\psi \quad \ldots(2).$$

The factor 2 is to be omitted for the opening term ($s=0$), on which depends the day's integral heat-supply, and so the element of the annual variation discussed in the previous paper. The other terms express the daily variation divided into harmonic components, whose periods are the day and its exact sub-multiples.

Integrating by parts, for the general case

$$\left.\begin{aligned}\chi_s &= \frac{\cos\lambda\cos\delta}{2s}\left(\frac{\sin(s-1)\psi_1}{s-1} - \frac{\sin(s+1)\psi_1}{s+1}\right)\\ \text{and for the case } s=1,\quad \chi_1 &= \frac{\cos\lambda\cos\delta}{2}(\psi_1 - \tfrac{1}{2}\sin 2\psi_1)\end{aligned}\right\}\ldots(3).$$

As ψ_1 is known in terms of λ and δ, these formulae give numerical values of the coefficients for an assigned latitude and date of the year. But a general expression for χ_s may be obtained in a series of associated Laplace's functions with $\sin\lambda$ and $\sin\delta$ as arguments, applicable to the whole globe and taking proper account of the discontinuity that occurs in arctic regions. For greater generality this is found for

$$\chi_{p,s} = \int_0^{\psi_1}(\sin\lambda\sin\delta + \cos\lambda\cos\delta\cos\psi)^p\cos s\psi\, d\psi \ldots(4),$$

the resemblance of which to the integral expression for an associated function, viz.,

$$\int_0^\pi (x+\sqrt{x^2-1}\cos\psi)^n \cos s\psi \, d\psi,$$

strikes the eye at once. Physically this corresponds to an absorbing atmosphere in which the fraction of light or heat transmitted is $(\cos I)^{p-1}$, depending on the angle between the zenith and the sun's direction; one factor $\cos I$ being due to the oblique exposure of a surface.

§ 2. First notice that the conditions for arctic and non-arctic regions are embraced in the statement that all values ψ between 0 and π, which make $\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi$ positive, are admitted in the integration. During the period of total day ψ_1 is continuously π, and during the period of total night it is 0.

Now if we expand in zonal harmonics a function which $=x^p$ for positive values of x, and vanishes for negative values, the coefficient of $P_n(x)$ in the expansion is

$$\left.\begin{aligned} A_{p,n} &= \frac{2n+1}{2}\int_0^1 x^p P_n(x)\,dx \\ &= \frac{2n+1}{2}\times\frac{p(p-2)\ldots(p-n+2)}{(p+1)(p+3)\ldots(p+n+1)} \text{ when } n \text{ is even} \\ &= \frac{2n+1}{2}\times\frac{(p-1)(p-3)\ldots(p-n+2)}{(p+2)(p+4)\ldots(p+n+1)} \quad \text{,, ,, ,, odd} \end{aligned}\right\} \ldots(5).$$

(See Todhunter's *Functions of Laplace*, pp. 18 and 19.) Introduce this expansion in (4), writing $\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi$ for x, and we get

$$\chi_{p,s}=\int_0^\pi \Sigma A_{p,n} P_n(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)\cos s\psi\,d\psi,$$

the integration being now from 0 to π, because we have replaced the original expression, with its discontinuity and variable limits, by the quasi-continuous expansion.

With $\mu=\sin\lambda$, $\mu'=\sin\delta$, we have

$$P_n(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi) = 2\Sigma\frac{\lfloor n-s}{\lfloor n+s}(1-\mu^2)^{\frac{s}{2}}(1-\mu'^2)^{\frac{s}{2}}P_n^{\,s}(\mu)P_n^{\,s}(\mu')\cos s\psi,$$

where $P_n^{\,s}(\mu)=\dfrac{d^sP_n}{d\mu^s}$, and the factor 2 is to be omitted for $s=0$.

Hence the expansion sought is

$$\chi_{p,s} = \pi \Sigma A_{p,n} \frac{\lfloor n-s}{\lfloor n+s} (1-\mu^2)^{\frac{s}{2}} (1-\mu'^2)^{\frac{s}{2}} P_n^s(\mu) P_n^s(\mu') \ldots (6).$$

The formulae quoted for $A_{p,n}$, though correct, are rather misleading for the case in which p is an integer; they suggest that differences in type turn on n being odd or even, whereas in reality they depend on $p-n$ being odd or even. Whether n is odd or even, the group of terms for which $p-n$ is *even* terminates with $n=p$, and the formula is

$$\frac{2n+1}{2} \frac{\lfloor p}{1.3 \ldots (n+p+1) . 2.4 \ldots (p-n)} \quad \ldots\ldots (7a).$$

When $p-n$ is odd and positive, the form is

$$\frac{2n+1}{2} \frac{\lfloor p}{2.4 \ldots (n+p+1) . 1.3 \ldots (p-n)} \quad \ldots\ldots (7b).$$

When $n=p+1$, it is

$$\frac{2n+1}{2} \frac{1}{2^{p+1}(p+1)} \quad \ldots\ldots\ldots\ldots\ldots (7c).$$

When $n-p$ is odd and greater than 1, it is

$$\frac{2n+1}{2} \frac{1.3 \ldots (n-p-2)}{2^p (2p+2)(2p+4) \ldots (p+n+1)} (-1)^{\frac{1}{2}(n-p-1)} \ldots (7d).$$

Thus up to $n=p+1$, all terms odd and even occur and all are positive, beyond this only terms for which $n-p$ is odd occur, alternately positive and negative, the first negative term being that for which $n=p+3$. All the terms for which $n \sim p$ is odd are embraced in the single formula

$$\frac{2n+1}{2} \frac{1.3 \ldots (n+p) . \lfloor p \, (-1)^{\frac{1}{2}(n+p+\overline{-1}\,|\,n)}}{2.4 \ldots (n+p+1)(n-p)(n-p+2) \ldots (n+p)},$$

which will be found to give correctly the three cases (*b*), (*c*) and (*d*).

The result (6) was obtained for $s=0$ in the previous paper by a much more troublesome method.

§ 3. Various sequence formulae may be established directly from the definition (4). Obviously

$$\chi_{p,s} = \chi_{p-1,s} \sin\lambda \sin\delta + \tfrac{1}{2} \cos\lambda \cos\delta \, (\chi_{p-1,s-1} + \chi_{p-1,s+1}) \ldots (8).$$

Also

$$\tfrac{1}{2}p\cos\lambda\cos\delta\,(\chi_{p-1,s-1}-\chi_{p-1,s+1})=\int_0^{\psi_1}p\sin\psi\sin s\psi\,.\,\cos\lambda\cos\delta\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p-1}\,d\psi$$

$$=\Big[-\sin s\psi\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p}\Big]_0^{\psi_1}+\int_0^{\psi_1}s\times(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p}\cos s\psi\,d\psi,$$

or
$$s\chi_{p,s}=\tfrac{1}{2}p\,.\,\cos\lambda\cos\delta\,(\chi_{p-1,s-1}-\chi_{p-1,s+1})\ldots\ldots\ldots(9).$$

From (8) and (9) we get

$$\tfrac{1}{2}p\cos\lambda\cos\delta\,\{(s-1)\,\chi_{p-1,s-1}-(s+1)\,\chi_{p-1,s+1}\}=(s^2-p)\,\chi_{p,s}+p\chi_{p-1,s}\sin\lambda\sin\delta\ldots\ldots(10),$$

which is wanted below.

Again

$$\frac{d}{d\psi}\{\sin\psi\cos s\psi\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p-1}\}$$

$$=(\cos\psi\cos s\psi-s\,.\sin\psi\sin s\psi)\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p-1}$$

$$-(p-1)\cos\lambda\cos\delta\sin^2\psi\cos s\psi\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p-2}.$$

Hence, writing

$$\cos\psi\cos s\psi-s.\sin\psi\sin s\psi=\tfrac{1}{2}\{(s+1)\cos(s+1)\psi-(s-1)\cos(s-1)\psi\},$$

and integrating

$$(p-1)\cos\lambda\cos\delta\int_0^{\psi_1}\sin^2\psi\cos s\psi\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p-2}d\psi=\tfrac{1}{2}\{(s+1)\,\chi_{p-1,s+1}-(s-1)\,\chi_{p-1,s-1}\},$$

or by (10)

$$p\,(p-1)\cos^2\lambda\cos^2\delta\int_0^{\psi_1}\sin^2\psi\cos s\psi\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p-2}d\psi=(p-s^2)\,\chi_{p,s}-p\chi_{p-1,s}\sin\lambda\sin\delta.$$

But evidently

$$\cos^2\lambda\cos^2\delta\int_0^{\psi_1}\cos^2\psi\cos s\psi\,(\sin\lambda\sin\delta+\cos\lambda\cos\delta\cos\psi)^{p-2}\,d\psi=\chi_{p,s}-2\chi_{p-1,s}\sin\lambda\sin\delta+\chi_{p-2,s}\sin^2\lambda\sin^2\delta.$$

Hence, multiplying this by $p\,(p-1)$, and adding to the previous equation,

$$p\,(p-1)\chi_{p-2,s}\cos^2\lambda\cos^2\delta=(p^2-s^2)\,\chi_{p,s}-p\,(2p-1)\chi_{p-1,s}\sin\lambda\sin\delta+p\,(p-1)\,\chi_{p-2,s}\sin^2\lambda\sin^2\delta,$$

or

$$p(p-1)(\cos^2\lambda-\sin^2\delta)\chi_{p-2,s}=(p^2-s^2)\cdot\chi_{p,s}-p(2p-1)\chi_{p-1,s}\sin\lambda\sin\delta \quad \ldots\ldots\ldots(11),$$

a sequence equation in which only the letter p changes.

The elimination of $\chi_{p,s}$ from (8) and (9) gives an equation in which only $p-1$ occurs, and if we raise by 1 the value of p this is

$$\tfrac{1}{2}\cos\lambda\cos\delta\{(s+p+1)\chi_{p,s+1}+(s-p-1)\chi_{p,s-1}\}+s\chi_{p,s}\sin\lambda\sin\delta=0 \quad \ldots\ldots\ldots\ldots(12),$$

the sequence equation in which only the letter s changes.

Further differentiating $\chi_{p,s}$ with regard to λ, we easily obtain

$$\cos\lambda\frac{d\chi_{p,s}}{d\lambda}+p\chi_{p,s}\sin\lambda=p\chi_{p-1,s}\sin\delta \quad \ldots\ldots\ldots(13),$$

and differentiating again

$$\cos\lambda\frac{d^2\chi_{p,s}}{d\lambda^2}+(p-1)\sin\lambda\frac{d\chi_{p,s}}{d\lambda}+p\chi_{p,s}\cos\lambda=p\sin\delta\frac{d\chi_{p-1,s}}{d\lambda}.$$

Multiply this by $\cos\lambda$, and substitute for $p\sin\lambda\cos\lambda\dfrac{d\chi_{p,s}}{d\lambda}$ and for $p\cos\lambda\sin\delta\dfrac{d\chi_{p-1,s}}{d\lambda}$ from (13) (in the latter case with p reduced to $p-1$), then

$$\cos^2\lambda\frac{d^2\chi_{p,s}}{d\lambda^2}-\sin\lambda\cos\lambda\frac{d\chi_{p,s}}{d\lambda}+p\chi_{p,s}\cos^2\lambda$$
$$=p^2\sin^2\lambda-p(2p-1)\chi_{p-1,s}\sin\lambda\sin\delta+p(p-1)\chi_{p-2,s}\sin^2\delta.$$

Simplify the right-hand member by the sequence equation (11) and divide by $\cos^2\lambda$,

$$\frac{d^2\chi_{p,s}}{d\lambda^2}-\tan\lambda\frac{d\chi_{p,s}}{d\lambda}+\left\{p(p+1)-\frac{s^2}{\cos^2\lambda}\right\}\chi_{p,s}=p(p-1)\chi_{p-2,s} \quad (14).$$

The left-hand member of this equation has the form of Laplace's equation for $\Theta_{p,s}$. In this case the function is linked to the alternate one by the right-hand member, which is zero for the Laplacian.

§ 4. The result (6) for the case in which $\sin\delta$ or $\mu'=0$, gives the expansion of $(1-\mu^2)^{\frac{p}{2}}$ in associated functions, an extension of the well-known expansion of $(1-\mu^2)^{\frac{1}{2}}$ in zonal harmonics. With $\mu'=0$,

$$\chi_{p,s}=\cos^p\lambda\int_0^{\frac{\pi}{2}}\cos^p\psi\cdot\cos s\psi\,d\psi=\frac{\pi\cdot\lfloor p\cos^p\lambda}{2^{p+1}\left\lfloor\frac{1}{2}(p-s)\right\rfloor\left\lfloor\frac{1}{2}(p+s)\right\rfloor}$$

when $p-s$ is even and ≥ 0,

$$= 0$$

when $p-s$ is even and < 0,

$$= \frac{\lfloor p \,.\, \cos^p \lambda \, (-1)^{\frac{1}{2}(p+s+\overline{-1}^s)}}{(s-p)\,(s-p+2)\ldots(s+p)}$$

when $p \sim s$ is odd.

On the right-hand of (6) the terms of the series which do not vanish are those for which $n-s$ is even, in which case

$$P_n^{\,s}(0) = \frac{\lfloor n+s \,(-1)^{\frac{1}{2}(n-s)}}{2^n \left\lfloor \frac{1}{2}(n-s) \right\rfloor \frac{1}{2}(n+s)}.$$

Taking $p-s$ even, $p-n$ is also even, and the expansion is finite, terminating with $n=p$. Thus s being not greater than p, n ranges from s to p, and

$$\frac{\pi \lfloor p \,(1-\mu^2)^{\frac{p}{2}}}{2^{p+1} \left\lfloor \frac{1}{2}(p-s) \right\rfloor \frac{1}{2}(p+s)} = \Sigma_{n=s}^{n=p} \frac{\pi}{2} \frac{(2n+1) \,.\, \lfloor p \, \lfloor n-s}{1.3\ldots(p+n+1).2.4\ldots(p-n) \lfloor n+s}$$

$$\times \frac{(-1)^{\frac{1}{2}(n-s)} \lfloor n+s}{2^n \left\lfloor \frac{1}{2}(n-s) \right\rfloor \frac{1}{2}(n+s)} (1-\mu^2)^{\frac{s}{2}} P_n^{\,s}(\mu),$$

or

$$(1-\mu^2)^{\frac{p}{2}} = \Sigma_{n=s}^{n=p} \frac{(-1)^{\frac{1}{2}(n-s)} \,.\, 2^p \,(2n+1) \,.\, \lfloor n-s \, \lfloor \frac{1}{2}(p-s) \, \lfloor \frac{1}{2}(p+s)}{2^n \, 1.3\ldots(n+p+1).2.4\ldots(p-n) \, \lfloor \frac{1}{2}(n-s) \, \left\lfloor \frac{1}{2}(n+s) \right.}$$

$$(1-\mu^2)^{\frac{s}{2}} P_n^{\,s}(\mu) \ldots\ldots\ldots (15).$$

With $p \sim s$ odd, $n \sim p$ is odd and the series infinite. Thus

$$\frac{\lfloor p \,.\, (1-\mu^2)^{\frac{p}{2}} (-1)^{\frac{1}{2}(p+s+\overline{-1}^s)}}{(s-p)\,(s-p+2)\ldots(s+p)}$$

$$= \Sigma_{n=s}^{n=\infty} \frac{\pi\,(2n+1)}{2.2.4\ldots(p+n+1)} \frac{\lfloor p \,.\, 1.3\ldots(n+p)\,(-1)^{\frac{1}{2}(n+p+\overline{-1}^n)}}{(n-p)\,(n-p+2)\ldots(n+p)}$$

$$\frac{\lfloor n-s}{\lfloor n+s} \cdot \frac{(-1)^{\frac{1}{2}(n-s)} \lfloor n+s}{\left\lfloor \frac{1}{2}(n-s) \right\rfloor \frac{1}{2}(n+s)} (1-\mu^2)^{\frac{s}{2}} P_n^{\,s}(\mu).$$

$$\therefore (1-\mu^2)^{\frac{p}{2}}$$

$$= \Sigma_{n=s}^{n=\infty} \frac{\pi(2n+1)1.3\ldots(n+p)(s-p)(s-p+2)\ldots(s+p) \lfloor n-s}{2^{n+1} 2.4\ldots(n+p+1)(n-p)(n-p+2)\ldots(n+p) \left\lfloor \frac{1}{2}(n-s) \right\rfloor \frac{1}{2}(n+s)}$$

$$(1-\mu^2)^{\frac{s}{2}} P_n^{\,s}(\mu) \ldots\ldots\ldots (16).$$

Remembering that

$$\int_0^1 (1-\mu^2)^s [P_n^s(\mu)]^2 d\mu = \frac{1}{2n+1} \frac{\lfloor n+s}{\lfloor n-s},$$

(15) implies

$$\int_0^1 (1-\mu^2)^{\frac{p+s}{2}} \frac{d^s P_n}{d\mu^s} d\mu = \frac{(-1)^{\frac{1}{2}(n-s)} \cdot 2^p \lfloor n+s \lfloor \frac{1}{2}(p-s) \lfloor \frac{1}{2}(p+s)}{2^n . 1.3...(n+p+1).2.4...(p-n) \lfloor \frac{1}{2}(n-s) \lfloor \frac{1}{2}(n+s)} \quad \ldots\ldots\ldots\ldots(17),$$

with n, p, s odd or even together, $n \geqq s$, $p \geqq s$, $p \geqq n$; but if with n, p, s odd or even together, $s > p$, then the integral vanishes.

In the same way (16) gives for n and s odd or even together and p the reverse, and either n or p the greater,

$$\int_0^1 (1-\mu^2)^{\frac{p+s}{2}} \frac{d^s P_n}{d\mu^s} d\mu$$

$$= \frac{\pi . 1 . 3 \ldots (n+p) \; n+s \, (s-p)(s-p+2)\ldots(s+p)}{2^{n+1} . 2 . 4 \ldots (n+p+1) \lfloor \frac{1}{2}(n-s) \lfloor \frac{1}{2}(n+s) (n-p)(n-p+2)\ldots(n+p)} \quad \ldots\ldots\ldots(18).$$

For the Θ notation the connecting formula is

$$\lfloor n-s \, (1-\mu^2)^{\frac{s}{2}} P_n^s(\mu) = 1 . 3 \ldots (2n-1) \, \Theta_{n,s}.$$

Two particular cases may be noted, viz. those for which $s = 0$; (16) then gives for $p = -1$, 0, and all positive integers*

$$\left(p = -1, \text{ opening term } \frac{\pi}{2}\right),$$

$$(1-\mu^2)^{\frac{2p+1}{2}} = \frac{\pi}{2} \cdot \frac{1 . 3 \ldots (2p+1)}{2 . 4 \ldots (2p+2)} + \frac{\pi}{2} (-1)^{p+1} \{1 . 3 \ldots (2p+1)\}^2$$

$$\sum_{n=1}^{n=\infty} \frac{(4n+1) 1 . 3 \ldots (2n-1) 1 . 3 \ldots (2n+2p+1)}{2 . 4 \ldots 2n . 2 . 4 \ldots (2n+2p+2)(2n-2p-1)(2n-2p+1)\ldots(2n+2p+1)} P_{2n}(\mu)$$

$$\ldots\ldots\ldots(19),$$

and similarly (15) gives

$$(1-\mu^2)^p = 2^p (\lfloor p)^2 . \sum_{n=0}^{n=p} \frac{(-1)^n (4n+1) \, P_{2n}(\mu)}{(2n+1)(2n+3)\ldots(2n+2p+1) \lfloor p \lfloor p-n} \quad \ldots\ldots(20).$$

* This expansion was obtained directly in *Messenger of Mathematics*, Oct. 1896, p. 92, but the reduction there given is only true for $2n > 2p+1$. The form above suits either case.

§ 5. Returning now to the particular case of the functions χ required for a diathermanous atmosphere, $p = 1$, and we need only write the s suffix; accordingly with $a_n = \dfrac{1.3 \ldots (2n-1)}{2.4 \ldots 2n}$ used for abbreviation,

$$\chi_1 = \frac{\pi}{4}\cos\lambda\cos\delta + \frac{\pi}{2}\cos\lambda\cos\delta \sum_{n=1}^{n=\infty} \frac{(-1)^{n-1}(4n+1)\,a_n}{(2n-1)(2n+2)} \cdot \frac{P'_{2n}(\sin\lambda)P'_{2n}(\sin\delta)}{2n(2n+1)}$$

$$\chi_2 = \frac{\pi}{2}\cos^2\lambda\cos^2\delta\Sigma \quad ,, \quad ,, \quad \frac{P''_{2n}(\sin\lambda)P''_{2n}(\sin\delta)}{(2n-1)2n(2n+1)(2n+2)}$$

$$\chi_3 = \frac{\pi}{2}\cos^3\lambda\cos^3\delta\Sigma \quad ,, \quad ,, \quad \frac{P'''_{2n}(\sin\lambda)P'''_{2n}(\sin\delta)}{(2n-2)(2n-1)2n(2n+1)(2n+2)(2n+3)}$$

$$\chi_4 = \frac{\pi}{2}\cos^4\lambda\cos^4\delta\Sigma \quad ,, \quad ,, \quad \frac{P''''_{2n}(\sin\lambda)P''''_{2n}(\sin\delta)}{(2n-3)(2n-2)(2n-1)2n(2n+1)(2n+2)(2n+3)(2n+4)}$$

.....(21)

First notice that all these coefficients vanish at the poles, as they should do, for there is no daily variation proper at the poles.

Again, $\sin\lambda$, $\sin\delta$ change signs, one in passing from northern to southern hemisphere, the other in passing through the equinoxes; $\cos\lambda$, $\cos\delta$ do not change sign.

Hence we see that χ_1 has a single even term $\frac{\pi}{4}\cos\lambda\cos\delta$, the remaining odd terms, $\chi_3\chi_5\ldots$ are entirely odd, $\chi_2\chi_4\ldots$ entirely even, conclusions which may be drawn directly from (3). When $\sin\delta$ is expressed in terms of solar longitude by the relation

$$\sin\delta = \sin\epsilon\sin\theta,$$

$\chi_3\chi_5\ldots$ and the odd part of χ_1 have the form

$$a_1\sin\theta + a_3\sin 3\theta + a_5\sin 5\theta + \ldots,$$

$\chi_2\chi_4$ and the even part of χ_1 have the form

$$a_0 + a_2\cos 2\theta + a_4\cos 4\theta + \ldots.$$

Where an odd power of $\cos\delta$ occurs, an elliptic integral is wanted to complete the harmonic expression, thus the even part of χ_1, viz.

$$\frac{\pi}{4}\cos\lambda\cos\delta = \cos\lambda\left(\frac{E}{2} + E_2\cos 2\theta + E_4\cos 4\theta\ldots\right)$$

$$= \cos\lambda\,(\cdot 7533 + \cdot 0324\cos 2\theta - \cdot 0003\cos 4\theta\ldots),$$

E_{2n} being used for the integral

$$\int_0^{\frac{\pi}{2}} \cos\delta \,.\, \cos 2n\theta \, d\theta,$$

or

$$\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2\epsilon \sin^2\theta} \,.\, \cos 2n\theta \, d\theta.$$

The series (21) converge only slowly; they have been used to confirm other work for the latitude of Greenwich and for latitude 60°, but in the absence of tables of associated functions calculated with reference to angle, the labour entailed is absolutely prohibitive. Accordingly in the case of the even functions $\chi_2 \chi_4 \ldots$ the formula (3) was used to give particular values for $\theta = 0°$, 30°, 60°, 90°. Knowing the form of the expansion, these are sufficient to determine $a_0\, a_2\, a_4\, a_6$ on the assumption that a_8 and subsequent coefficients are negligible. The tabulated results for χ_0 obtained in the previous paper (Q there) were obtained directly from finite formulae involving elliptic integrals, and shewed a rapid convergence in the numbers $a_0\, a_2 \ldots$, although in that case also the convergence of the zonal harmonics was slow. The expectation that this would also prove to be the case for $\chi_2 \ldots$ was confirmed by the small values obtained for a_6, so small that although a_6 was always included in the work, its values are not tabulated, as they only just come into the 4th place of decimals for latitudes 50° and 60°. For the odd functions $\chi_3 \chi_5 \ldots$ and the odd part of χ_1 only the values for 30°, 60°, 90° are available; sufficient to give $a_1\, a_3\, a_5$ on the assumption that subsequent terms may be neglected. Here also the values of a_5 turned out very small. The calculations were made to 6 places, and two are suppressed.

As regards the tables it should be mentioned that when

$$r^2 \frac{d\theta}{dt} = h$$

is used, formula (1) becomes

$$\frac{2H\, d\theta}{h} [\tfrac{1}{2}\chi_0 + \Sigma \chi_s \cos s\psi],$$

in which the χ's depend on θ as shewn in table (A). With one year for the unit of time $h = 2\pi ab$, also $d\theta = 2\pi\, dt$ [1 + small quantities depending on eccentricity].

The transformation to mean time is made as in § 17 of the previous paper, the product $\chi_s d\theta$ becoming $2\pi \chi_s' dt$, so that formula (1) then stands

$$\frac{2H}{ab}\, dt\, [\tfrac{1}{2}\chi_0' + \Sigma \chi_s' \cos s\psi],$$

and accordingly the χ_s' functions tabulated in (B) contain the correction for the varying distance of the sun. For abbreviation ϕ is written for $2\pi t$. The even terms give no difference between northern and southern hemispheres, for χ_3' the sign is to be changed throughout for the southern hemisphere, for χ_1' with both odd and even terms, the effects are more complicated and results given for both hemispheres.

(A) Table of χ's expressed as harmonic functions of sun's longitude measured from the vernal equinox.

$$\chi_1 = a_0 + a_1 \sin\theta + a_2 \cos 2\theta + a_3 \sin 3\theta + a_4 \cos 4\theta + \ldots$$

$\lambda =$ 0°	·7533	·0	·0324	·0	−·0003
10°	·7419	·0691	·0319	·0	−·0003
20°	·7079	·1357	·0305	·0002	−·0003
30°	·6524	·1976	·0280	·0008	−·0003
40°	·5771	·2516	·0248	·0015	−·0002
50°	·4842	·2946	·0208	·0038	−·0002
60°	·3767	·3190	·0162	·0095	−·0002

$$\chi_2 = a_0 + a_2 \cos 2\theta + a_4 \cos 4\theta.$$

$\lambda =$ 0°	·3197	+·0138	−·0001
10°	·3140	·0144	−·0006
20°	·2952	·0183	−·0002
30°	·2648	·0242	−·0003
40°	·2227	·0330	−·0004
50°	·1689	·0455	−·0002
60°	·1037	·0616	+·0017

$$\chi_3 = a_1 \sin\theta + a_3 \sin 3\theta.$$

$\lambda =$ 10°	−·0229	−·0
20°	−·0441	−·0004
30°	−·0618	−·0016
40°	−·0733	−·0042
50°	−·0737	−·0097
60°	−·0531	−·0204

$$\chi_4 = a_0 + a_2 \cos 2\theta + a_4 \cos 4\theta.$$

$\lambda =$ 0°	−·0639	−·0028	+·0
10°	−·0618	−·0039	+·0001
20°	−·0550	−·0078	−·0001
30°	−·0439	−·0139	+·0000
40°	−·0289	−·0217	−·0001
50°	−·0115	−·0290	−·0025
60°	+·0025	−·0272	−·0087

(B) Table of χ_s' expressed in mean time.

$\phi = 2\pi t$, t measured from perihelion, unit time 1 year.

χ_1'

$\lambda =$ 0°	$\cdot7533 + \cdot0270\cos(\phi + 0°\ 43') - \cdot0321\cos(2\phi + 21°\ 50') - \cdot0016\cos(3\phi + 22°\ \ ) - \cdot0003\cos(4\phi + 44°)$
10° N.	$\cdot7419 - \cdot0438\cos(\phi + 17°\ \ ) - \cdot0339\cos(2\phi + 21°\ 28') - \cdot0017\cos(3\phi + 21°\ 31') - \cdot0003\cos(4\phi + 44°)$
S.	$\quad + \cdot0943\cos(\phi + 8°\ 11') - \cdot0293\cos(2\phi + 23°\ 8') - \cdot0015\cos(3\phi + 22°\ 31') - \cdot0003\cos(4\phi + 44°)$
20° N.	$\cdot7079 - \cdot1119\cos(\phi + 13°\ 17') - \cdot0343\cos(2\phi + 20°\ 46') - \cdot0014\cos(3\phi + 26°\ \ ) - \cdot0003\cos(4\phi + 44°)$
S.	$\quad + \cdot1597\cos(\phi + 8°\ 50') - \cdot0261\cos(2\phi + 24°\ 12') - \cdot0016\cos(3\phi + 24°\ 12') - \cdot0003\cos(4\phi + 44°)$
30° N.	$\cdot6524 - \cdot1756\cos(\phi + 12°\ 20') - \cdot0342\cos(2\phi + 20°\ 4') - \cdot0032\cos(3\phi + 26°\ 15') - \cdot0003\cos(4\phi + 44°)$
S.	$\quad + \cdot2196\cos(\phi + 9°\ 56') - \cdot0212\cos(2\phi + 25°\ 45') + \cdot0005\cos(3\phi + 55°\ 31') - \cdot0002\cos(4\phi + 44°)$
40° N.	$\cdot5771 - \cdot2321\cos(\phi + 11°\ 54') - \cdot0329\cos(2\phi + 19°\ 22') - \cdot0003\cos(3\phi - 62°\ 7') - \cdot0002\cos(4\phi + 44°)$
S.	$\quad + \cdot2711\cos(\phi + 10°\ 14') - \cdot0164\cos(2\phi + 28°\ 1') - \cdot0025\cos(3\phi + 29°\ 44') - \cdot0002\cos(4\phi + 44°)$
50° N.	$\cdot4842 - \cdot2782\cos(\phi + 11°\ 37') - \cdot0303\cos(2\phi + 18°\ 36') + \cdot0025\cos(3\phi + 39°\ 21') - \cdot0002\cos(4\phi + 44°)$
S.	$\quad + \cdot3108\cos(\phi + 10°\ 26') - \cdot0110\cos(2\phi + 32°\ 24') - \cdot0045\cos(3\phi + 31°\ 50') - \cdot0002\cos(4\phi + 44°)$
60° N.	$\cdot3767 - \cdot3062\cos(\phi + 11°\ 27') - \cdot0286\cos(2\phi + 17°\ 59') - \cdot0211\cos(3\phi + 32°\ 32') - \cdot0001\cos(4\phi + 44°)$
	$+ \cdot3316\cos(\phi + 10°\ 35') - \cdot0059\cos(2\phi + 42°\ 54') + \cdot0199\cos(3\phi + 33°\ 4') - \cdot0001\cos(4\phi + 44°)$

χ_2'

$\lambda =$ 0°	$\cdot3197 + \cdot0110\cos(\phi + 0°\ 20') - \cdot0137\cos(2\phi + 22°\ 14') - \cdot0007\cos(3\phi + 22°) - \cdot0001\cos(4\phi + 44°)$
10°	$\cdot3140 + \cdot0108\cos(\phi + 0°\ 30') - \cdot0143\cos(2\phi + 22°\ 13') - \cdot0007\cos(3\phi + 22°) - \cdot0006\cos(4\phi + 44°)$
20°	$\cdot2952 + \cdot0102\cos(\phi + 0°\ 40') - \cdot0182\cos(2\phi + 22°\ 10') - \cdot0009\cos(3\phi + 22°) - \cdot0002\cos(4\phi + 44°)$
30°	$\cdot2648 + \cdot0091\cos(\phi + 0°\ 47') - \cdot0242\cos(2\phi + 22°\ 7') - \cdot0012\cos(3\phi + 22°) - \cdot0003\cos(4\phi + 44°)$
40°	$\cdot2227 + \cdot0080\cos(\phi + 1°\ 31') - \cdot0339\cos(2\phi + 22°\ 4') - \cdot0018\cos(3\phi + 22°) - \cdot0004\cos(4\phi + 44°)$
50°	$\cdot1689 + \cdot0064\cos(\phi + 2°\ 34') - \cdot0455\cos(2\phi + 22°\ 2') - \cdot0023\cos(3\phi + 22°) - \cdot0002\cos(4\phi + 44°)$
60°	$\cdot1037 + \cdot0045\cos(\phi + 5°\ \ ) - \cdot0616\cos(2\phi + 22°\ 1') - \cdot0032\cos(3\phi + 22°) + \cdot0017\cos(4\phi + 44°)$

χ_3'

$\lambda = 10°$ $\cdot 0229 \cos(\phi + 11°) + \cdot 0008 \cos(2\phi + 11°) - \cdot 0001 \cos(3\phi + 33°)$

$20°$ $\cdot 0441 \cos(\phi + 11°) + \cdot 0015 \cos(2\phi + 11°) - \cdot 0004 \cos(3\phi + 33°)$

$30°$ $\cdot 0618 \cos(\phi + 11°) + \cdot 0020 \cos(2\phi + 11°) - \cdot 0016 \cos(3\phi + 33°)$

$40°$ $\cdot 0733 \cos(\phi + 11°) + \cdot 0024 \cos(2\phi + 11°) - \cdot 0042 \cos(3\phi + 33°)$

$50°$ $\cdot 0737 \cos(\phi + 11°) + \cdot 0025 \cos(2\phi + 11°) - \cdot 0097 \cos(3\phi + 33°)$

$60°$ $\cdot 0531 \cos(\phi + 11°) + \cdot 0018 \cos(2\phi + 11°) - \cdot 0204 \cos(3\phi + 33°)$

χ_4'

$\lambda = 0°$ $-\cdot 0639 - \cdot 0022 \cos(\phi + 0° 20') + \cdot 0028 \cos(2\phi + 22° 12') + \cdot 0001 \cos(3\phi + 22°) - \cdot 0000 \cos(4\phi + 44°)$

$-\cdot 0618 - \cdot 0021 \cos(\phi + 0° 38') + \cdot 0039 \cos(2\phi + 22°) + \cdot 0001 \cos(3\phi + 22°) - \cdot 0000 \cos(4\phi + 44°)$

$-\cdot 0550 - \cdot 0019 \cos(\phi + 1° 40') + \cdot 0078 \cos(2\phi + 22°) + \cdot 0004 \cos(3\phi + 22°) - \cdot 0001 \cos(4\phi + 44°)$

$-\cdot 0439 - \cdot 0017 \cos(\phi + 3° 16') + \cdot 0139 \cos(2\phi + 22°) + \cdot 0007 \cos(3\phi + 22°) - \cdot 0003 \cos(4\phi + 44°)$

$-\cdot 0289 - \cdot 0013 \cos(\phi + 5° 54') + \cdot 0217 \cos(2\phi + 22°) + \cdot 0011 \cos(3\phi + 22°) - \cdot 0005 \cos($ $)$

$-\cdot 0115 - \cdot 0009 \cos(\phi + 12° 16') + \cdot 0290 \cos(2\phi + 22°) + \cdot 0015 \cos(3\phi + 22°) - \cdot 0025 \cos($ $)$

$+\cdot 0025 - \cdot 0004 \cos(\phi + 26° 40') + \cdot 0271 \cos(2\phi + 22°) + \cdot 0014 \cos(3\phi + 22°) - \cdot 0087 \cos($ $)$

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CAMBRIDGE: PRINTED BY J. AND C. F. CLAY, AT THE UNIVERSITY PRESS.

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